THE UNIVERSITY OF BRITISH COLUMBIA

Department of Mechanical Engineering

Sessional Examination - December 2012

Mech 260 - Introduction to Mechanics of Materials - Sections 101& 102

Instructors: C. W. de Silva (101) and F. Sassani (102) *Time: 2.5 hours*

Closed book/notes. One 8.5”x11” fact sheet *(double-sided) prepared by student is allowed* *This question paper contains* ***3 pages***

Non-programmable non-graphing calculator allowed. *Alternatively, you may use only the* ***“arithmetic mode”*** *of a programmable calculator.* ***No communication devices are allowed.***

Clearly state all your assumptions and ***show all your steps of the derivations/computations****. Define any new variables or parameters that you use, which are not given in the problem. Draw a box around your answers and* ***clearly indicate units****, and* ***explain the results****. For each question, up to 1 bonus point will be given for orderly and neat presentation. (Bold text above, carry grading points.)*

Question 1

(Total 25 points) A mechanical structure consists of an overhead beam (assumed rigid) which is hinged (smooth) at its mid-span, and supported on a bronze rod and a steel rod (see Figure 1). The top end of the bronze rod is pin-jointed (smooth) to the left end of the beam. The bottom end of the bronze rod is hinged (smooth) to a rigid base. The bottom end of the steel rod is also hinged (smooth) to the rigid base, at the same horizontal level. The top segment of the steel rod has threads and passes loosely through a hole at the right end of the beam. The two rods are vertical, and initially there are no loads on these rods (Neglect the weight of the beam). The nut of the threaded segment of the steel is lightly tightened initially (with negligible force on the beam and the rod). The following parameter values are given:

Effective length of the bronze rod and the steel rod, $L = 1.0$ m

Area of X-section of the bronze rod, $A_b = 20$ mm^2

Area of X-section of the steel rod, $A_s = 10$ mm^2

Young’s modulus of bronze, $E_b = 100$ GPa

Young’s modulus of steel, $E_s = 200$ GPa

Coefficient of thermal expansion of bronze, $\alpha_b = 17 \times 10^{-6}$ /°C

Coefficient of thermal expansion of steel, $\alpha_s = 12 \times 10^{-6}$ /°C

Pitch of the threading on the steel rod (i.e., axial movement of the nut when rotated through a full turn), $p = 0.5$ mm.

a) From the initial unstrained conditions when the nut is snugly in contact with the beam, the steel rod is tensioned by tightening the nut through two full turns ($n = 2$). Determine the resulting axial normal stresses in the bronze rod and the steel rod. ***(15 Points)***

b) After the nut is tightened as in Part (a), the ambient temperature of the structure is increased through $\Delta T = 20^\circ \mathrm{C}$. Determine the axial normal stresses in the two rods after this temperature increase. ***(10 Points)***

Note 1: The two vertical rods are two-force members. Also, neglect the weight and any thermal deformation of the rigid beam. Angles of rotation, due to the deformation of the rods, are very small.

Note 2: You must give the details of all your steps in solving the problems. The answers must be expressed only in terms of the given parameters. If you use new parameters, they must be clearly defined and then eliminated at the end.

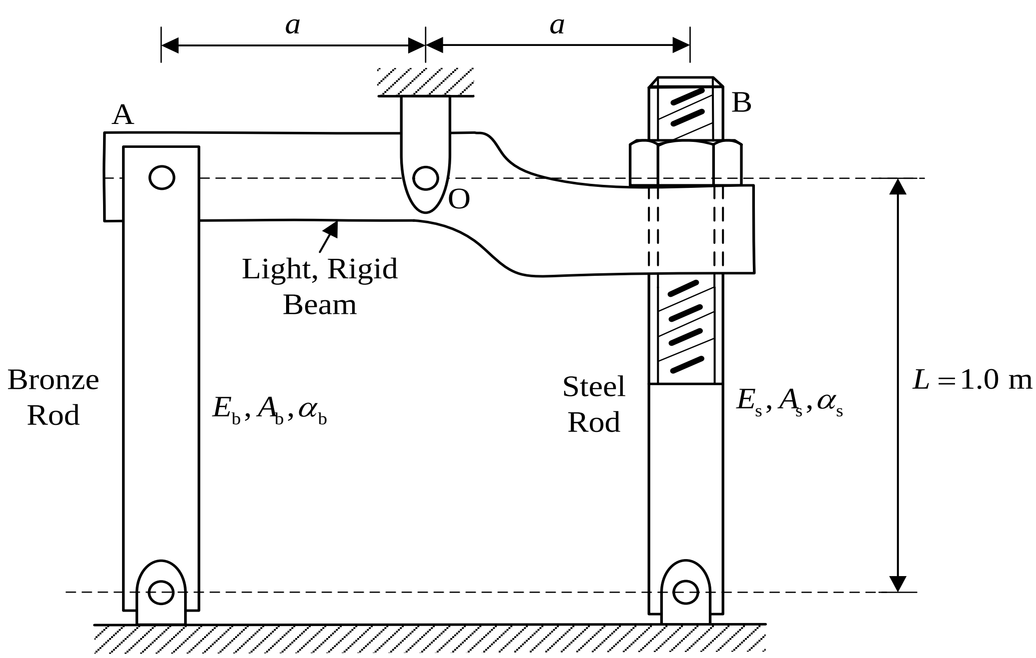


Figure 1: A mechanical structure with smooth joints.

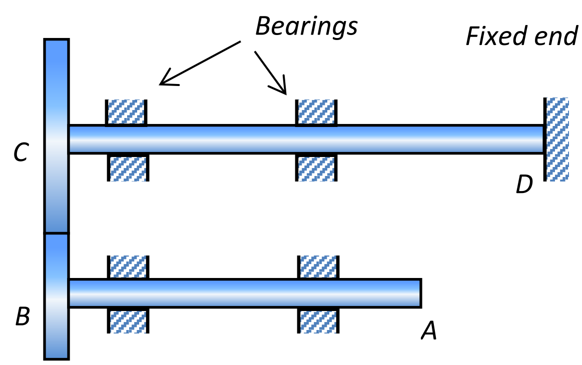
Question 2

(Total 25 points) Two solid steel shafts are connected through two rigid gears and used as a torsional spring as shown in Figure 2. Knowing that for each shaft the modulus of rigidity, ***G***= *12x10*^{6} *[psi]*, and that allowable shear stress τ_{allow}= *8 [ksi]*, determine:

a) The largest torque, ***T*** *[lb-ft]*, which may be applied to the end *A* of the shaft *AB*. ***(11 points)***

b) The corresponding angle through which the end *A* of shaft *AB* rotates. ***(11 points)***

c) The torsional spring constant, *K [lb-ft/Rad]*, at end *A*. ***(3 points)***



Diameter of Shaft *AB = 0.50 [in]*
Diameter of shaft *CD = 0.50 [in]*
Length of shaft *AB = 24 [in]*
Length of shaft *CD = 36 [in]*
Radius of gear *B = 3 [in]*
Radius of gear *C = 5 [in]*

Figure 2

Question 3

(Total 25 points) For the beam and loading shown in Figure 3:

a) Draw Shear force and Bending moment diagrams ***(8 points)***

b) Determine the maximum shear stress in *[MPa]* due to shear force, and clearly state/show its <u>complete</u> location in the beam. ***(9 points)***

c) Determine the maximum tensile and compressive normal stresses in *[MPa]* due to bending moment, and clearly state/show their location in the beam. ***(8 points)***

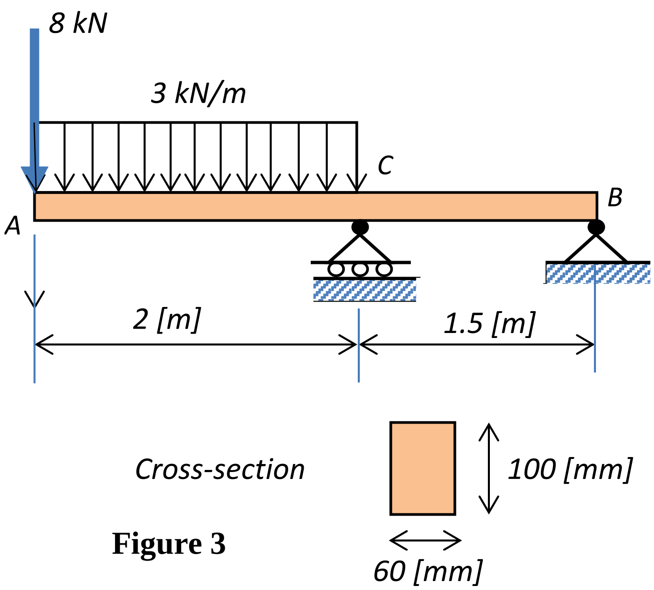


Figure 3

Question 4

(Total 25 points) A uniform beam ABC of length $2L$ is hinged (smooth) at end A and is supported on a smooth roller at its mid-point B. End C is free (overhung), as shown in Figure 4. A concentrated moment M_o is applied to the beam at B, in the clock-wise direction.

a) Determine the support reactions at A and B. ***(5 Points)***

b) Determine a complete expression for the vertical deflection v of the beam (of its elastic curve) from the original horizontal position, when the external load M_o is applied. The deflection v should be expressed as a function of x, where x = the distance to any general location along the beam from end A. *Note*: Use the method of direct integration. ***(15 Points)***

c) From the result of Part (b) determine the vertical deflection at the free end C of the beam. Verify this result by using the slope of the beam (elastic curve) at B. ***(5 Points)***

Given: I = 2nd Moment of area of the beam X-section about its neutral axis

E = Young's modulus of the beam material.

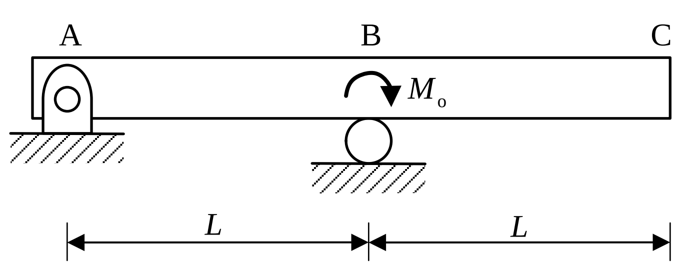


Figure 4: A uniform beam supported by a hinge and a smooth roller.