UNIVERSITY OF WATERLOO
FINAL EXAM
FALL TERM 2009

Student Name (Print Legibly) ______________________________

(FAMILY NAME) (GIVEN NAME)

Signature ______________________________

Student ID Number ______________________________

COURSE NUMBER	MATH 127
COURSE TITLE	Calculus 1 for the Sciences
COURSE SECTION(s)	001 002 003 004 005 006 007 008 009
DATE OF EXAM	Thursday, December 17th, 2009
TIME PERIOD	16:00 - 18:30
DURATION OF EXAM	2.5 hours
NUMBER OF EXAM PAGES (Including this sheet)	10
INSTRUCTORS	**(please indicate your section)**

☐ 001 Wentang Kuo (9:30)
☐ 002 Bernard Anderson (10:30)
☐ 003 Shengda Hu (1:30)
☐ 004 Paula Smith (12:30)
☐ 005 Maria Wesslen (8:30)
☐ 006 Timothy Rees (1:30)
☐ 007 Andrew Beltaos (11:30)
☐ 008 Joe West (10:30)
☐ 009 Maria Wesslen (10:30)

EXAM TYPE	Closed Book
ADDITIONAL MATERIALS ALLOWED	NONE **(NO CALCULATORS)**

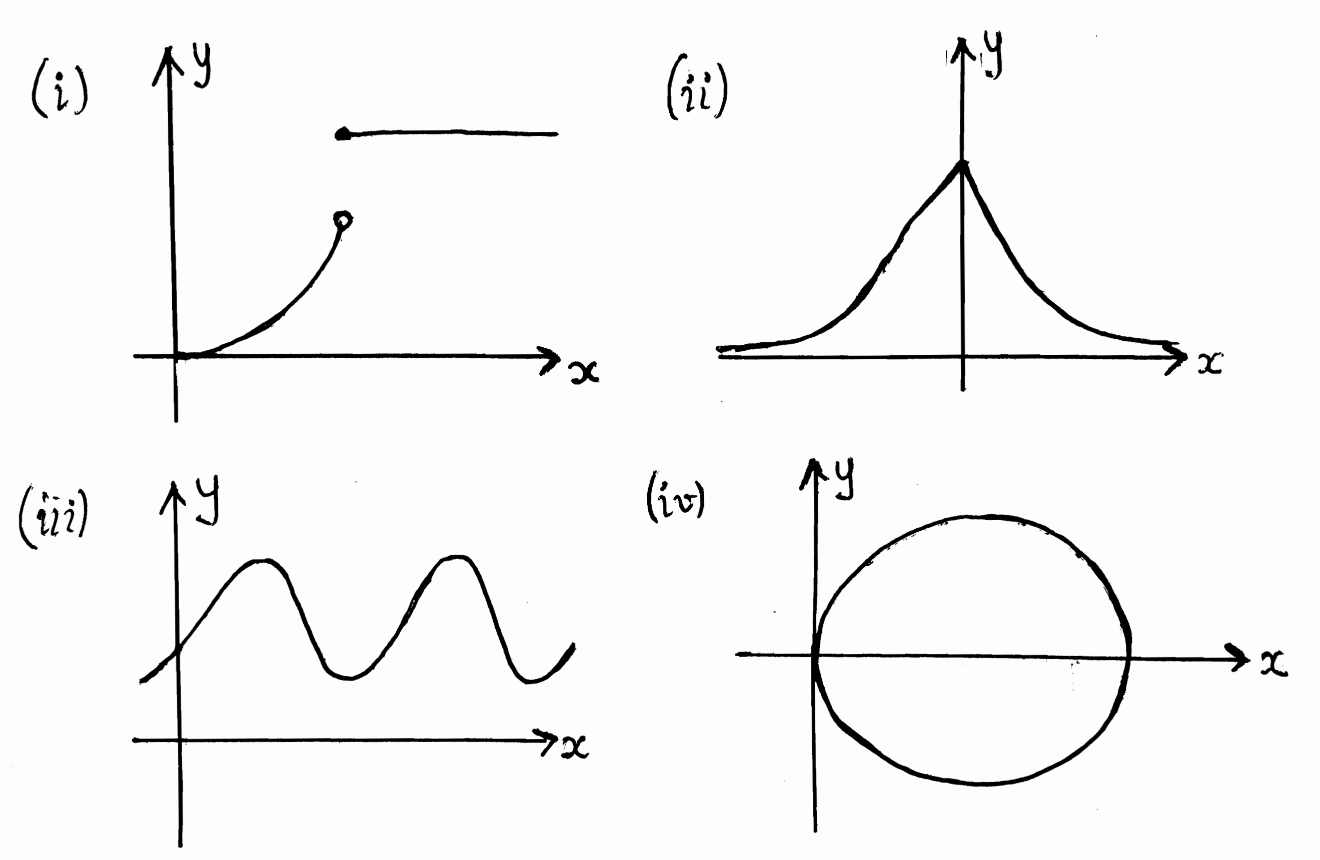
Notes:

1. Fill in your name, ID number, section, and sign the paper. *Don't write formulas on this page.*
2. Answer all questions in the space provided. Ask for extra sheets if required.
3. Check that there are 10 sheets.
4. **Your grade will be influenced by how clearly you express your ideas, and how well you organize your solutions.**

Marking Scheme:

Question	Mark	Out of
Cover sheet		1
1		17
2		12
3		10
4		12
5		18
6		14
7		12
8		4
Total		100

1. Various short questions.
   (a)-(c) Consider the following four graphs:



[2] (a) Which are the graph of a function? (i) (ii) (iii) (iv) ← (Just circle all that apply.)

[2] (b) Which functions are continuous? (i) (ii) (iii) (iv) ← (Just circle all that apply.)

[2] (c) Which functions are differentiable (on their domains)? (i) (ii) (iii) (iv) ← (circle...)

[3] (d) Write down the definition of the derivative of $f(x)$ at $x = a$. *Hint:* it involves a limit.

[3] (e) Find the average value of $f(x) = x^2$ over the interval $[0, 3]$.

[3] (f) Find $\frac{dy}{dx}$ if $y = \arctan(2x) + e^y$.

[2] (g) Evaluate $\arcsin(\sin(\frac{2\pi}{3}))$.

2. Evaluate the following limits.

[3] (a) $\lim_{x\to\infty} \frac{x^2+4}{x^2+x+2}$

[3] (b) $\lim_{x\to 0} \frac{\tan x}{x}$

[3] (c) $\lim_{x\to 0^+} x \ln x$

[3] (d) $\lim_{x\to\pi} (\sin x)^x$

3. The rate of decrease of the amount of a certain medicine in a patient's bloodstream is proportional to the amount of medicine present in the bloodstream.

[2] (a) If $m(t) = m_0 e^{-kt}$ is the amount of medicine present at time t where m_0 and k are positive constants, show that $\frac{dm}{dt} = -km$. (This is essentially what the first sentence is saying.)

[8] (b) Suppose that a patient gets their first dose of 50 mg on Thursday at 5:00pm. At 7:00pm, it is observed that 30 mg remains. How much drug will remain in the bloodstream when the TV show 'The Office' airs at 9:00pm?

4. Consider the function $f(x) = x^2 e^{-x^3}$.

[10] (a) Give a detailed sketch of this curve, but **don't do** an analysis of concavity.

[2] (b) How many inflection points do you think this function has? Just an answer is sufficient; you are not required to actually *find* the points.

5. Applications of the derivative.

[8] (a) Use the linear approximation to find an approximate value of $\sqrt{99}$.

[10] (b) A cylindrical can with height h and radius r is to be used to store coconut milk. It is to be made with 6π square centimetres of tin. Find the height h and radius r which maximizes the volume of the can.
Hint: The volume of a cylinder is $\pi r^2 h$ and the surface area of the side wall of a cylinder is $2\pi rh$. The can will also have a top and a bottom, of course.

6. Evaluate the following integrals.

[3] (a) $\displaystyle\int \left(\frac{1}{x} - x^2\right) \, \mathrm{dx}$

[4] (b) $\displaystyle\int_0^{\pi/2} e^{\sin x} \cos x \, \mathrm{dx}$

[4] (c) $\displaystyle\int \frac{t}{\sqrt{1+t^2}} \, \mathrm{dt}$

[3] (d) $\displaystyle\int_{-4007}^{4007} x^4 \sin^3 x \, \mathrm{dx}$

7. (Applications of integration.) Consider the region R bounded by the curves $y = x$, $y = 1/x$, and $x = 2$.

[5] (a) Sketch R in the xy-plane and find its area.

[5] (b) What is the volume of the solid generated by rotating R about the x-axis?

[2] (c) Consider the solid generated by rotating R about the y-axis. Will its volume be less or greater than the volume found in part (b)? Why? (You need not necessarily find this volume.)

[4] 8. [1] A circular disk of radius r is used in an evaporator and is rotated in a vertical plane. It is to be partially submerged in the liquid so as to maximize the exposed wetted area of the disk. Show that the center of the disk should be positioned at a height $r/\sqrt{1+\pi^2}$ above the surface of the liquid. *Hint:* The fundamental theorem of calculus may come into play here.

[1] Caution: This problem is only worth 4 marks, and is intended as a challenging problem for those finished with the rest of the exam.

ROUGH WORK and IDENTITIES

(You may tear off this sheet — it will not be marked.)

1. $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B \quad \Rightarrow \quad \sin 2\theta = 2 \sin\theta \cos\theta$

2. $\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B \quad \Rightarrow \quad \begin{cases} \cos 2\theta &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta \end{cases}$