There are 5 long answer questions. **You must show all your work.**
(non-programmable, non-graphing calculators)

Q1 Find all solutions of the following linear system.

$$x_1 + 3x_2 + 4x_3 = 7$$
$$3x_1 + 9x_2 + 7x_3 = 6$$

(a) (4 Marks) What's the augmented matrix of this linear system?

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right]$$

(b) (6 Marks) Row reduce the augmented matrix in (a) to reduced echelon form.

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 3 & 9 & 7 & 6 \end{array}\right] \xrightarrow{R(2)+(-3)R1} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & -5 & -15 \end{array}\right] \quad 2$$

$$\xrightarrow{R2*(-\frac{1}{5})} \left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 0 & 1 & 3 \end{array}\right] 2 \xrightarrow{R1+(-4)R2} \left[\begin{array}{ccc|c} 1 & 3 & 0 & -5 \\ 0 & 0 & 1 & 3 \end{array}\right] 2$$

Q2 $\vec{u} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{v} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

(a) (4 Marks) Compute $\vec{u} + \vec{v}$.

$$\vec{u}+\vec{v} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} + \begin{bmatrix} -3 \\ -1 \end{bmatrix} \underset{2}{=} \begin{bmatrix} -1-3 \\ 2-1 \end{bmatrix} = \underset{2}{\begin{bmatrix} -4 \\ 1 \end{bmatrix}}$$

(b) (6 Marks) Compute $-2\vec{v}$, then find $\vec{u} - 2\vec{v}$.

$$-2\vec{v} \underset{2}{=} \begin{bmatrix} (-2)\times(-3) \\ (-2)\times(-1) \end{bmatrix} \underset{1}{=} \begin{bmatrix} 6 \\ 2 \end{bmatrix}, \quad \vec{u} - 2\vec{v} = \begin{bmatrix} -1+6 \\ 2+2 \end{bmatrix} \underset{2}{} = \begin{bmatrix} 5 \\ 4 \end{bmatrix} \ 1$$

Q3 (5 Marks) Let $\vec{a}_1 = \begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix}$, $\vec{a}_2 = \begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$. For what value of h is $\vec{b}$ in the span$\{\vec{a}_1, \vec{a}_2\}$.

If $\vec{b}$ is in span $\{\vec{a}_1, \vec{a}_2\}$, then $\vec{b} = x_1\vec{a}_1 + x_2\vec{a}_2$

i.e. $x_1\begin{bmatrix} 1 \\ 3 \\ -1 \end{bmatrix} + x_2\begin{bmatrix} -5 \\ -8 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -5 \\ h \end{bmatrix}$

Therefore, the augmented matrix is.

$$\left[\begin{array}{cc|c} 1 & -5 & 3 \\ 3 & -8 & -5 \\ -1 & 2 & h \end{array}\right] \xrightarrow[R_2+(-3)R_1]{R_3+R_1} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 7 & -14 \\ 0 & -3 & h+3 \end{array}\right] \xrightarrow{R_2\times(\frac{1}{7})} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & -3 & h+3 \end{array}\right]$$

$$\xrightarrow{R_3+3R_2} \left[\begin{array}{cc|c} 1 & -5 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & h-3 \end{array}\right] \Rightarrow \quad h-3=0 \iff h=3$$

Q4 (5 Marks) Compute $A\vec{x}$, where $A = \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$.

$$A\vec{x} = \begin{bmatrix} 1 & 2 \\ -3 & 1 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times(-2) + 2\times 3 \\ (-3)\times(-2) + 1\times 3 \\ 1\times(-2) + 6\times 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 9 \\ 16 \end{bmatrix}$$

Q5 Given $A = \begin{bmatrix} 1 & 2 & -3 \\ 2 & 1 & -3 \\ -1 & 1 & 0 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 5 \\ 13 \\ -8 \end{bmatrix}$

(a) (6 Marks) Find all solutions in parametric vector form of the homogeneous system $A\vec{x} = \vec{0}$.

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 2 & 1 & -3 & 0 \\ -1 & 1 & 0 & 0 \end{array}\right] \xrightarrow[R_3 + R_1]{R_2 + (-2)R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \times (-\frac{1}{3})} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & -3 & 0 \end{array}\right] \xrightarrow{R_3 + (-3)R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$ 3

$$\xrightarrow{R_1 + (-2)R_2} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \Rightarrow \begin{cases} x_1 - x_3 = 0 \\ x_2 - x_3 = 0 \\ 0 = 0 \end{cases} \text{ i.e. } \begin{cases} x_1 = x_3 \\ x_2 = x_3 \\ x_3 = \text{free} \end{cases}$$ 2

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_3 \\ x_3 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ 1

(b) (4 Marks) If $\begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix}$ is one solution of the non-homogenous system $A\vec{x} = \vec{b}$, write out all solutions in parametric vector form of $A\vec{x} = \vec{b}$ (without solving the augmented matrix).

All solutions for $A\vec{x} = \vec{b}$ is

$$\vec{x} = \begin{bmatrix} 7 \\ -1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ (by Th6 on page 46)