The UNIVERSITY *of* WESTERN ONTARIO

APPLIED MATHEMATICS 1411

Final Examination (Part 2)

Date: 18 December 2010 TIME: 120 minutes

Name:____

STUDENT NUMBER								
0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5
6	6	6	6	6	6	6	6	6
7	7	7	7	7	7	7	7	7
8	8	8	8	8	8	8	8	8
9	9	9	9	9	9	9	9	9

This is an open book examination.

There are 6 questions. All questions are of equal weight.

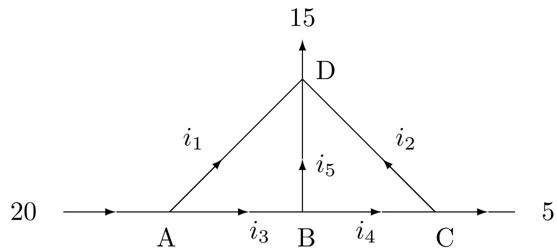
When using a calculator, quote all answers to 4 significant figures.

All answers must be written in the space provided in this booklet.

THIS SPACE FOR EXAMINERS ONLY

1	2	3	4	5	6	7	8

1. Consider the pipe network below, with numbers being flow rates.



(a) Apply conservation of mass at the points A,B,C,D to obtain equations for i_1, i_2, i_3, i_4, i_5.

(b) Find all the solutions of the equations (including the possibility that there is none).

(c) The pipe AD becomes blocked, forcing $i_1 = 0$. Find all the solutions of the new set of equations (including the possibility that there is none).

2. Consider the transformation given by

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \tag{1}$$

Use the left-hand axes to plot vectors $\mathbf{x} = (x_1, x_2)$. Use the right-hand axes to plot vectors $\mathbf{y} = (y_1, y_2)$.

(a) Plot and label the square described by the points
$P : \mathbf{x} = (-1, 1)$, $Q : \mathbf{x} = (1, 1)$, $R : \mathbf{x} = (1, -1)$ and $S : \mathbf{x} = (-1, -1)$.

(b) Plot the image of the square under the transformation (1).

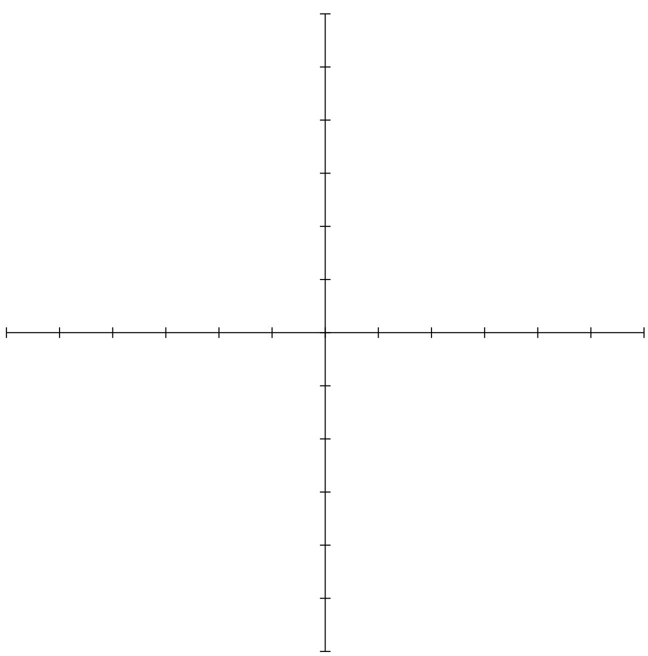
(c) Plot the point $W : \mathbf{y} = (1, 3)$ on the right-hand axes (the $\mathbf{y}$ axes).

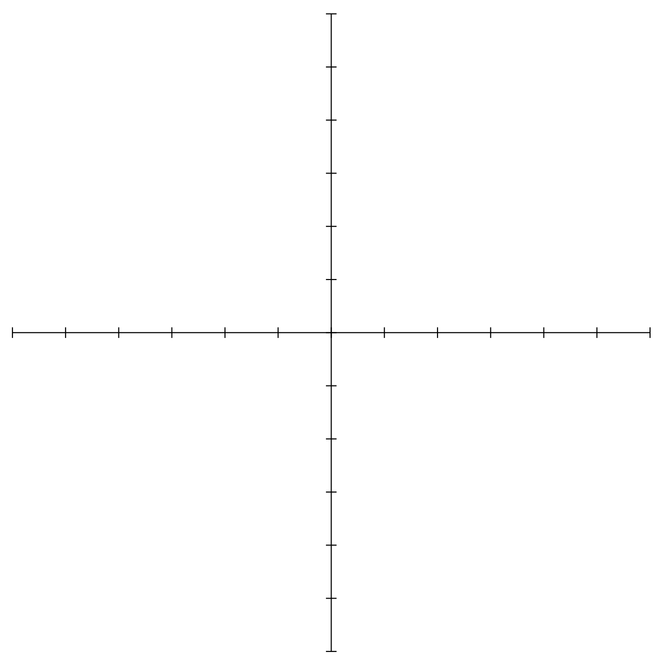
(d) Plot the corresponding point (the pre-image) W on the left axes.

(e) Calculate the angle between the lines PQ and QR in the image plane, i.e. in the axes that are used to plot (y_1, y_2) values.

(f) Let the origins of either set of axes be O. By comparing the vectors OP, OQ in the $\mathbf{x}$-plane with their images in the $\mathbf{y}$-plane, deduce the eigenvectors and eigenvalues of

$$A = \begin{pmatrix} -1 & 2 \\ 2 & -1 \end{pmatrix}$$





3. (a) Find the eigenvalues and eigenvectors of the matrix

$$A = \begin{pmatrix} 3 & 1 & 1 \\ -4 & -2 & -5 \\ 2 & 2 & 5 \end{pmatrix} .$$

You may use the fact that one eigenvalue equals 3.

(b) What matrix P converts A to a diagonal matrix using a similarity transformation?

4. Follow the steps below to use the method of least squares to obtain the best linear fit to the points (x, y) given by

x	-2	0	2	4
y	1	3	-5	-11

Let the straight line be $y = ax + b$.

(a) Write down 4 equations for the coefficients a, b.

(b) Show that the equations do not have a solution.

(c) Convert the equations to matrix form $AX = B$.

(d) Calculate the normal equations for the least-squares approximate solution for a, b.

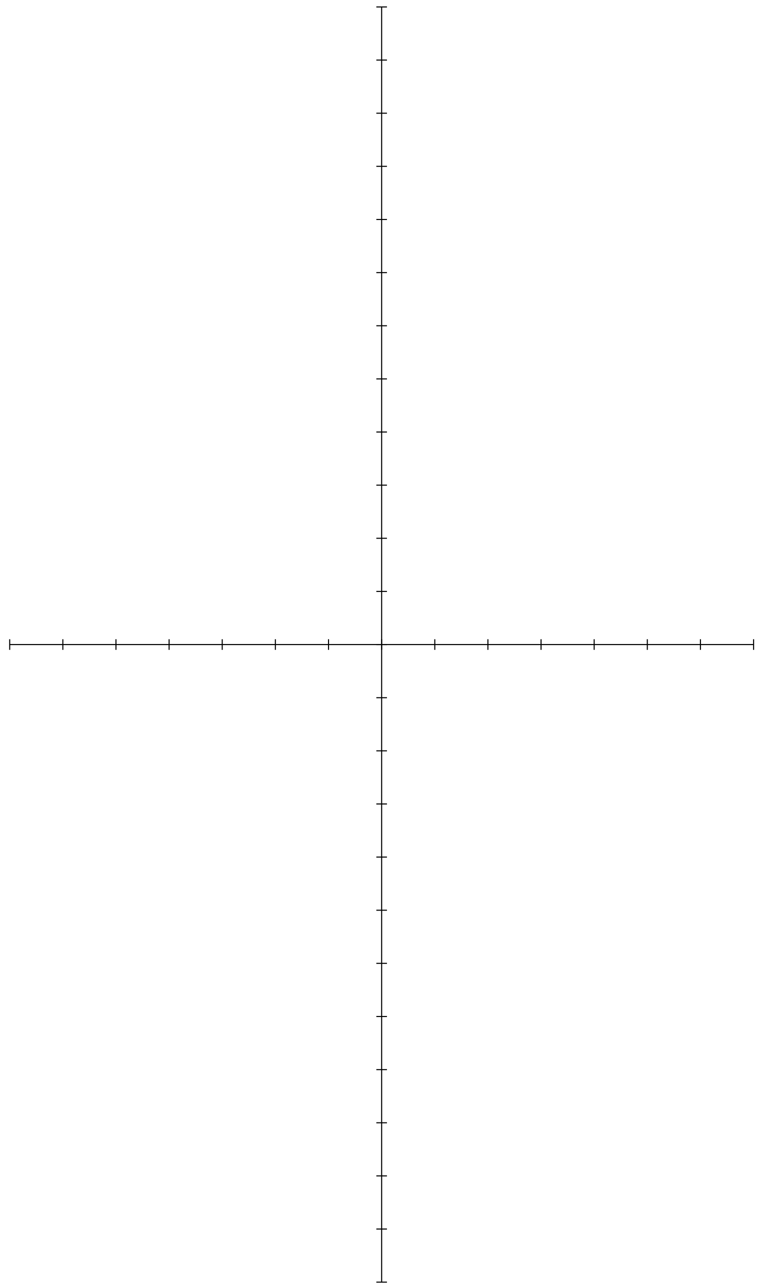
(e) Solve the equations.

(f) Plot the data points and your best fit on the axes below, on the next page.

(g) Use your solution to calculate the residual $\|AX - B\|$ for the equations you found above in part (4c).

(h) Compare the residual in part (4g) with $\|B\|$. Say which is greater and explain why.

Space for answers to question 4 and 5.



5. It is required to fit exactly a cubic curve through the points (x, y) given by

x	-2	-1	0	2	3
y	1	W	3	-5	-9

Notice that the y coordinate of the point at $x = -1$ is unknown.

Let the cubic curve be $y = px^3 + qx^2 + rx + s$.

(a) Write down 5 equations for the coefficients p, q, r, s and the unknown W.

(b) Solve the equations, by any method.

(c) Plot the data points and your cubic on the axes above, on the previous page.

6. (a) For the matrix below, find the inverse or show the matrix is singular.

$$\begin{pmatrix} 1 & 6 & 4 \\ 2 & 4 & -1 \\ -1 & 2 & 5 \end{pmatrix}$$

(b) For the matrix below, find the inverse or show the matrix is singular.

$$\begin{pmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{pmatrix}$$