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CVG 3141 – MECHANICS OF MATERIALS II

Mid-Term Examination (Closed book) — **November 4th, 2010**
Dr. B. Martín-Pérez — Time: **1 hour & 20 minutes**

1. A structural steel (E = 200 GPa and σ_y = 220 MPa) bar has a diameter of 50 mm, is 5 m long, and supports an axial compressive load P. End A is fixed. The support at B permits movement in the x and z directions but no rotation about z. Determine the maximum load P that can be applied to the bar if a safety factor n_{cr} = 2 against failure by buckling is specified. (40% of total mark)

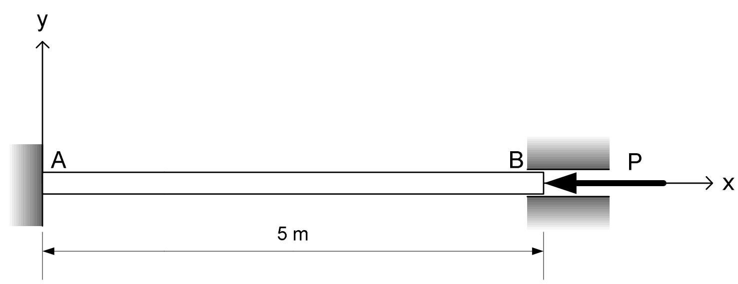
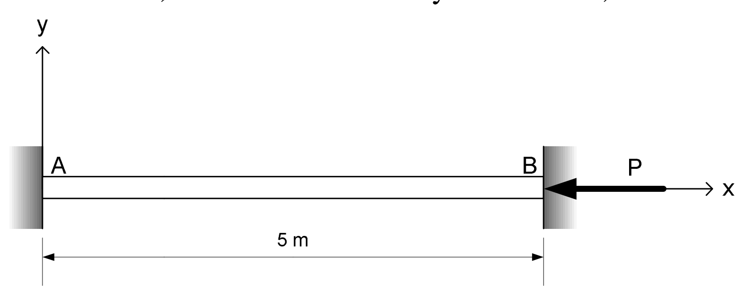


Figure 1

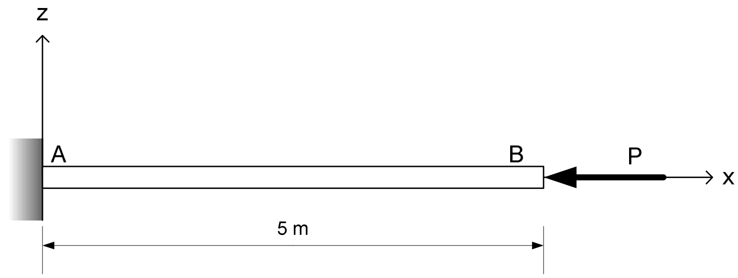
Based on the problem statement, the boundary conditions of the bar in the x-y plane are different from those in the the x-z plane. Let's consider the x-y plane first. Since no rotation about z is permitted at end B, this end is essentially fixed. Thus, K = 0.5.



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 EI_z}{0.25L^2} = \frac{\pi^2 \times 200\times10^3 \times 3.06796\times10^5}{0.25\times5000^2} = 96.9 \text{ kN}$$

$$I_z = \frac{\pi d^4}{64} = \frac{\pi\times50^4}{64} = 3.06796\times10^5 \text{ mm}^4$$

In the *x*-*z* plane, end *B* is free. Thus, $K = 2$.



$$P_{cr} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 EI_y}{4L^2} = \frac{\pi^2 \times 200\times10^3 \times 3.06796\times10^5}{4\times5000^2} = 6.05 \text{ kN (governs)}$$

$$I_y = I_z = \frac{\pi d^4}{64} = \frac{\pi \times 50^4}{64} = 3.06796\times10^5 \text{ mm}^4$$

Buckling will occur first in the *x*-*z* plane. Therefore, the maximum load that can be applied to the bar is:

$$P_{\max} = \frac{P_{cr}}{n_{cr}} = \frac{6.05}{2} = \underline{3.03 \text{ kN}}$$

Check for stress: $\sigma_{cr} = \frac{P_{cr}}{A} = \frac{6.05\times10^3}{\pi\times25^2} = 3.08 \text{ MPa} < \sigma_y$

2. For the beam with cross section shown in Figure 2, determine:
   a. The orientation of the neutral axis (show its location on a sketch of the cross section);
   b. The maximum tensile and compressive flexural stresses in the cross section; and,
   c. The shape factor *k* assuming bending takes place with respect to the *z*-axis.

(60% of total mark)

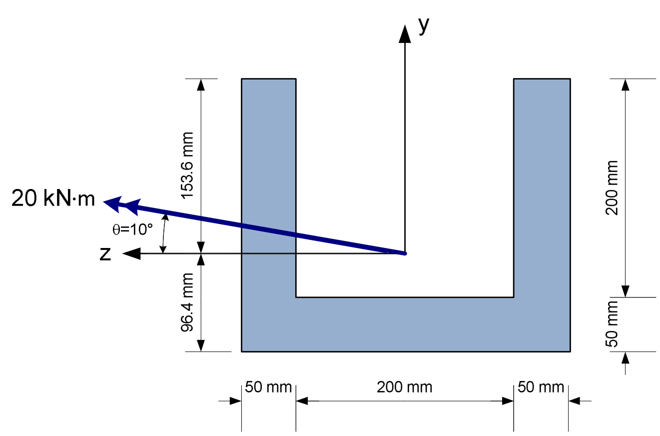


Figure 2

a. Orientation of the neutral axis

$$\bar{y} = \frac{\sum \bar{y}_i A_i}{\sum A_i} = \frac{(125\times300\times250)-(150\times200\times200)}{(300\times250)-(200\times200)} = 96.4 \text{ mm from the bottom}$$

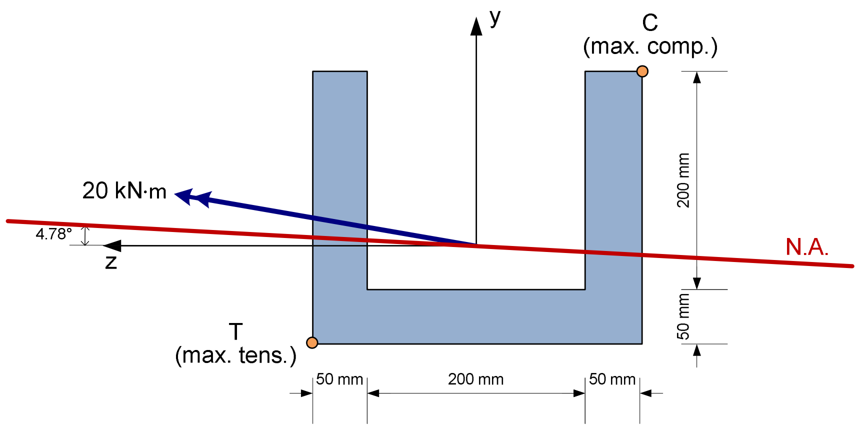
$\theta = 10°$

Since the y-axis is an axis of symmetry, it is also a principal axis. Therefore, the z-axis is a principal axis as well, and $I_{yz} = 0$.

$$I_z = \frac{300\times250^3}{12} + \left(300\times250\times28.6^2\right) - \frac{200\times200^3}{12} - \left(200\times200\times53.6^2\right) = 203.7203\times10^6 \text{ mm}^4$$

$$I_y = \frac{250\times300^3}{12} - \frac{200\times200^3}{12} = 429.1667\times10^6 \text{ mm}^4$$

$$\tan\alpha = \frac{I_z}{I_y}\tan\theta = \frac{203.7203}{429.1667}\tan10^\circ = 0.0837 \quad \Rightarrow \quad \alpha = \tan^{-1}(0.0837) = \underline{4.78°}$$ (clockwise from z-axis)



b. Maximum tensile and compressive flexural stresses in the cross section

$$M_z = 20\times\cos10^\circ = 19.7 \text{ kN·m}$$

$$M_y = 20\times\sin10^\circ = 3.47 \text{ kN·m}$$

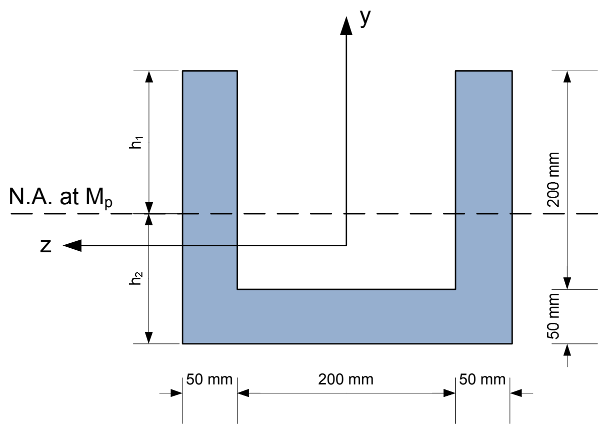
The maximum compressive stress occurs at C (see figure above), whereas the maximum tensile stress occurs at T.

$$\sigma_x^C = -\frac{M_z y^C}{I_z} + \frac{M_y z^C}{I_y} = -\frac{19.7\times10^6\times153.6}{203.7203\times10^6} + \frac{3.47\times10^6\times(-150)}{429.1667\times10^6} = \underline{-16.1 \text{ MPa}}$$

$$\sigma_x^T = -\frac{M_z y^C}{I_z} + \frac{M_y z^C}{I_y} = -\frac{19.7\times10^6 \times(-96.4)}{203.7203\times10^6} + \frac{3.47\times10^6 \times150}{429.1667\times10^6} = \underline{10.5 \text{ MPa}}$$

c. <u>Shape factor *k* assuming bending takes place with respect to the *z*-axis</u>

When the section reaches M_p, the neutral axis divides the cross section into two equal areas.



$$S = \frac{I_z}{c} = \frac{203.7203\times10^6}{153.6} = 1.3263\times10^6 \text{ mm}^3$$

$$A = (300\times250) - (200\times200) = 35{,}000 \text{ mm}^2$$

$$2h_1 \times 50 = \frac{A}{2} = 17{,}500 \quad \Rightarrow \quad h_1 = 175 \text{ mm},\ h_2 = 250 - 175 = 75 \text{ mm}$$

$$\bar{y}_1 = \frac{175}{2} = 87.5 \text{ mm},\ \bar{y}_2 = \frac{(37.5\times300\times75) - (12.5\times200\times25)}{(300\times75) - (200\times25)} = 44.6 \text{ mm}$$

$$Z = \frac{A(\bar{y}_1 + \bar{y}_2)}{2} = \frac{35{,}000\times(87.5 + 44.6)}{2} = 2.3117\times10^6 \text{ mm}^3$$

$$k = \frac{M_p}{M_y} = \frac{Z}{S} = \frac{2.3117}{1.3263} = \underline{1.743}$$