

## CHAPTER 8

### FAILURE

#### PROBLEM SOLUTIONS

##### Principles of Fracture Mechanics

- 8.1 What is the magnitude of the maximum stress that exists at the tip of an internal crack having a radius of curvature of  $2.5 \times 10^{-4}$  mm and a crack length of  $2.6 \times 10^{-2}$  mm when a tensile stress of 170 MPa is applied?

##### Solution

This problem asks that we compute the magnitude of the maximum stress that exists at the tip of an internal crack. Equation 8.1 is employed to solve this problem, as

$$\begin{aligned}\sigma_m &= 2\sigma_0 \left( \frac{a}{\rho_t} \right)^{1/2} \\ &= (2)(170 \text{ MPa}) \left[ \frac{2.6 \times 10^{-2} \text{ mm}}{2.5 \times 10^{-4} \text{ mm}} \right]^{1/2} = 2452 \text{ MPa}\end{aligned}$$

- 8.2 Estimate the theoretical fracture strength of a brittle material if it is known that fracture occurs by the propagation of an elliptically shaped surface crack of length 0.28 mm and having a tip radius of curvature of  $1.2 \times 10^{-3}$  mm when a stress of 1200 MPa is applied.

Solution

In order to estimate the theoretical fracture strength of this material it is necessary to calculate  $\sigma_m$  using Equation 8.1 given that  $\sigma_0 = 1200$  MPa,  $a = 0.28$  mm, and  $\rho_t = 1.2 \times 10^{-3}$  mm. Thus,

$$\begin{aligned}\sigma_m &= 2\sigma_0 \left( \frac{a}{\rho_t} \right)^{1/2} \\ &= (2)(1200 \text{ MPa}) \left[ \frac{0.28 \text{ mm}}{1.2 \times 10^{-3} \text{ mm}} \right]^{1/2} = 3.7 \times 10^4 \text{ MPa} = 37 \text{ GPa}\end{aligned}$$

- 8.3 If the specific surface energy for soda-lime glass is  $0.30 \text{ J/m}^2$ , using data contained in Table 12.5, compute the critical stress required for the propagation of a surface crack of length  $0.05 \text{ mm}$ .

Solution

We may determine the critical stress required for the propagation of an surface crack in soda-lime glass using Equation 8.3; taking the value of  $69 \text{ GPa}$  (Table 12.5) as the modulus of elasticity, we get

$$\sigma_c = \left[ \frac{2E\gamma_s}{\pi a} \right]^{1/2}$$
$$= \left[ \frac{(2)(69 \times 10^9 \text{ N/m}^2)(0.30 \text{ N/m})}{(\pi)(0.05 \times 10^{-3} \text{ m})} \right]^{1/2} = 16.2 \times 10^6 \text{ N/m}^2 = 16.2 \text{ MPa}$$

- 8.4 A polystyrene component must not fail when a tensile stress of 1.25 MPa is applied. Determine the maximum allowable surface crack length if the surface energy of polystyrene is 0.50 J/m<sup>2</sup>. Assume a modulus of elasticity of 3.0 GPa.

Solution

The maximum allowable surface crack length for polystyrene may be determined using Equation 8.3; taking 3.0 GPa as the modulus of elasticity, and solving for  $a$ , leads to

$$a = \frac{2E\gamma_s}{\pi\sigma_c^2} = \frac{(2)(3 \times 10^9 \text{ N/m}^2)(0.50 \text{ N/m})}{(\pi)(1.25 \times 10^6 \text{ N/m}^2)^2}$$
$$= 6.1 \times 10^{-4} \text{ m} = 0.61 \text{ mm}$$

- 8.5 A specimen of a 4340 steel alloy having a plane strain fracture toughness of  $45 \text{ MPa}\sqrt{\text{m}}$  is exposed to a stress of 1000 MPa. Will this specimen experience fracture if it is known that the largest surface crack is 0.76 mm long? Why or why not? Assume that the parameter  $Y$  has a value of 1.0.

Solution

This problem asks us to determine whether or not the 4340 steel alloy specimen will fracture when exposed to a stress of 1000 MPa, given the values of  $K_{Ic}$ ,  $Y$ , and the largest value of  $a$  in the material. This requires that we solve for  $\sigma_c$  from Equation 8.6. Thus

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{45 \text{ MPa}\sqrt{\text{m}}}{(1.0)\sqrt{(\pi)(0.76 \times 10^{-3} \text{ m})}} = 921 \text{ MPa}$$

Therefore, fracture will most likely occur because this specimen will tolerate a stress of 927 MPa before fracture, which is less than the applied stress of 1000 MPa.

- 8.6 Some aircraft component is fabricated from an aluminum alloy that has a plane strain fracture toughness of  $35 \text{ MPa}\sqrt{\text{m}}$ . It has been determined that fracture results at a stress of 250 MPa when the maximum (or critical) internal crack length is 2.0 mm. For this same component and alloy, will fracture occur at a stress level of 325 MPa when the maximum internal crack length is 1.1 mm? Why or why not?

Solution

We are asked to determine if an aircraft component will fracture for a given fracture toughness ( $35 \text{ MPa}\sqrt{\text{m}}$ ), stress level (325 MPa), and maximum internal crack length (1.1 mm), given that fracture occurs for the same component using the same alloy for another stress level and internal crack length. It first becomes necessary to solve for the parameter  $Y$ , using Equation 8.5, for the conditions under which fracture occurred (i.e.,  $\sigma = 250 \text{ MPa}$  and  $2a = 2.0 \text{ mm}$ ). Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{35 \text{ MPa}\sqrt{\text{m}}}{(250 \text{ MPa})\sqrt{(\pi)\left(\frac{2 \times 10^{-3} \text{ m}}{2}\right)}} = 2.50$$

Now we will solve for the product  $Y\sigma\sqrt{\pi a}$  for the other set of conditions, so as to ascertain whether or not this value is greater than the  $K_{Ic}$  for the alloy. Thus,

$$\begin{aligned} Y\sigma\sqrt{\pi a} &= (2.50)(325 \text{ MPa})\sqrt{(\pi)\left(\frac{1.1 \times 10^{-3} \text{ m}}{2}\right)} \\ &= 33.8 \text{ MPa}\sqrt{\text{m}} \end{aligned}$$

Therefore, fracture *will not* occur since this value ( $33.8 \text{ MPa}\sqrt{\text{m}}$ ) is less than the  $K_{Ic}$  of the material,  $35 \text{ MPa}\sqrt{\text{m}}$ .

- 8.7 Suppose that a wing component on an aircraft is fabricated from an aluminum alloy that has a plane strain fracture toughness of  $40 \text{ MPa}\sqrt{\text{m}}$ . It has been determined that fracture results at a stress of 365 MPa when the maximum internal crack length is 2.6 mm. For this same component and alloy, compute the stress level at which fracture will occur for a critical internal crack length of 4.0 mm.

Solution

This problem asks us to determine the stress level at which an a wing component on an aircraft will fracture for a given fracture toughness ( $40 \text{ MPa}\sqrt{\text{m}}$ ) and maximum internal crack length (4.0 mm), given that fracture occurs for the same component using the same alloy at one stress level (365 MPa) and another internal crack length (2.6 mm). It first becomes necessary to solve for the parameter  $Y$  for the conditions under which fracture occurred using Equation 8.5. Therefore,

$$Y = \frac{K_{Ic}}{\sigma\sqrt{\pi a}} = \frac{40 \text{ MPa}\sqrt{\text{m}}}{(365 \text{ MPa})\sqrt{(\pi)\left(\frac{2.6 \times 10^{-3} \text{ m}}{2}\right)}} = 1.71$$

Now we will solve for  $\sigma_c$  using Equation 8.6 as

$$\sigma_c = \frac{K_{Ic}}{Y\sqrt{\pi a}} = \frac{40 \text{ MPa}\sqrt{\text{m}}}{(1.71)\sqrt{(\pi)\left(\frac{4 \times 10^{-3} \text{ m}}{2}\right)}} = 295 \text{ MPa}$$

- 8.8 A large plate is fabricated from a steel alloy that has a plane strain fracture toughness of  $55 \text{ MPa}\sqrt{\text{m}}$ . If, during service use, the plate is exposed to a tensile stress of 200 MPa, determine the minimum length of a surface crack that will lead to fracture. Assume a value of 1.0 for  $Y$ .

Solution

For this problem, we are given values of  $K_{Ic}$  ( $55 \text{ MPa}\sqrt{\text{m}}$ ),  $\sigma$  (200 MPa), and  $Y$  (1.0) for a large plate and are asked to determine the minimum length of a surface crack that will lead to fracture. All we need do is to solve for  $a_c$  using Equation 8.7; therefore

$$a_c = \frac{1}{\pi} \left( \frac{K_{Ic}}{Y\sigma} \right)^2 = \frac{1}{\pi} \left[ \frac{55 \text{ MPa}\sqrt{\text{m}}}{(1.0)(200 \text{ MPa})} \right]^2 = 0.024 \text{ m} = 24 \text{ mm}$$

- 8.9 Calculate the maximum internal crack length allowable for a 7075-T651 aluminum alloy (Table 8.1) component that is loaded to a stress one-half of its yield strength. Assume that the value of  $Y$  is 1.35.

Solution

This problem asks us to calculate the maximum internal crack length allowable for the 7075-T651 aluminum alloy in Table 8.1 given that it is loaded to a stress level equal to one-half of its yield strength. For this alloy,  $K_{Ic} = 24 \text{ MPa}\sqrt{\text{m}}$ ; also,  $\sigma = \sigma_y/2 = (495 \text{ MPa})/2 = 248 \text{ MPa}$ . Now solving for  $2a_c$  using Equation 8.7 yields

$$2a_c = \frac{2}{\pi} \left( \frac{K_{Ic}}{Y\sigma} \right)^2 = \frac{2}{\pi} \left[ \frac{24 \text{ MPa}\sqrt{\text{m}}}{(1.35)(248 \text{ MPa})} \right]^2 = 0.0033 \text{ m} = 3.3 \text{ mm}$$

- 8.10 A structural component in the form of a wide plate is to be fabricated from a steel alloy that has a plane strain fracture toughness of  $77.0 \text{ MPa}\sqrt{\text{m}}$  and a yield strength of  $1400 \text{ MPa}$ . The flaw size resolution limit of the flaw detection apparatus is  $4.1 \text{ mm}$ . If the design stress is one half of the yield strength and the value of  $Y$  is  $1.0$ , determine whether or not a critical flaw for this plate is subject to detection.

Solution

This problem asks that we determine whether or not a critical flaw in a wide plate is subject to detection given the limit of the flaw detection apparatus ( $4.1 \text{ mm}$ ), the value of  $K_{Ic}$  ( $77 \text{ MPa}\sqrt{\text{m}}$ ), the design stress ( $\sigma_y/2$  in which  $\sigma_y = 1400 \text{ MPa}$ ), and  $Y = 1.0$ . We first need to compute the value of  $a_c$  using Equation 8.7; thus

$$a_c = \frac{1}{\pi} \left( \frac{K_{Ic}}{Y\sigma} \right)^2 = \frac{1}{\pi} \left[ \frac{77 \text{ MPa}\sqrt{\text{m}}}{(1.0) \left( \frac{1400 \text{ MPa}}{2} \right)} \right]^2 = 0.0039 \text{ m} = 3.9 \text{ mm}$$

Therefore, the critical flaw is *not* subject to detection since this value of  $a_c$  ( $3.9 \text{ mm}$ ) is less than the  $4.1 \text{ mm}$  resolution limit.

- 8.11 After consultation of other references, write a brief report on one or two nondestructive test techniques that are used to detect and measure internal and/or surface flaws in metal alloys.

The student should do this problem on his/her own.

## Impact Fracture Testing

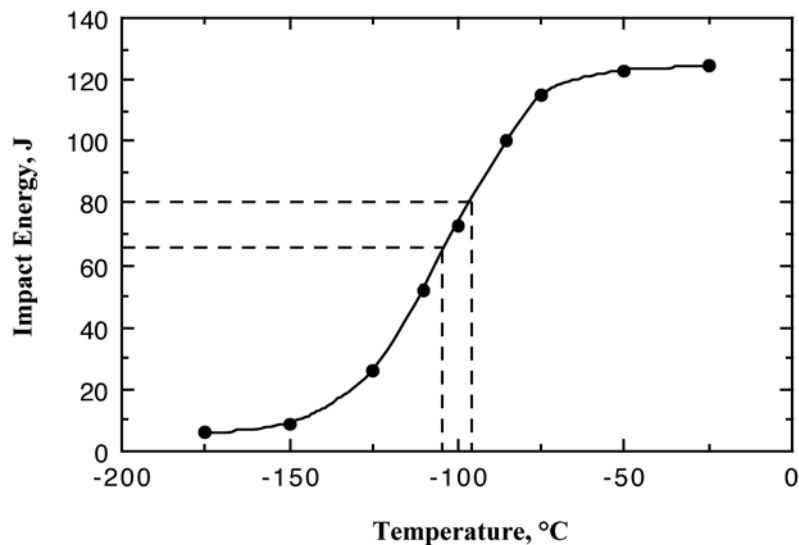
8.12 Following is tabulated data that were gathered from a series of Charpy impact tests on a ductile cast iron.

<i>Temperature (°C)</i>	<i>Impact Energy (J)</i>
-25	124
-50	123
-75	115
-85	100
-100	73
-110	52
-125	26
-150	9
-175	6

- Plot the data as impact energy versus temperature.
- Determine a ductile-to-brittle transition temperature as that temperature corresponding to the average of the maximum and minimum impact energies.
- Determine a ductile-to-brittle transition temperature as that temperature at which the impact energy is 80 J.

### Solution

- The plot of impact energy versus temperature is shown below.



- The average of the maximum and minimum impact energies from the data is

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$$\text{Average} = \frac{124 \text{ J} + 6 \text{ J}}{2} = 65 \text{ J}$$

As indicated on the plot by the one set of dashed lines, the ductile-to-brittle transition temperature according to this criterion is about  $-105^{\circ}\text{C}$  (168 K).

(c) Also, as noted on the plot by the other set of dashed lines, the ductile-to-brittle transition temperature for an impact energy of 80 J is about  $-95^{\circ}\text{C}$  (178 K).

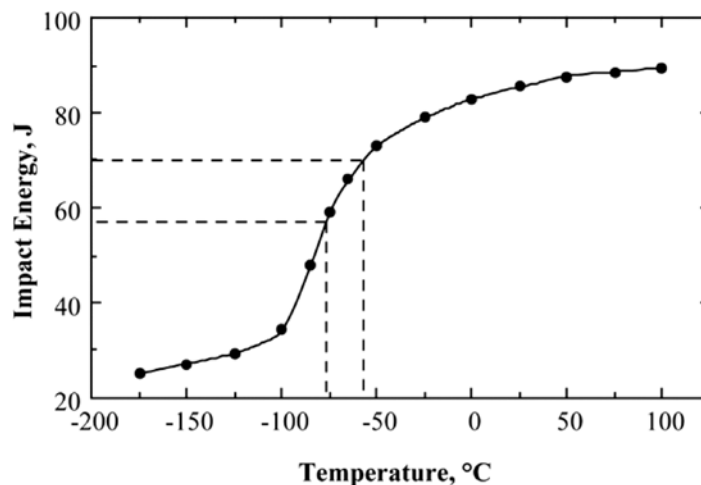
8.13 Following is tabulated data that were gathered from a series of Charpy impact tests on a tempered 4140 steel alloy.

<i>Temperature (°C)</i>	<i>Impact Energy (J)</i>
100	89.3
75	88.6
50	87.6
25	85.4
0	82.9
-25	78.9
-50	73.1
-65	66.0
-75	59.3
-85	47.9
-100	34.3
-125	29.3
-150	27.1
-175	25.0

- Plot the data as impact energy versus temperature.
- Determine a ductile-to-brittle transition temperature as that temperature corresponding to the average of the maximum and minimum impact energies.
- Determine a ductile-to-brittle transition temperature as that temperature at which the impact energy is 70 J.

Solution

The plot of impact energy versus temperature is shown below.



(b) The average of the maximum and minimum impact energies from the data is

$$\text{Average} = \frac{89.3 \text{ J} + 25 \text{ J}}{2} = 57.2 \text{ J}$$

As indicated on the plot by the one set of dashed lines, the ductile-to-brittle transition temperature according to this criterion is about  $-75^{\circ}\text{C}$  (198 K).

(c) Also, as noted on the plot by the other set of dashed lines, the ductile-to-brittle transition temperature for an impact energy of 70 J is about  $-55^{\circ}\text{C}$  (218K).

## Cyclic Stresses (Fatigue)

### The S-N Curve

8.14 A fatigue test was conducted in which the mean stress was 50 MPa and the stress amplitude was 225 MPa.

- Compute the maximum and minimum stress levels.
- Compute the stress ratio.
- Compute the magnitude of the stress range.

#### Solution

(a) Given the values of  $\sigma_m$  (50 MPa) and  $\sigma_a$  (225 MPa) we are asked to compute  $\sigma_{\max}$  and  $\sigma_{\min}$ . From Equation 8.14

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2} = 50 \text{ MPa}$$

Or,

$$\sigma_{\max} + \sigma_{\min} = 100 \text{ MPa}$$

Furthermore, utilization of Equation 8.16 yields

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = 225 \text{ MPa}$$

Or,

$$\sigma_{\max} - \sigma_{\min} = 450 \text{ MPa}$$

Simultaneously solving these two expressions leads to

$$\sigma_{\max} = 275 \text{ MPa}$$

$$\sigma_{\min} = -175 \text{ MPa}$$

(b) Using Equation 8.17 the stress ratio  $R$  is determined as follows:

$$R = \frac{\sigma_{\min}}{\sigma_{\max}} = \frac{-175 \text{ MPa}}{275 \text{ MPa}} = -0.64$$

(c) The magnitude of the stress range  $\sigma_r$  is determined using Equation 8.15 as

$$\sigma_r = \sigma_{\max} - \sigma_{\min} = 275 \text{ MPa} - (-175 \text{ MPa}) = 450 \text{ MPa}$$

- 8.15 A cylindrical 1045 steel bar (Figure 8.34) is subjected to repeated compression-tension stress cycling along its axis. If the load amplitude is 22,000 N, compute the minimum allowable bar diameter to ensure that fatigue failure will not occur. Assume a factor of safety of 2.0.

Solution

From Figure 8.34, the fatigue limit stress amplitude for this alloy is 310 MPa. Stress is defined in Equation 6.1 as  $\sigma = \frac{F}{A_0}$ . For a cylindrical bar

$$A_0 = \pi \left( \frac{d_0}{2} \right)^2$$

Substitution for  $A_0$  into the Equation 6.1 leads to

$$\sigma = \frac{F}{A_0} = \frac{F}{\pi \left( \frac{d_0}{2} \right)^2} = \frac{4F}{\pi d_0^2}$$

We now solve for  $d_0$ , taking stress as the fatigue limit divided by the factor of safety. Thus

$$\begin{aligned} d_0 &= \sqrt{\frac{4F}{\pi \left( \frac{\sigma}{N} \right)}} \\ &= \sqrt{\frac{(4)(22,000 \text{ N})}{(\pi) \left( \frac{310 \times 10^6 \text{ N/m}^2}{2} \right)}} = 13.4 \times 10^{-3} \text{ m} = 13.4 \text{ mm} \end{aligned}$$

8.16 An 8.0 mm diameter cylindrical rod fabricated from a red brass alloy (Figure 8.34) is subjected to reversed tension–compression load cycling along its axis. If the maximum tensile and compressive loads are +7500 N and –7500 N, respectively, determine its fatigue life. Assume that the stress plotted in Figure 8.34 is stress amplitude.

Solution

We are asked to determine the fatigue life for a cylindrical red brass rod given its diameter (8.0 mm) and the maximum tensile and compressive loads (+7500 N and –7500 N, respectively). The first thing that is necessary is to calculate values of  $\sigma_{\max}$  and  $\sigma_{\min}$  using Equation 6.1. Thus

$$\begin{aligned}\sigma_{\max} &= \frac{F_{\max}}{A_0} = \frac{F_{\max}}{\pi \left(\frac{d_0}{2}\right)^2} \\ &= \frac{7500 \text{ N}}{\left(\pi\right)\left(\frac{8.0 \times 10^{-3} \text{ m}}{2}\right)^2} = 150 \times 10^6 \text{ N/m}^2 = 150 \text{ MPa}\end{aligned}$$

$$\begin{aligned}\sigma_{\min} &= \frac{F_{\min}}{\pi \left(\frac{d_0}{2}\right)^2} \\ &= \frac{-7500 \text{ N}}{\left(\pi\right)\left(\frac{8.0 \times 10^{-3} \text{ m}}{2}\right)^2} = -150 \times 10^6 \text{ N/m}^2 = -150 \text{ MPa} \quad (-22,500 \text{ psi})\end{aligned}$$

Now it becomes necessary to compute the stress amplitude using Equation 8.16 as

$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2} = \frac{150 \text{ MPa} - (-150 \text{ MPa})}{2} = 150 \text{ MPa}$$

From Figure 8.34,  $f$  for the red brass, the number of cycles to failure at this stress amplitude is about  $1 \times 10^5$  cycles.

8.17 A 12.5 mm diameter cylindrical rod fabricated from a 2014-T6 alloy (Figure 8.34) is subjected to a repeated tension–compression load cycling along its axis. Compute the maximum and minimum loads that will be applied to yield a fatigue life of  $1.0 \times 10^7$  cycles. Assume that the stress plotted on the vertical axis is stress amplitude, and data were taken for a mean stress of 50 MPa.

Solution

This problem asks that we compute the maximum and minimum loads to which a 12.5 mm diameter 2014-T6 aluminum alloy specimen may be subjected in order to yield a fatigue life of  $1.0 \times 10^7$  cycles; Figure 8.34 is to be used assuming that data were taken for a mean stress of 50 MPa. Upon consultation of Figure 8.34, a fatigue life of  $1.0 \times 10^7$  cycles corresponds to a stress amplitude of 160 MPa. Or, from Equation 8.16

$$\sigma_{\max} - \sigma_{\min} = 2\sigma_a = (2)(160 \text{ MPa}) = 320 \text{ MPa}$$

Since  $\sigma_m = 50 \text{ MPa}$ , then from Equation 8.14

$$\sigma_{\max} + \sigma_{\min} = 2\sigma_m = (2)(50 \text{ MPa}) = 100 \text{ MPa}$$

Simultaneous solution of these two expressions for  $\sigma_{\max}$  and  $\sigma_{\min}$  yields

$$\sigma_{\max} = +210 \text{ MPa}$$

$$\sigma_{\min} = -110 \text{ MPa}$$

Now, inasmuch as  $\sigma = \frac{F}{A_0}$  (Equation 6.1), and  $A_0 = \pi\left(\frac{d_0}{2}\right)^2$  then

$$F_{\max} = \frac{\sigma_{\max} \pi d_0^2}{4} = \frac{(210 \times 10^6 \text{ N/m}^2) (\pi)(12.5 \times 10^{-3} \text{ m})^2}{4} = 25,800 \text{ N}$$

$$F_{\min} = \frac{\sigma_{\min} \pi d_0^2}{4} = \frac{(-110 \times 10^6 \text{ N/m}^2) (\pi)(12.5 \times 10^{-3} \text{ m})^2}{4} = -13,500 \text{ N}$$

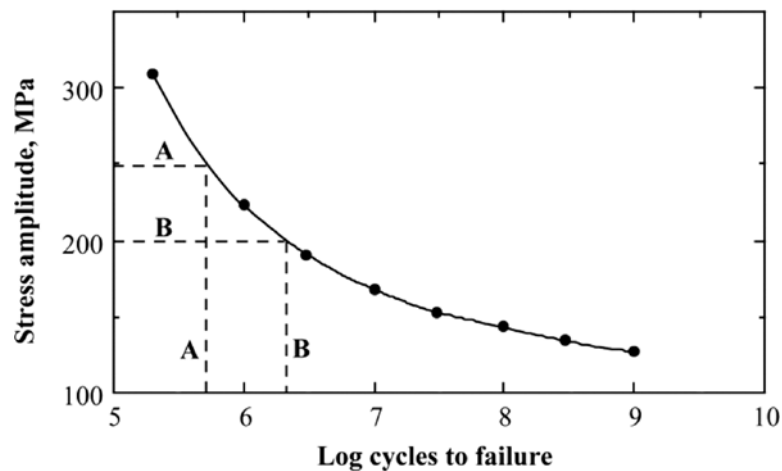
8.18 The fatigue data for a brass alloy are given as follows:

<i>Stress Amplitude (MPa)</i>	<i>Cycles to Failure</i>
310	$2 \times 10^5$
223	$1 \times 10^6$
191	$3 \times 10^6$
168	$1 \times 10^7$
153	$3 \times 10^7$
143	$1 \times 10^8$
134	$3 \times 10^8$
127	$1 \times 10^9$

- Make an  $S-N$  plot (stress amplitude versus logarithm cycles to failure) using these data.
- Determine the fatigue strength at  $5 \times 10^5$  cycles.
- Determine the fatigue life for 200 MPa.

Solution

- The fatigue data for this alloy are plotted below.



- As indicated by the “A” set of dashed lines on the plot, the fatigue strength at  $5 \times 10^5$  cycles [ $\log(5 \times 10^5) = 5.7$ ] is about 250 MPa.
- As noted by the “B” set of dashed lines, the fatigue life for 200 MPa is about  $2 \times 10^6$  cycles (i.e., the log of the lifetime is about 6.3).

- 8.19 Suppose that the fatigue data for the brass alloy in Problem 8.18 were taken from torsional tests, and that a shaft of this alloy is to be used for a coupling that is attached to an electric motor operating at 1500 rpm. Give the maximum torsional stress amplitude possible for each of the following lifetimes of the coupling: (a) 1 year, (b) 1 month, (c) 1 day, and (d) 2 hours.

Solution

For each lifetime, first compute the number of cycles, and then read the corresponding fatigue strength from the above plot.

(a) Fatigue lifetime =  $(1 \text{ yr})(365 \text{ days/yr})(24 \text{ h/day})(60 \text{ min/h})(1500 \text{ cycles/min}) = 7.9 \times 10^8$  cycles. The stress amplitude corresponding to this lifetime is about 130 MPa.

(b) Fatigue lifetime =  $(30 \text{ days})(24 \text{ h/day})(60 \text{ min/h})(1500 \text{ cycles/min}) = 6.5 \times 10^7$  cycles. The stress amplitude corresponding to this lifetime is about 145 MPa.

(c) Fatigue lifetime =  $(24 \text{ h})(60 \text{ min/h})(1500 \text{ cycles/min}) = 2.2 \times 10^6$  cycles. The stress amplitude corresponding to this lifetime is about 195 MPa.

(d) Fatigue lifetime =  $(2 \text{ h})(60 \text{ min/h})(1500 \text{ cycles/min}) = 1.8 \times 10^5$  cycles. The stress amplitude corresponding to this lifetime is about 315 MPa.

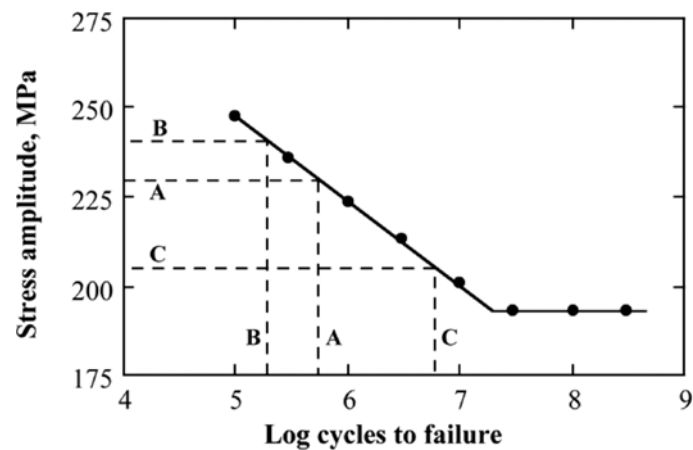
8.20 The fatigue data for a ductile cast iron are given as follows:

<i>Stress Amplitude [MPa]</i>	<i>Cycles to Failure</i>
248	$1 \times 10^5$
236	$3 \times 10^5$
224	$1 \times 10^6$
213	$3 \times 10^6$
201	$1 \times 10^7$
193	$3 \times 10^7$
193	$1 \times 10^8$
193	$3 \times 10^8$

- Make an  $S-N$  plot (stress amplitude versus logarithm cycles to failure) using these data.
- What is the fatigue limit for this alloy?
- Determine fatigue lifetimes at stress amplitudes of 230 MPa and 175 MPa.
- Estimate fatigue strengths at  $2 \times 10^5$  and  $6 \times 10^6$  cycles.

Solution

- The fatigue data for this alloy are plotted below.



- (b) The fatigue limit is the stress level at which the curve becomes horizontal, which is 193 MPa.
- (c) As noted by the “A” set of dashed lines, the fatigue lifetime at a stress amplitude of 230 MPa is about  $5 \times 10^5$  cycles ( $\log N = 5.7$ ). From the plot, the fatigue lifetime at a stress amplitude of 230 MPa is about 50,000 cycles ( $\log N = 4.7$ ). At 175 MPa the fatigue lifetime is essentially an infinite number of cycles since this stress amplitude is below the fatigue limit.
- (d) As noted by the “B” set of dashed lines, the fatigue strength at  $2 \times 10^5$  cycles ( $\log N = 5.3$ ) is about 240 MPa; and according to the “C” set of dashed lines, the fatigue strength at  $6 \times 10^6$  cycles ( $\log N = 6.78$ ) is about 205 MPa.

- 8.21 Suppose that the fatigue data for the cast iron in Problem 8.20 were taken for bending-rotating tests, and that a rod of this alloy is to be used for an automobile axle that rotates at an average rotational velocity of 750 revolutions per minute. Give maximum lifetimes of continuous driving that are allowable for the following stress levels: (a) 250 MPa, (b) 215 MPa, (c) 200 MPa, and (d) 150 MPa.

Solution

For each stress level, first read the corresponding lifetime from the above plot, then convert it into the number of cycles.

- (a) For a stress level of 250 MPa, the fatigue lifetime is approximately 90,000 cycles. This translates into  $(9 \times 10^4 \text{ cycles})(1 \text{ min}/750 \text{ cycles}) = 120 \text{ min}$ .
- (b) For a stress level of 215 MPa, the fatigue lifetime is approximately  $2 \times 10^6$  cycles. This translates into  $(2 \times 10^6 \text{ cycles})(1 \text{ min}/750 \text{ cycles}) = 2670 \text{ min} = 44.4 \text{ h}$ .
- (c) For a stress level of 200 MPa, the fatigue lifetime is approximately  $1 \times 10^7$  cycles. This translates into  $(1 \times 10^7 \text{ cycles})(1 \text{ min}/750 \text{ cycles}) = 1.33 \times 10^4 \text{ min} = 222 \text{ h}$ .
- (d) For a stress level of 150 MPa, the fatigue lifetime is essentially infinite since we are below the fatigue limit [193 MPa].

8.22 Three identical fatigue specimens (denoted A, B, and C) are fabricated from a nonferrous alloy. Each is subjected to one of the maximum-minimum stress cycles listed below; the frequency is the same for all three tests.

<i>Specimen</i>	$\sigma_{\max}$ (MPa)	$\sigma_{\min}$ (MPa)
A	+450	-350
B	+400	-300
C	+340	-340

- (a) Rank the fatigue lifetimes of these three specimens from the longest to the shortest.  
 (b) Now justify this ranking using a schematic  $S-N$  plot.

Solution

In order to solve this problem, it is necessary to compute both the mean stress and stress amplitude for each specimen. Since from Equation 8.14, mean stresses are the specimens are determined as follows:

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$

$$\sigma_m(\text{A}) = \frac{450 \text{ MPa} + (-350 \text{ MPa})}{2} = 50 \text{ MPa}$$

$$\sigma_m(\text{B}) = \frac{400 \text{ MPa} + (-300 \text{ MPa})}{2} = 50 \text{ MPa}$$

$$\sigma_m(\text{C}) = \frac{340 \text{ MPa} + (-340 \text{ MPa})}{2} = 0 \text{ MPa}$$

Furthermore, using Equation 8.16, stress amplitudes are computed as

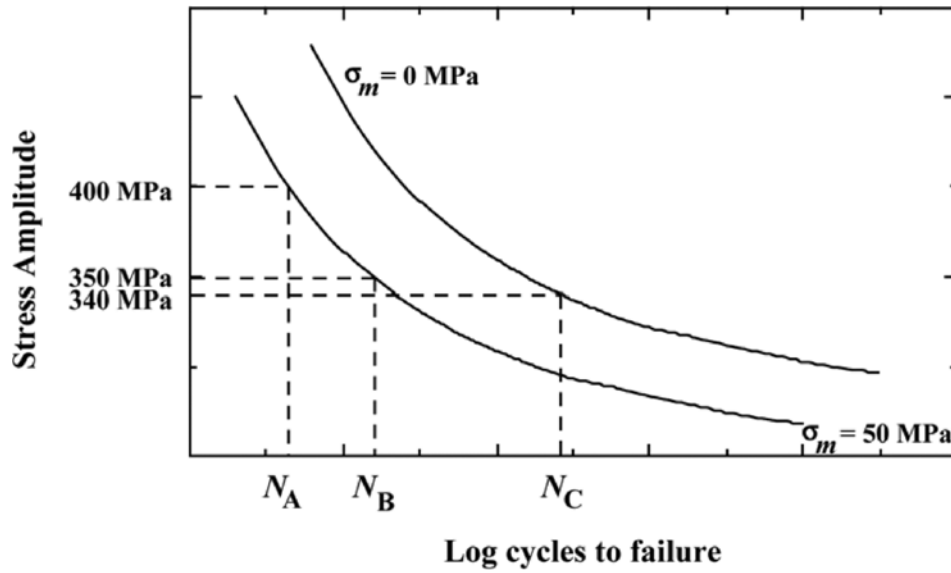
$$\sigma_a = \frac{\sigma_{\max} - \sigma_{\min}}{2}$$

$$\sigma_a(\text{A}) = \frac{450 \text{ MPa} - (-350 \text{ MPa})}{2} = 400 \text{ MPa}$$

$$\sigma_a(\text{B}) = \frac{400 \text{ MPa} - (-300 \text{ MPa})}{2} = 350 \text{ MPa}$$

$$\sigma_a(\text{C}) = \frac{340 \text{ MPa} - (-340 \text{ MPa})}{2} = 340 \text{ MPa}$$

On the basis of these results, the fatigue lifetime for specimen C will be greater than specimen B, which in turn will be greater than specimen A. This conclusion is based upon the following  $S-N$  plot on which curves are plotted for two  $\sigma_m$  values.



8.23 Cite five factors that may lead to scatter in fatigue life data.

Solution

Five factors that lead to scatter in fatigue life data are (1) specimen fabrication and surface preparation, (2) metallurgical variables, (3) specimen alignment in the test apparatus, (4) variation in mean stress, and (5) variation in test cycle frequency.

## **Crack Initiation and Propagation**

### **Factors That Affect Fatigue Life**

8.24 Briefly explain the difference between fatigue striations and beachmarks both in terms of (a) size and (b) origin.

#### Solution

- (a) With regard to size, beachmarks are normally of macroscopic dimensions and may be observed with the naked eye; fatigue striations are of microscopic size and it is necessary to observe them using electron microscopy.
- (b) With regard to origin, beachmarks result from interruptions in the stress cycles; each fatigue striation corresponds to the advance of a fatigue crack during a single load cycle.

8.25 List four measures that may be taken to increase the resistance to fatigue of a metal alloy.

Solution

Four measures that may be taken to increase the fatigue resistance of a metal alloy are:

- (1) Polish the surface to remove stress amplification sites.
- (2) Reduce the number of internal defects (pores, etc.) by means of altering processing and fabrication techniques.
- (3) Modify the design to eliminate notches and sudden contour changes.
- (4) Harden the outer surface of the structure by case hardening (carburizing, nitriding) or shot peening.

## Generalized Creep Behavior

8.26 Give the approximate temperature at which creep deformation becomes an important consideration for each of the following metals: nickel, copper, iron, tungsten, lead, and aluminum.

### Solution

Creep becomes important at about  $0.4T_m$ ,  $T_m$  being the absolute melting temperature of the metal. (The melting temperatures in degrees Celsius are found in the front section of the book.)

$$\text{For Ni, } 0.4T_m = (0.4)(1455 + 273) = 691 \text{ K or } 418^\circ\text{C}$$

$$\text{For Cu, } 0.4T_m = (0.4)(1085 + 273) = 543 \text{ K or } 270^\circ\text{C}$$

$$\text{For Fe, } 0.4T_m = (0.4)(1538 + 273) = 725 \text{ K or } 450^\circ\text{C}$$

$$\text{For W, } 0.4T_m = (0.4)(3410 + 273) = 1473 \text{ K or } 1200^\circ\text{C}$$

$$\text{For Pb, } 0.4T_m = (0.4)(327 + 273) = 240 \text{ K or } -33^\circ\text{C}$$

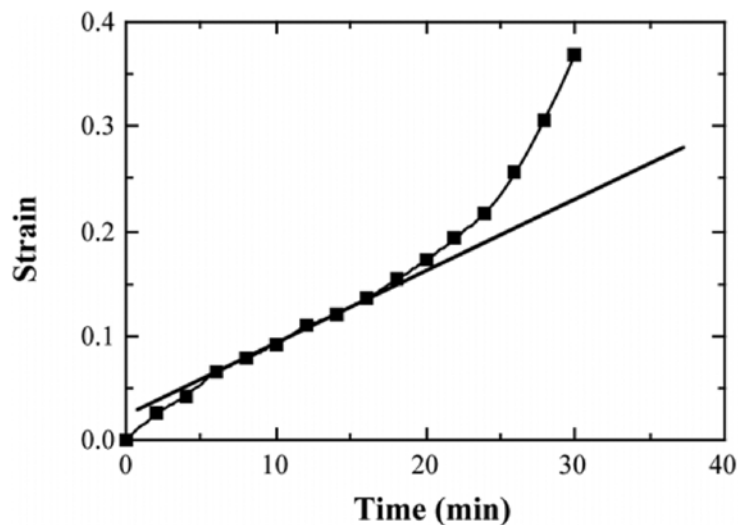
$$\text{For Al, } 0.4T_m = (0.4)(660 + 273) = 373 \text{ K or } 100^\circ\text{C}$$

8.27 The following creep data were taken on an aluminum alloy at 400°C (673 K) and a constant stress of 25 MPa. Plot the data as strain versus time, then determine the steady-state or minimum creep rate. *Note:* The initial and instantaneous strain is not included.

<i>Time</i> (min)	<i>Strain</i>	<i>Time</i> (min)	<i>Strain</i>
0	0.000	16	0.135
2	0.025	18	0.153
4	0.043	20	0.172
6	0.065	22	0.193
8	0.078	24	0.218
10	0.092	26	0.255
12	0.109	28	0.307
14	0.120	30	0.368

#### Solution

These creep data are plotted below



The steady-state creep rate ( $\Delta\epsilon/\Delta t$ ) is the slope of the linear region (i.e., the straight line that has been superimposed on the curve) as

$$\frac{\Delta\epsilon}{\Delta t} = \frac{0.230 - 0.09}{30 \text{ min} - 10 \text{ min}} = 7.0 \times 10^{-3} \text{ min}^{-1}$$

## Stress and Temperature Effects

8.28 A specimen 760 mm long of an S-590 alloy (Figure 8.31) is to be exposed to a tensile stress of 80 MPa at 815°C (1088 K). Determine its elongation after 5000 h. Assume that the total of both instantaneous and primary creep elongations is 1.5 mm.

### Solution

From the 815°C line in Figure 8.31, the steady state creep rate  $\dot{\epsilon}_s$  is about  $5.5 \times 10^{-6} \text{ h}^{-1}$  at 80 MPa. The steady state creep strain,  $\epsilon_s$ , therefore, is just the product of  $\dot{\epsilon}_s$  and time as

$$\begin{aligned}\epsilon_s &= \dot{\epsilon}_s \times (\text{time}) \\ &= (5.5 \times 10^{-6} \text{ h}^{-1})(5,000 \text{ h}) = 0.0275\end{aligned}$$

Strain and elongation are related as in Equation 6.2; solving for the steady state elongation,  $\Delta l_s$ , leads to

$$\Delta l_s = l_0 \epsilon_s = (760 \text{ mm})(0.0275) = 20.9 \text{ mm}$$

Finally, the total elongation is just the sum of this  $\Delta l_s$  and the total of both instantaneous and primary creep elongations [i.e., 1.5 mm]. Therefore, the total elongation is  $20.9 \text{ mm} + 1.5 \text{ mm} = 22.4 \text{ mm}$ .

8.29 For a cylindrical S-590 alloy specimen (Figure 8.31) originally 10 mm in diameter and 505 mm long, what tensile load is necessary to produce a total elongation of 145 mm after 2,000 h at 730°C (1003 K)? Assume that the sum of instantaneous and primary creep elongations is 8.6 mm.

Solution

It is first necessary to calculate the steady state creep rate so that we may utilize Figure 8.31 in order to determine the tensile stress. The steady state elongation,  $\Delta l_s$ , is just the difference between the total elongation and the sum of the instantaneous and primary creep elongations; that is,

$$\Delta l_s = 145 \text{ mm} - 8.6 \text{ mm} = 136.4 \text{ mm}$$

Now the steady state creep rate,  $\dot{\epsilon}_s$  is just

$$\begin{aligned} \dot{\epsilon}_s &= \frac{\Delta \epsilon}{\Delta t} = \frac{\frac{\Delta l_s}{l_0}}{\Delta t} = \frac{\frac{136.4 \text{ mm}}{505 \text{ mm}}}{2,000 \text{ h}} \\ &= 1.35 \times 10^{-4} \text{ h}^{-1} \end{aligned}$$

Employing the 730°C line in Figure 8.31, a steady state creep rate of  $1.35 \times 10^{-4} \text{ h}^{-1}$  corresponds to a stress  $\sigma$  of about 200 MPa [since  $\log(1.35 \times 10^{-4}) = -3.87$ ]. From this we may compute the tensile load using Equation 6.1 as

$$\begin{aligned} F &= \sigma A_0 = \sigma \pi \left( \frac{d_0}{2} \right)^2 \\ &= (200 \times 10^6 \text{ N/m}^2) (\pi) \left( \frac{10.0 \times 10^{-3} \text{ m}}{2} \right)^2 = 15,700 \text{ N} \end{aligned}$$

8.30 If a component fabricated from an S-590 alloy (Figure 8.30) is to be exposed to a tensile stress of 300 MPa at 650°C (923 K), estimate its rupture lifetime.

Solution

This problem asks us to calculate the rupture lifetime of a component fabricated from an S-590 alloy exposed to a tensile stress of 300 MPa at 650°C. All that we need do is read from the 650°C line in Figure 8.30 the rupture lifetime at 300 MPa; this value is about 600 h.

8.31 A cylindrical component constructed from an S-590 alloy (Figure 8.30) has a diameter of 12.5 mm. Determine the maximum load that may be applied for it to survive 500 h at 925°C (1198 K).

Solution

We are asked in this problem to determine the maximum load that may be applied to a cylindrical S-590 alloy component that must survive 500 h at 925°C (1198 K). From Figure 8.30, the stress corresponding to 500 h is about 50 MPa. Since stress is defined in Equation 6.1 as  $\sigma = F/A_0$ , and for a cylindrical specimen,  $A_0 = \pi\left(\frac{d_0}{2}\right)^2$ , then

$$\begin{aligned} F &= \sigma A_0 = \sigma \pi \left(\frac{d_0}{2}\right)^2 \\ &= (50 \times 10^6 \text{ N/m}^2) (\pi) \left(\frac{12.5 \times 10^{-3} \text{ m}}{2}\right)^2 = 6133 \text{ N} \end{aligned}$$

8.32 From Equation 8.19, if the logarithm of  $\dot{\epsilon}_s$  is plotted versus the logarithm of  $\sigma$ , then a straight line should result, the slope of which is the stress exponent  $n$ . Using Figure 8.31, determine the value of  $n$  for the S-590 alloy at 925°C, and for the initial (i.e., lower-temperature) straight line segments at each of 650°C, 730°C, and 815°C.

Solution

The slope of the line from a  $\log \dot{\epsilon}_s$  versus  $\log \sigma$  plot yields the value of  $n$  in Equation 8.19; that is

$$n = \frac{\Delta \log \dot{\epsilon}_s}{\Delta \log \sigma}$$

We are asked to determine the values of  $n$  for the creep data at the four temperatures in Figure 8.31 [i.e., at 925°C (1198 K), and for the initial (i.e., lower-temperature) straight line segments at each of 650°C (923 K), 730°C (1003 K), and 815°C (1088 K)]. This is accomplished by taking ratios of the differences between two  $\log \dot{\epsilon}_s$  and  $\log \sigma$  values. (Note: Figure 8.31 plots  $\log \sigma$  versus  $\log \dot{\epsilon}_s$ ; therefore, values of  $n$  are equal to the reciprocals of the slopes of the straight-line segments.)

Thus for 650°C (923 K)

$$n = \frac{\Delta \log \dot{\epsilon}_s}{\Delta \log \sigma} = \frac{\log(10^{-1}) - \log(10^{-5})}{\log(545 \text{ MPa}) - \log(240 \text{ MPa})} = 11.2$$

While for 730°C (1003 K)

$$n = \frac{\Delta \log \dot{\epsilon}_s}{\Delta \log \sigma} = \frac{\log(1) - \log(10^{-6})}{\log(430 \text{ MPa}) - \log(125 \text{ MPa})} = 11.2$$

And at 815°C (1088 K)

$$n = \frac{\Delta \log \dot{\epsilon}_s}{\Delta \log \sigma} = \frac{\log(1) - \log(10^{-6})}{\log(320 \text{ MPa}) - \log(65 \text{ MPa})} = 8.7$$

And, finally at 925°C (1198 K)

$$n = \frac{\Delta \log \dot{\epsilon}_s}{\Delta \log \sigma} = \frac{\log(10^2) - \log(10^{-5})}{\log(350 \text{ MPa}) - \log(44 \text{ MPa})} = 7.8$$

- 8.33 (a) Estimate the activation energy for creep (i.e.,  $Q_c$  in Equation 8.20) for the S-590 alloy having the steady-state creep behavior shown in Figure 8.31. Use data taken at a stress level of 300 MPa and temperatures of 650°C and 730°C. Assume that the stress exponent  $n$  is independent of temperature. (b) Estimate  $\dot{\epsilon}_s$  at 600°C and 300 MPa.

Solution

(a) We are asked to estimate the activation energy for creep for the S-590 alloy having the steady-state creep behavior shown in Figure 8.31, using data taken at  $\sigma = 300$  MPa and temperatures of 650°C (923 K) and 730°C (1003 K). Since  $\sigma$  is a constant, Equation 8.20 takes the form

$$\dot{\epsilon}_s = K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right) = K_2' \exp\left(-\frac{Q_c}{RT}\right)$$

where  $K_2'$  is now a constant. (Note: the exponent  $n$  has about the same value at these two temperatures per Problem 8.32.) Taking natural logarithms of the above expression

$$\ln \dot{\epsilon}_s = \ln K_2' - \frac{Q_c}{RT}$$

For the case in which we have creep data at two temperatures (denoted as  $T_1$  and  $T_2$ ) and their corresponding steady-state creep rates ( $\dot{\epsilon}_{s1}$  and  $\dot{\epsilon}_{s2}$ ), it is possible to set up two simultaneous equations of the form as above, with two unknowns, namely  $K_2'$  and  $Q_c$ . Solving for  $Q_c$  yields

$$Q_c = - \frac{R(\ln \dot{\epsilon}_{s1} - \ln \dot{\epsilon}_{s2})}{\left[\frac{1}{T_1} - \frac{1}{T_2}\right]}$$

Let us choose  $T_1$  as 650°C (923 K) and  $T_2$  as 730°C (1003 K); then from Figure 8.31, at  $\sigma = 300$  MPa,  $\dot{\epsilon}_{s1} = 8.9 \times 10^{-5} \text{ h}^{-1}$  and  $\dot{\epsilon}_{s2} = 1.3 \times 10^{-2} \text{ h}^{-1}$ . Substitution of these values into the above equation

leads to

$$\begin{aligned} Q_c &= - \frac{(8.31 \text{ J/mol} \cdot \text{K}) \left[ \ln (8.9 \times 10^{-5}) - \ln (1.3 \times 10^{-2}) \right]}{\left[ \frac{1}{923 \text{ K}} - \frac{1}{1003 \text{ K}} \right]} \\ &= 480,000 \text{ J/mol} \end{aligned}$$

(b) We are now asked to estimate  $\dot{\epsilon}_s$  at 600°C (873 K) and 300 MPa. It is first necessary to determine the value of  $K'_2$ , which is accomplished using the first expression above, the value of  $Q_c$ , and one value each of  $\dot{\epsilon}_s$  and  $T$  (say  $\dot{\epsilon}_{s_1}$  and  $T_1$ ). Thus,

$$K'_2 = \dot{\epsilon}_{s_1} \exp\left(\frac{Q_c}{RT_1}\right)$$

$$= (8.9 \times 10^{-5} \text{ h}^{-1}) \exp\left[\frac{480,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(923 \text{ K})}\right] = 1.34 \times 10^{23} \text{ h}^{-1}$$

Now it is possible to calculate  $\dot{\epsilon}_s$  at 600°C (873 K) and 300 MPa as follows:

$$\dot{\epsilon}_s = K'_2 \exp\left(-\frac{Q_c}{RT}\right)$$

$$= (1.34 \times 10^{23} \text{ h}^{-1}) \exp\left[-\frac{480,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(873 \text{ K})}\right]$$

$$= 2.47 \times 10^{-6} \text{ h}^{-1}$$

8.34 Steady-state creep rate data are given below for nickel at 1000°C (1273 K):

$\dot{\epsilon}_s$ ( $s^{-1}$ )	$\sigma$ [MPa]
$10^{-4}$	15
$10^{-6}$	4.5

If it is known that the activation energy for creep is 272,000 J/mol, compute the steady-state creep rate at a temperature of 850°C (1123 K) and a stress level of 25 MPa.

Solution

Taking natural logarithms of both sides of Equation 8.20 yields

$$\ln \dot{\epsilon}_s = \ln K_2 + n \ln \sigma - \frac{Q_c}{RT}$$

With the given data there are two unknowns in this equation—namely  $K_2$  and  $n$ . Using the data provided in the problem statement we can set up two independent equations as follows:

$$\ln(1 \times 10^{-4} \text{ s}^{-1}) = \ln K_2 + n \ln(15 \text{ MPa}) - \frac{272,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(1273 \text{ K})}$$

$$\ln(1 \times 10^{-6} \text{ s}^{-1}) = \ln K_2 + n \ln(4.5 \text{ MPa}) - \frac{272,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(1273 \text{ K})}$$

Now, solving simultaneously for  $n$  and  $K_2$  leads to  $n = 3.825$  and  $K_2 = 466 \text{ s}^{-1}$ . Thus it is now possible to solve for  $\dot{\epsilon}_s$  at 25 MPa and 1123 K using Equation 8.20 as

$$\begin{aligned} \dot{\epsilon}_s &= K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right) \\ &= (466 \text{ s}^{-1}) (25 \text{ MPa})^{3.825} \exp\left[-\frac{272,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(1123 \text{ K})}\right] \end{aligned}$$

$$2.28 \times 10^{-5} \text{ s}^{-1}$$

8.35 Steady-state creep data taken for a stainless steel at a stress level of 70 MPa are given as follows:

$\dot{\epsilon}_s$ ( $s^{-1}$ )	$T$ (K)
$1.0 \times 10^{-5}$	977
$2.5 \times 10^{-3}$	1089

If it is known that the value of the stress exponent  $n$  for this alloy is 7.0, compute the steady-state creep rate at 1250 K and a stress level of 50 MPa.

Solution

Taking natural logarithms of both sides of Equation 8.20 yields

$$\ln \dot{\epsilon}_s = \ln K_2 + n \ln \sigma - \frac{Q_c}{RT}$$

With the given data there are two unknowns in this equation--namely  $K_2$  and  $Q_c$ . Using the data provided in the problem statement we can set up two independent equations as follows:

$$\ln(1.0 \times 10^{-5} \text{ s}^{-1}) = \ln K_2 + (7.0) \ln(70 \text{ MPa}) - \frac{Q_c}{(8.31 \text{ J/mol} \cdot \text{K})(977 \text{ K})}$$

$$\ln(2.5 \times 10^{-3} \text{ s}^{-1}) = \ln K_2 + (7.0) \ln(70 \text{ MPa}) - \frac{Q_c}{(8.31 \text{ J/mol} \cdot \text{K})(1089 \text{ K})}$$

Now, solving simultaneously for  $K_2$  and  $Q_c$  leads to  $K_2 = 2.55 \times 10^5 \text{ s}^{-1}$  and  $Q_c = 436,000 \text{ J/mol}$ . Thus, it is now possible to solve for  $\dot{\epsilon}_s$  at 50 MPa and 1250 K using Equation 8.20 as

$$\begin{aligned} \dot{\epsilon}_s &= K_2 \sigma^n \exp\left(-\frac{Q_c}{RT}\right) \\ &= (2.55 \times 10^5 \text{ s}^{-1})(50 \text{ MPa})^{7.0} \exp\left[-\frac{436,000 \text{ J/mol}}{(8.31 \text{ J/mol} \cdot \text{K})(1250 \text{ K})}\right] \\ &= 0.118 \text{ s}^{-1} \end{aligned}$$

## Alloys for High-Temperature Use

8.36 Cite three metallurgical/processing techniques that are employed to enhance the creep resistance of metal alloys.

### Solution

Three metallurgical/processing techniques that are employed to enhance the creep resistance of metal alloys are (1) solid solution alloying, (2) dispersion strengthening by using an insoluble second phase, and (3) increasing the grain size or producing a grain structure with a preferred orientation.

## DESIGN PROBLEMS

8.D1 Each student (or group of students) is to obtain an object/structure/component that has failed. It may come from your home, an automobile repair shop, a machine shop, etc. Conduct an investigation to determine the cause and type of failure (i.e., simple fracture, fatigue, creep). In addition, propose measures that can be taken to prevent future incidents of this type of failure. Finally, submit a report that addresses the above issues.

Each student or group of students is to submit their own report on a failure analysis investigation that was conducted.

## Principles of Fracture Mechanics

- 8.D2 (a) For the thin-walled spherical tank discussed in Design Example 8.1, on the basis of critical crack size criterion [as addressed in part (a)], rank the following polymers from longest to shortest critical crack length: nylon 6,6 (50% relative humidity), polycarbonate, poly(ethylene terephthalate), and poly(methyl methacrylate). Comment on the magnitude range of the computed values used in the ranking relative to those tabulated for metal alloys as provided in Table 8.3. For these computations, use data contained in Tables B.4 and B.5 in Appendix B.
- (b) Now rank these same four polymers relative to maximum allowable pressure according to the leak-before-break criterion, as described in the (b) portion of Design Example 8.1. As above, comment on these values in relation to those for the metal alloys that are tabulated in Table 8.4.

### Solution

- (a) This portion of the problem calls for us to rank four polymers relative to critical crack length in the wall of a spherical pressure vessel. In the development of Design Example 8.1, it was noted that critical crack length is proportional to the square of the  $K_{Ic}$ - $\sigma_y$  ratio. Values of  $K_{Ic}$  and  $\sigma_y$  as taken from Tables B.4 and B.5 are tabulated below. (Note: when a range of  $\sigma_y$  or  $K_{Ic}$  values is given, the average value is used.)

Material	$K_{Ic}$ (MPa $\sqrt{m}$ )	$\sigma_y$ (MPa)
Nylon 6,6	2.75	51.7
Polycarbonate	2.2	62.1
Poly(ethylene terephthalate)	5.0	59.3
Poly(methyl methacrylate)	1.2	63.5

On the basis of these values, the four polymers are ranked per the squares of the  $K_{Ic}$ - $\sigma_y$  ratios as follows:

Material	$\left(\frac{K_{Ic}}{\sigma_y}\right)^2$ (mm)
PET	7.11
Nylon 6,6	2.83
PC	1.26
PMMA	0.36

These values are smaller than those for the metal alloys given in Table 8.3, which range from 0.93 to 43.1 mm.

(b) Relative to the leak-before-break criterion, the  $K_{Ic}^2/\sigma_y$  ratio is used. The four polymers are ranked according to values of this ratio as follows:

Material	$\frac{K_{Ic}^2}{\sigma_y}$ (MPa · m)
PET	0.422
Nylon 6,6	0.146
PC	0.078
PMMA	0.023

These values are all smaller than those for the metal alloys given in Table 8.4, which values range from 1.2 to 11.2 MPa-m.

## Data Extrapolation Methods

8.D3 An S-590 alloy component (Figure 8.32) must have a creep rupture lifetime of at least 100 days at 500°C (773 K). Compute the maximum allowable stress level.

### Solution

This problem asks that we compute the maximum allowable stress level to give a rupture lifetime of 100 days for an S-590 iron component at 773 K. It is first necessary to compute the value of the Larson-Miller parameter as follows:

$$\begin{aligned} T(20 + \log t_r) &= (773 \text{ K})\{20 + \log [(100 \text{ days})(24 \text{ h/day})]\} \\ &= 18.1 \times 10^3 \end{aligned}$$

From the curve in Figure 8.32, this value of the Larson-Miller parameter corresponds to a stress level of \_\_\_\_\_ about 530 MPa.

8.D4 Consider an S-590 alloy component (Figure 8.32) that is subjected to a stress of 200 MPa. At what temperature will the rupture lifetime be 500 h?

Solution

We are asked in this problem to calculate the temperature at which the rupture lifetime is 500 h when an S-590 iron component is subjected to a stress of 200 MPa. From the curve shown in Figure 8.32, at 200 MPa, the value of the Larson-Miller parameter is  $22.5 \times 10^3$  (K-h). Thus,

$$\begin{aligned} 22.5 \times 10^3 \text{ (K-h)} &= T(20 + \log t_r) \\ &= T[20 + \log(500 \text{ h})] \end{aligned}$$

Or, solving for  $T$  yields  $T = 991 \text{ K}$  (718°C).

8.D5 For an 18-8 Mo stainless steel (Figure 8.35), predict the time to rupture for a component that is subjected to a stress of 80 MPa at 700°C (973 K).

Solution

This problem asks that we determine, for an 18-8 Mo stainless steel, the time to rupture for a component that is subjected to a stress of 80 MPa at 700°C (973 K). From Figure 8.35, the value of the Larson-Miller parameter at 80 MPa is about  $23.5 \times 10^3$ , for  $T$  in K and  $t_r$  in h. Therefore,

$$\begin{aligned} 23.5 \times 10^3 &= T(20 + \log t_r) \\ &= 973(20 + \log t_r) \end{aligned}$$

And, solving for  $t_r$

$$24.15 = 20 + \log t_r$$

which leads to  $t_r = 1.42 \times 10^4$  h = 1.6 yr.

8.D6 Consider an 18-8 Mo stainless steel component (Figure 8.35) that is exposed to a temperature of 500°C (773 K). What is the maximum allowable stress level for a rupture lifetime of 5 years? 20 years?

Solution

We are asked in this problem to calculate the stress levels at which the rupture lifetime will be 5 years and 20 years when an 18-8 Mo stainless steel component is subjected to a temperature of 500°C (773 K). It first becomes necessary to calculate the value of the Larson-Miller parameter for each time. The values of  $t_r$  corresponding to 5 and 20 years are  $4.38 \times 10^4$  h and  $1.75 \times 10^5$  h, respectively. Hence, for a lifetime of 5 years

$$T(20 + \log t_r) = 773[20 + \log (4.38 \times 10^4)] = 19.05 \times 10^3$$

And for  $t_r = 20$  years

$$T(20 + \log t_r) = 773[20 + \log (1.75 \times 10^5)] = 19.51 \times 10^3$$

Using the curve shown in Figure 8.35, the stress values corresponding to the five- and twenty-year lifetimes are approximately 260 MPa and 225 MPa, respectively.