Solution-Question 1

(a)

Free-body diagram of the beam is shown in Figure S1.

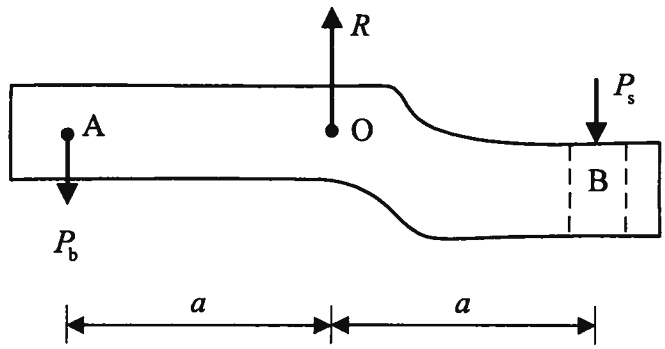


Figure S1: FBD of the beam.

Note: All the forces are vertical and there are no moments (because of the two-force members and smooth joints).

P_b = tension in the bronze rod

P_s = tension in the steel rod

Equilibrium of the Beam:

$$\circlearrowright \sum M_O = 0: \quad P_b \times a - P_s \times a = 0 \quad \rightarrow \quad P_b = P_s \qquad \text{(i)}$$

A second equation (e.g., force balance) will introduce a new unknown R (reaction at O) and will not lead to solution of the axial forces in the two rods $\rightarrow$ statically indeterminate system.

A compatibility condition is needed to solve the problem.

Note: Tightening the nut will pull the steel rod (upward) generating its tension P_s. The equal downward reaction on the nut will be balanced by the upward reaction from the beam on the nut. Hence, the downward force on the beam by the nut is P_s, as shown in Figure S1.

Tension in the bronze rod will extend it through δ_b, where,

$$\delta_b = \frac{P_b L}{E_b A_b} \qquad \text{(ii)}$$

As the end A of the beam moves up by δ_b, the end B will move down by δ_b (because OA = OB).

Tension in the steel rod will extend it through δ_s where,

$$\delta_s = \frac{P_s L}{E_s A_s} \qquad \text{(iii)}$$

The extensions in the two rods would generate a gap of $\delta_b + \delta_s$ between the nut and the top surface of the beam. This gap is removed by the two full turns of the nut. Hence, the compatibility condition is,

$$\delta_b + \delta_s = np \qquad \text{(iv)}$$

where

n = number of turns of the nut

p = pitch of the thread.

Substitute (i)-(iii) into (iv):

$$P_b L\left(\frac{1}{E_b A_b} + \frac{1}{E_s A_s}\right) = np$$

Substitute numerical values:

$$P_b(\mathrm{N}) \times 1.0\mathrm{m} \times \left(\frac{1}{100\times10^9(\mathrm{Pa})\times20\times10^{-6}(\mathrm{m}^2)} + \frac{1}{200\times10^9(\mathrm{Pa})\times10\times10^{-6}(\mathrm{m}^2)}\right) = 2\times0.5\times10^{-3}(\mathrm{m})$$

$$\rightarrow \frac{P_b}{10^6}\left(\frac{1}{2}+\frac{1}{2}\right) = 1\times10^{-3} \quad \rightarrow \quad P_b = 1\times10^3\,\mathrm{N} = P_s$$

Normal (tensile) stress in the bronze rod,

$$\sigma_b = \frac{P_b}{A_b} = \frac{1\times10^3(\mathrm{N})}{20\times10^{-6}(\mathrm{m}^2)} = 50\times10^6 \text{ Pa} = 50 \text{ MPa}$$

Normal (tensile) stress in the steel rod,

$$\sigma_s = \frac{P_s}{A_s} = \frac{1\times10^3 (\text{N})}{10\times10^{-6} (\text{m}^2)} = 100\times10^6 \text{ Pa} = 100 \text{ MPa}$$

(b)

After the temperature rise of $\Delta T = 20^\circ\text{C}$, the new quantities are denoted by $()^*$.

We have:

$$P_b^* = P_s^* \qquad \text{(i)}^*$$

$$\delta_b^* = (\frac{P_b^*}{A_b E_b} + \alpha_b \cdot \Delta T) L \qquad \text{(ii)}^*$$

$$\delta_s^* = (\frac{P_s^*}{A_s E_s} + \alpha_s \cdot \Delta T) L \qquad \text{(iii)}^*$$

$$\delta_b^* + \delta_s^* = np \qquad \text{(iv)}^*$$

Substitute (i)*- (iii)* and (iv)*:

$$P_b^* L (\frac{1}{E_b A_b} + \frac{1}{E_s A_s}) = np - (\alpha_b + \alpha_s) \cdot \Delta T \cdot L$$

Substitute numerical values (and make use of the previous computations):

$$\frac{P_b^*}{10^6} = 1\times10^{-3} - (17\times10^{-6} + 12\times10^{-6}) \times 20 \times 10 = 0.42\times10^{-3}$$

$$\rightarrow \quad P_b^* = 0.42\times10^3 \text{ N} = P_s^*$$

$$\sigma_b^* = \frac{P_b^*}{A_b} = \frac{0.42\times10^3 (\text{N})}{20\times10^{-6} (\text{m}^2)} = 21\times10^6 \text{ Pa} = 21 \text{ MPa}$$

$$\sigma_s^* = \frac{P_s^*}{A_s} = \frac{0.42\times10^3 (\text{N})}{10\times10^{-6} (\text{m}^2)} = 42\times10^6 \text{ Pa} = 42 \text{ MPa}$$

Solution
Question 2

a) Note: The students may start their solution from either of the two shafts. So, I call them Approach 1 and Approach 2.

☆ Approach 1: Starting from shaft AB

$$\tau_{allow,AB} = \left.\frac{T\,c}{J}\right)_{AB} \quad \text{or} \quad T_{allow,AB} = \left.\frac{\tau J}{c}\right)_{AB}$$

$$T_{Allow,AB} = \frac{8\times10^3\ [lb/in^2]\cdot\frac{\pi}{2}(0.25)^4\ [in^4]}{0.25\ [in]}$$

$$T_{Allow,AB} = 196.35\ [lb\cdot in] \Longrightarrow T_{Allow,AB} = 16.36\ [lb\cdot ft]$$

#① Max Torque as far as the shaft AB only is concerned.

With the gear ratio, now we find the effect of $T_{Allow,AB}$ on shaft CD:

No sign is needed.

$$T_{CD} = T_{Allow,AB}\cdot\frac{R_C}{R_B} = 16.36\ [lb\,ft]\times\frac{5\ [in]}{3\ [in]} \qquad T_{CD} = 27.27\ [lb\cdot ft]$$

#②

Since both shafts are identical, without calculation we see that $T_{CD} = 27.27$ [lb-ft] is too high for shaft CD.

That is: $T_{Allow,CD} = 16.36$ [lb-ft]

So, "CD" is the controlling shaft. #③

We now work backward to find the $T_{Max,AB}$.

$$T_{Max,AB} = T_{Allow,CD}\cdot\frac{R_B}{R_C}$$

$$T_{Max,AB} = 16.36\ [lb\text{-}ft]\ \frac{3\ [in]}{5\ [in]}$$

For the assembly:

$$T_{max} = 9.82\ [lb\ ft]$$ ← #④

at Point ^

Final Answer

(i.e. between equations ① & ④ we have selected ④.)

★ Approach 2: starting from shaft CD

$$\tau_{allow, CD} = \frac{T \cdot C}{J}\Big)_{CD} \quad \text{or} \quad T_{allow, CD} = \frac{\tau J}{C}\Big)_{CD}$$

$$T_{allow, CD} = \frac{8 \times 10^3 \, [lb/in^2] \cdot \frac{\pi}{2} (0.25)^4 \, [in^4]}{0.25 \, [in]}$$

$T_{allow, CD} = 196.35$ [lb-in] ⟹ $T_{Allow, CD} = 16.36$ [lb-ft] #①

with gear ratio, we now find the effect of $T_{Allow\ CD}$ on shaft AB:

$$T_{AB} = T_{Allow, CD} \cdot \frac{R_B}{R_C} = 16.35 \, [lb\text{-}ft] \cdot \frac{3 \, [in]}{5 \, [in]}$$

$T_{AB} = 9.82$ [lb-ft] #②

Since both shafts are identical, without calculation we know that: $T_{Allow, AB}$ is also 16.36 [lb-ft]. #③

Therefore $T_{AB} = 9.82$ [lb-ft] is ok.

So, between equations ② and ③ we select ②.

Note: if we select ③

Then

$$T_{CD} = 16.36 \times \frac{5}{3}$$

$T_{CD} = 27.27$ [lb-ft]

too high. (> 16.36)

Thus, for the assembly:

$T_{max} = 9.82$ [lb-ft] #④

At point A

Final answer.

b)

T_{max} @ A : $T = 9.82$ [lb-ft]

$$\phi_{AB} = \frac{TL}{JG}\Big)_{AB} \qquad \phi_{CD} = \frac{TL}{JG}\Big)_{CD}$$

$$\phi_{AB} = \frac{9.82\,[lb\text{-}ft]\cdot[12\,\tfrac{in}{ft}] \times 24\,[in]}{\frac{\pi}{2}(0.25)^4 \cdot 12\times10^6\,[lb/in^2]} \qquad \phi_{AB} = 0.04\,[Rad]$$

$$\phi_{CD} = \frac{16.36\,[lb\text{-}ft]\cdot[12\,in/ft] \times 36\,[in]}{\frac{\pi}{2}(0.25)^4 \cdot 12\times10^6\,[lb/in^2]} \qquad \phi_{CD} = 0.06\,[Rad]$$

$$\phi_{CD} = \phi_C = 0.06\,[Rad]$$

$$\phi_B = \phi_C \times \frac{R_C}{R_B} = 0.06 \times \frac{5}{3} = 0.10\,[Rad]$$

Total: $\phi_A = \phi_B + \phi_{AB} = 0.10 + 0.04$

$$\boxed{\phi_A = 0.14\,[Rad]}$$

Final answer ←

C)

$K = \dfrac{T}{\phi}$] For any available data.

$$K = \frac{T_{max,A}}{\phi_A} = \frac{9.82\,[lb\text{-}ft]}{0.14\,[Rad]}$$

$$\boxed{K = 70\,[lbft/Rad]}$$

Final answer. ←

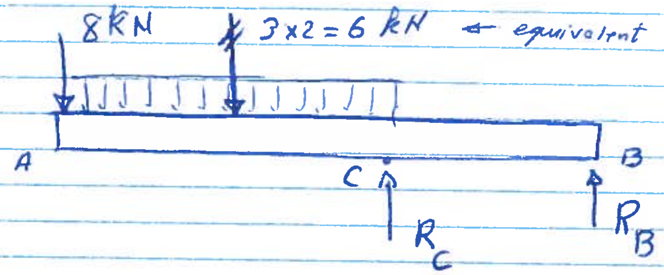
Note: Due to rounding off answers may vary slightly.

End of Question 2

Solution

Question 3

a) First we find the reaction. FBD:



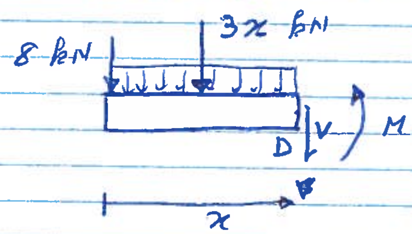
$\circlearrowleft + \sum M_C = 0$ $\quad R_B \times 1.5 + 6 \times 1 + 8 \times 2 = 0$ $\quad R_B = -14.67$ kN

or $\boxed{R_B = 14.67 \text{ kN}} \downarrow$ downward

$+\uparrow \sum F_y = 0$ $\quad R_C - 6 - 8 - 14.67 = 0$

$\boxed{R_C = 28.67 \text{ kN}} \uparrow$ upward

Cut 1: Between A and C:



$+\uparrow \sum F_y = 0$

$-8 - 3x - V = 0$

$\boxed{V = -8 - 3x}$ Between A & C

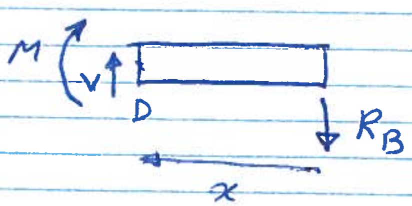
$\circlearrowleft + \sum M_O = 0$

$8 \times x + 3x \times \frac{x}{2} + M = 0$

$\boxed{M = -\frac{3}{2}x^2 - 8x}$ Between A & C

Cut #2 Between C and B,

& we consider the R.H.S. FBD:



Note: x measured from B to the left

$+\uparrow \sum F_y = 0$

$V - R_B = 0$

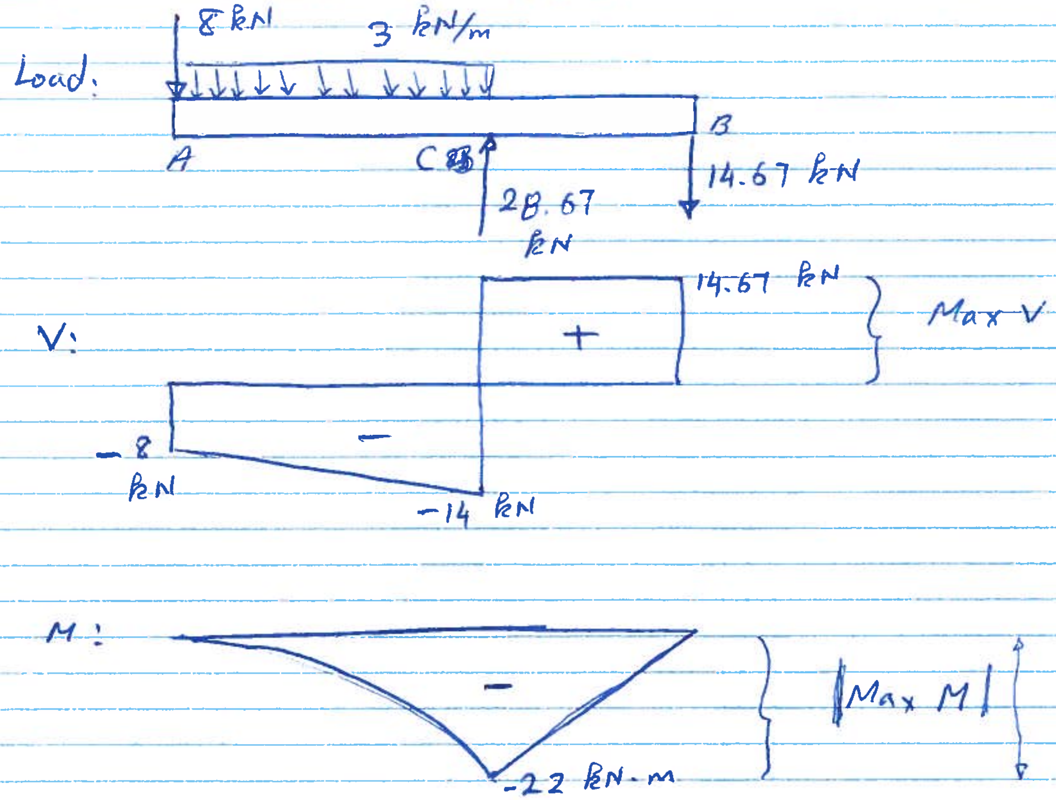
$\boxed{V = 14.67}$ Between C & B

$\circlearrowleft + \sum M_D = 0$

$-M - R_B x = 0$

$\boxed{M = -14.67x}$ Between C & B

Diagrams:



b) For rectangular cross-section:

$$\tau_{max} = \frac{VQ}{I_z t} = \frac{3V}{2A}$$

$$\tau_{max} = \frac{3}{2} \cdot \frac{14.67 \times 10^3 [N]}{60[mm] \cdot 100[mm]} = 3.67 \left[\frac{N}{mm^2}\right] = 3.67 [MPa]$$

$$\boxed{\tau_{max} = 3.67 [MPa]}$$

Location is on N.A. Between C and B (all range)

Final Answer.

c) $$\sigma_{max} = \frac{M_{max} C}{I}$$

$$\sigma_{max} = \frac{22 \times 10^3 [N.m] \times \frac{1000}{1}[mm/m] \times \frac{100}{2}[mm]}{\frac{1}{12}\, 60[mm] \times 100^3 [mm^3]}$$

$$\sigma_{max} = 220 \left[\frac{N}{mm^2}\right] = 220 [MPa]$$

$$\boxed{\sigma_{max} = 220 [MPa]}$$

Location is Top layer of beam at point C

Final answer.

Solution-Question 4

(a)

Free-body diagram of the beam is shown in Figure S4.

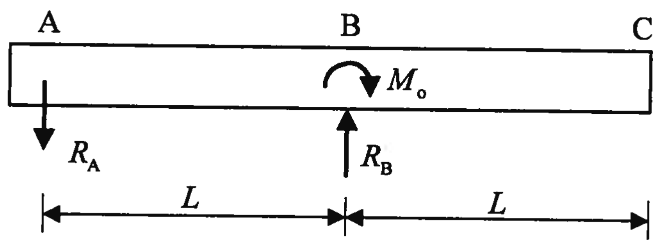


Figure S4: FBD of the beam.

Equations of Equilibrium:

$$\circlearrowright \sum M_B = 0: \quad R_A \times L - M_o = 0 \quad \rightarrow R_A = \frac{M_o}{L} \qquad \text{(i)}$$

$$\uparrow \sum F_y = 0: \; -R_A + R_B = 0 \quad \rightarrow R_B = R_A = \frac{M_o}{L} \qquad \text{(ii)}$$

(b)

We use the method of direct integration for the two segments AB and BC of the beam, separately, as given in Table S4.

Table S4: Steps of determining the beam deflection.

Quantity	AB	BC
$M = EI\dfrac{d^2v}{dx^2}$	$-R_A x$	$-R_A x + M_o + R_B \times (x-L)$
Integrate: $EI\dfrac{dv}{dx}$	$-\dfrac{R_A}{2}x^2 + A_1$	$-\dfrac{R_A}{2}x^2 + M_o \times (x-L) + \dfrac{R_B}{2}(x-L)^2 + B_1$
$\dfrac{dv}{dx}$ is the same at B from both expressions: (at $x = L$)	$\rightarrow$ $A_1 =$	B_1
Integrate: $EIv =$	$-\dfrac{R_A x^3}{6} + A_1 x + A_2$	$-\dfrac{R_A}{6}x^3 + \dfrac{M_o}{2} \times (x-L)^2 + \dfrac{R_B}{6} \times (x-L)^3 + A_1 x + B_2$
v is the same at B from both expressions:	$\rightarrow$ $A_2 =$	B_2
$v = 0$ at $x = 0$	$\rightarrow$ $A_2 = 0$ $\rightarrow$	$B_2 = 0$
$v = 0$ at $x = L$	$\rightarrow$ $-\dfrac{R_A L^3}{6} + A_1 L = 0$ $\rightarrow$	$A_1 = \dfrac{R_A L^2}{6}$

For $x = 0$ to L:

$$v = \frac{R_A x}{6EI}(L^2 - x^2)$$

$$\rightarrow \quad v = \frac{M_o x}{6EIL}(L^2 - x^2) \text{ for } x = 0 \text{ to } L \qquad \text{(ii)} \qquad \text{(From (i))}$$

For $x = L$ to $2L$:

$$v = \frac{1}{6EI}[-R_A x^3 + 3M_o(x-L)^2 + R_B(x-L)^3 + R_A L^2 x]$$

$$= \frac{1}{6EIL}[-M_o x^3 + 3M_o L(x-L)^2 + M_o(x-L)^3 + M_o L^2 x]$$

$$= \frac{M_o}{6EIL}[-x^3 + 3L(x-L)^2 + (x-L)^3 + L^2 x]$$

$$= \frac{M_o}{6EIL}[-x^3 + 3L(x^2 - 2xL + L^2) + (x^3 - 3x^2 L + 3xL^2 - L^3) + L^2 x]$$

$$\rightarrow \qquad v = \frac{M_o L}{3EI}(L - x) \qquad \text{for } x = L \text{ to } 2L \qquad \text{(iii)}$$

(c)

From (iii),

v at $x = 2L$ is,

$$v_C = \frac{M_o L}{3EI}(L - 2L) = -\frac{M_o L^2}{3EI}$$

From Table S4, $\frac{dv}{dx}$ at $x = L$ is

$$\left.\frac{dv}{dx}\right|_B = \frac{1}{EI}\left[-\frac{R_A L^2}{2} + A_1\right] = \frac{1}{EI}\left[-\frac{R_A L^2}{2} + \frac{R_A L^2}{6}\right] = -\frac{R_A L^2}{3EI} = -\frac{M_o L}{3EI}$$

$v_C = v_B + \left.\frac{dv}{dx}\right|_B \times L$ {Because, slope is constant, which can be confirmed by differentiating (iii) or by the fact that $M = 0$ from B to C}

with $v_B = 0$.

$$\rightarrow v_C = -\frac{M_o L}{3EI} \times L = -\frac{M_o L^2}{3EI}$$

This confirms the previous result for v_C.