Q1 Let $A = \begin{bmatrix} 2 & -5 \\ 3 & -2 \end{bmatrix}$.

(a) (5 Marks) Compute $3I_2 - A$

2

Sol: $3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 2 & -5 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -5 \\ 3 & -2 \end{bmatrix}$

1

$= \begin{bmatrix} 3-2 & 0-(-5) \\ 0-3 & 3-(-2) \end{bmatrix} = \begin{bmatrix} 1 & 5 \\ -3 & 5 \end{bmatrix}$ 2.

(b) (5 Marks) Compute $(3I_2)A$

Sol. $(3I_2)\cdot A = 3A = 3\begin{bmatrix} 2 & -5 \\ 3 & -2 \end{bmatrix} = \begin{bmatrix} 2\times 3 & (-5)\times 3 \\ 3\times 3 & (-2)\times 3 \end{bmatrix}$

$= \begin{bmatrix} 6 & -15 \\ 9 & -6 \end{bmatrix}$

Q2 Find the inverses of the following matrices.

(a) (5 Marks) $A = \begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$

M2. $\left[\begin{array}{cc|cc} 8 & 6 & 1 & 0 \\ 5 & 4 & 0 & 1 \end{array}\right]$

$\det A = 8\times 4 - 5\times 6 = 2$ 2

$A^{-1} = \frac{1}{2}\begin{bmatrix} 4 & -6 \\ -5 & 8 \end{bmatrix} \left(= \underbrace{\begin{bmatrix} 2 & -3 \\ -\frac{5}{2} & 4 \end{bmatrix}}_{3} \right)$

(b) (6 Marks) $A = \begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ -3 & 1 & 4 & 0 & 1 & 0 \\ 2 & -3 & 4 & 0 & 0 & 1 \end{array}\right] \xrightarrow[R3+(-2)R1]{R2+3R1} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & -3 & 8 & -2 & 0 & 1 \end{array}\right]$$ 1

1

$$\xrightarrow{R3+3R2} \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 1 & 0 & 0 \\ 0 & 1 & -2 & 3 & 1 & 0 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

1

$$\xrightarrow[R2+R3]{R1+R3} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 2 & 7 & 3 & 1 \end{array}\right]$$

1

$$\xrightarrow{R3\times(1/2)} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 8 & 3 & 1 \\ 0 & 1 & 0 & 10 & 4 & 1 \\ 0 & 0 & 1 & \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{array}\right]$$

A^{-1}

2

$A^{-1} = \begin{bmatrix} 8 & 3 & 1 \\ 10 & 4 & 1 \\ \frac{7}{2} & \frac{3}{2} & \frac{1}{2} \end{bmatrix}$

Q3 Given a matrix A and an echelon form of A. $A = \begin{bmatrix} 4 & 5 & 9 & -2 \\ 6 & 5 & 1 & 12 \\ 3 & 4 & 8 & -3 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 6 & -5 \\ 0 & 1 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{array}{cccc} \uparrow & \uparrow & & \\ P & P & & \end{array}$

(a) (4 Marks) Find the basis for Col A.

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ 6 \\ 3 \end{bmatrix}, \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix} \right\}$$

(2) (2)

(b) (5 Marks) Find the basis for Nul A.

solve $A\vec{x} = \vec{0}$ (0)

$$\left[\begin{array}{cccc|c} 4 & 5 & 9 & -2 & 0 \\ 6 & 5 & 1 & 12 & 0 \\ 3 & 4 & 8 & -3 & 0 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & 6 & -5 & 0 \\ 0 & 1 & 5 & -6 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$ (1)

$$\begin{cases} x_1 = 4x_3 - 7x_4 \\ x_2 = -5x_3 + 6x_4 \\ x_3 = \text{free} \\ x_4 = \text{free} \end{cases} \qquad \vec{x} = x_3 \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$ (2)

$$\mathcal{B} = \left\{ \begin{bmatrix} 4 \\ -5 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ 6 \\ 0 \\ 1 \end{bmatrix} \right\}$$ (2)

(c) (2 Marks) What's the rank A?

rank $A = 2$

(d) (2 Marks) What's the dimension of Nul A?

$\dim(\text{Nul } A) = 2$

Q5 (6 Marks) Consider the basis $B = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} \right\}$ for $\mathcal{R}^2$. If $[\vec{x}]_B = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, what is $\vec{x}$?

$$\bar{x} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

(3) (3)