Sauder School of Business **COMM 371 Sep-Dec 2011**
Finance Division

Name: .... **Student ID:** ....

Section: .... **Instructor:** ....

Final Exam

- You have **150 minutes** to complete this exam.
- Answer all questions inside the exam booklet. Make sure your answers are **clearly indicated**.
- You are to answer these questions **without consulting anyone**.
- Be neat and **show your work towards answers wherever necessary.**
- Answers without work towards answers receive no credit. Wrong answers with partially correct work may receive partial credit.
- Manage your time wisely. Do not write more than it is really necessary. **Short, unambiguous, and well structured** answers are the best.
- The exam consists of **7 problems**.
- Total number of marks on this exam is **100**.
- The formula sheet is attached. The last two pages of the booklet are left empty for your notes.

Instructor use only:	
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
Question 6	
Question 7	
Question 8	
Total	

Question 1: For each part of the question (a), (b), and (c), indicate the security with a higher current (i.e., $t = 0$) price. Explain briefly but pricesely the reasoning behind your answer.

a. *[3 marks]* Bond A with coupon rate 5% and YTM 6%, vs. bond B with coupon rate 6% and YTM 6%. Both bonds pay coupons annually, have 5 years to maturity, and have face value $100.

b. *[4 marks]* Bond A with coupon rate 5%, YTM 6%, and 5 years to maturity, vs. bond B with coupon rate 5%, YTM 6%, and 15 years to maturity. Both bonds pay coupons annually and have face value $100.

c. *[4 marks]* European put option P_1 on a stock with strike price $65, or European put option P_2 on the same stock but with strike price $55. Options P_1 and P_2 have the same time to maturity.

Question 2: The following bonds are currently (at year 0) traded in the market.

Bond	Coupon rate (%)	Maturity	Price
A	0	3 years	69.00
B	5	4 years	70.90
C	10	4 years	84.57

a. *[8 marks]* Consider a new bond, called a *C371* bond, with the following characteristics: it pays 1% annual coupons in years 1 and 2, pays 11% coupons in years 3 and 4, and returns the face value in year 4. What should be the current arbitrage-free price of this bond?

Assume that all bonds have face value \$100. Coupon payments are annual, and bid-ask spreads are zero.

Question 3: You are working for an asset-management company, and are in the process of determining the asset allocation for one of your clients. The investments under consideration are T-bills, the S&P500 index (market portfolio), and an index of companies in the "environmental management sector." Your company's research department is particularly excited about that sector's potential.

The return on T-bills is 3%. The expected return on the S&P500 index is 9%, and the S&P500 index's standard deviation is 20%. The beta of the environmental index is 1.2, and the environmental index's standard deviation is 35%. The CAPM holds.

Your objective is to achieve an expected return of 7% with the lowest possible standard deviation.

a. *[3 marks]* Suppose that you decide to invest only in T-bills and the S&P500 index. What weights should you choose?

b. *[3 marks]* Compute the standard deviation of the portfolio in part (a).

c. *[4 marks]* Suppose that you decide to also consider the environmental index, and you assume that the environmental index's expected return is given by the CAPM. What weights should you choose (for T-bills, the S&P500, and the environmental index)?

Question 3 (cont'd):

d. *[4 marks]* Suppose that you become convinced by the research department that the environmental index has an alpha of 3%. Based on this, you decide to give the index a weight of 20%. What weights should you choose for T-bills and the S&P500, so that your expected return is still 7%?

e. *[5 marks]* Compute the standard deviation of the portfolio in part (d), and show that it is smaller than that of the portfolio in part (a). Briefly explain why this result obtains.

Question 4: Stocks A, B, and C have the same expected return 10% and the same standard deviation 30%.

a. *[2 marks]* Compute the standard deviation of the equally weighted portfolio if the correlation between all pairs of stocks is zero.

b. *[4 marks]* Compute the standard deviation of the equally weighted portfolio if the correlation between all pairs of stocks is 0.4.

c. *[4 marks]* Determine all portfolios on the portfolio frontier one can create using stocks A, B, and C.

Question 5: Suppose that the 12-month spot rate is 3%, the 15-month spot rate is 3.5%, and the 24-month spot rate is 4%. All rates are annual rates with annual compounding.

a. *[7 marks]* Determine the no-arbitrage forward price for a bond with annual coupon rate 10%, face value \$100, and maturity two years. The maturity of the forward contract is 15 months.

b. *[9 marks]* Assume that a MADBANK is willing to trade (buy/sell) the forward from part (a) with you at price \$120. Assume also that you can trade riskless zero-coupon bonds with face value \$100 with any maturity and that you can trade the 2-year coupon bond from part (a). Set up an abritrage trade. Give details: What assets you buy/sell at what quatities? What is the arbitrage profit per unit of trade at the time you are entering into it (i.e, at $t = 0$)?

Question 6: The price today of SAUDER.com stock is \$40. After 6 months, it will either increase by 25% or decrease by 20%. Over the next 6 month period the stock price will again either increase by 25% or decrease by 20%. The stock does not pay any dividends over the next year. The 6 month interest rate is 10% (for both the 6 month periods). In answering the following questions, assume that there are no arbitrage opportunities.

a. *[6 marks]* Calculate the price today of a 1 year European put option on this stock which has a strike price of \$40.

b. *[6 marks]* Calculate the price today of a 1 year American put option on this stock which has a strike price of \$40.

c. *[9 marks]* An *Asian* option is an option whose payoff depends on the average value of the stock price over the life of the option. (Early exercise of the option is not allowed.) One example is a *fixed strike Asian call option*, whose payoff at maturity is $\max[A - X, 0]$, where A is the average value of the stock price and X is the strike price. Calculate the value today of a 1-year fixed strike Asian call option on this stock with a strike price of $X = \$35$ (include today's stock price when you calculate the average price over the year).

Question 7: A *butterfly* spread is a combination of option positions that involves three strike prices. To create a butterfly spread, a trader purchases an option with a low strike price and an option with a high strike price, and sells two options with an intermediate strike price. For this problem, assume that the intermediate strike price is halfway between the low and the high strike prices and that the options are European. Denote the intermediate strike price by X, the low strike price by $X - a$, and the high strike price by $X + a$, $a > 0$.

a. *[5 marks]* Graph the payoff diagram at maturity of the butterfly spread in which the underlying options are *call* options.

b. *[5 marks]* Graph the payoff diagram at maturity of the butterfly spread in which the underlying options are *put* options.

c. *[5 marks]* Using put-call parity, show whether the initial investment (i.e., the amount of money you need to pay at $t = 0$) to create the butterfly spread is higher/lower/the same when you use puts instead of calls.

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Formula Sheet for the Final Exam

- **Present value** of the cash flow stream assuming **constant** interest rate is

$$PV_0 = C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \frac{C_3}{(1+r)^3} + ... + \frac{C_T}{(1+r)^T}.$$

- **Present value** of the cash flow stream assuming **time-dependent** interest rate is

$$PV_0 = C_0 + \frac{C_1}{1+r_1} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + ... + \frac{C_T}{(1+r_T)^T}.$$

- **Future value** of the cash flow stream assuming **constant** interest rate is

$$FV_T = C_0(1+r)^T + C_1(1+r)^{T-1} + C_2(1+r)^{T-2} + ... + C_T.$$

- **Future value** of the cash flow stream assuming **time-dependent** interest rate is

$$FV_T = C_0(1 +_0 r_T)^T + C_1(1 +_1 r_{T-1})^{T-1} + C_2(1 +_2 r_{T-2})^{T-2} + ... + C_T.$$

- **Perpetuity** is an asset that pays a **fixed** sum each year (time period) **forever**

$$PV_0 = \frac{C_1}{1+r} + \frac{C_1}{(1+r)^2} + \frac{C_1}{(1+r)^3} + ... = \sum_{i=1}^{\infty} \frac{C_1}{(1+r)^i} = \frac{C_1}{r}.$$

- **Perpetuity with growing payments** is an asset that pays an **increasing** sum (at constant rate g) each year (time period) **forever**

$$PV_0 = \frac{C_1}{1+r} + \frac{C_1(1+g)}{(1+r)^2} + \frac{C_1(1+g)^2}{(1+r)^3} + ... = \sum_{i=1}^{\infty} \frac{C_1(1+g)^{i-1}}{(1+r)^i} = \frac{C_1}{r-g}.$$

- **Annuity** is an asset that pays a **fixed** sum each year (time period) for a **specified number of years** (time periods) (from year 1 to year T)

$$PV_0 = \frac{C_1}{1+r} + \frac{C_1}{(1+r)^2} + \frac{C_1}{(1+r)^3} + .. + \frac{C_1}{(1+r)^T} = \sum_{i=1}^{\infty} \frac{C_1}{(1+r)^i}$$

$$PV_0 = \sum_{i=1}^{\infty} \frac{C_1}{(1+r)^i} - \sum_{j=T+1}^{\infty} \frac{C_1}{(1+r)^j} = \frac{C_1}{r} - \frac{1}{(1+r)^T}\frac{C_1}{r} = \frac{C_1}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

- **Realized return** from holding an asset is

$$R_{t-T,t} = \frac{D_t + (P_t - P_{t-T})}{P_{t-T}},$$

where P_t is the price of the asset at time t, P_{t-T} is the price of the asset at time $t-T$ ($T>0$), and D_t is dividend (or other income) received at time t.

- **EAR** that corresponds to **APR** with T compounding periods in a year is

$$EAR = \left(1 + \frac{APR}{T}\right)^T - 1.$$

- Relation between **nominal** rate i and **real** rate r is

$$1 + r = \frac{1+i}{1+\pi},$$

where π is the **inflation** rate during the year (time period).

- Relation between the t-year and $t+T$-year **spot** rates and the **forward** rate ${}_tf_T$ between years t and $t+T$ is

$$(1 + {}_tf_T)^T = \frac{(1+r_{t+T})^{t+T}}{(1+r_t)^t}.$$

- The present value of the **bond with semiannual coupon rate** $c\%$ and T years to maturity when spot rates are quoted as semiannual APRs is

$$PV = \frac{\frac{c}{2}}{1 + \frac{r_{0.5}}{2}} + \frac{\frac{c}{2}}{\left(1 + \frac{r_1}{2}\right)^2} + \cdots + \frac{100 + \frac{c}{2}}{\left(1 + \frac{r_T}{2}\right)^{2T}}.$$

- The **Macaulay duration** (D) of the annual coupon bond with coupon rate $c\%$ and T years to maturity is

$$D = \sum_{t=1}^{T} w_t t,$$

where

$$w_t = \frac{c}{(1+r)^t}\frac{1}{P} \quad \text{for} \quad t = 1, ..., T-1$$
$$w_T = \frac{100+c}{(1+r)^T}\frac{1}{P}$$

and P is the bond price.

- The **modified duration** (D^*) of a bond is

$$D^* = \frac{D}{1+r},$$

where D is the Macaulay duration of the bond.

- For a small interest rate movement Δr, the **change in the investment's value** P is approximately

$$\Delta P \approx -PD^* \Delta r,$$

where D^* is the modified duration of the investment.

- For a data series X_1, ..., X_N:
  - The **sample average** is

$$\overline{X} = \frac{X_1 + \cdots + X_N}{N}.$$

  - The **sample variance** is

$$s(X)^2 = \frac{(X_1 - \overline{X})^2 + \cdots + (X_N - \overline{X})^2}{N - 1}.$$

  - The **sample standard deviation** is

$$s(X) = \sqrt{s(X)^2}.$$

- For a **random variable** Z that takes the values Z_1, ..., Z_K, with probabilities p_1, ..., p_K:
  - The **expectation** of Z is

$$E(Z) = p_1 Z_1 + \cdots + p_K Z_K.$$

  - The **variance** of Z is

$$V(Z) = p_1(Z_1 - E(Z))^2 + \cdots + p_K(Z_K - E(Z))^2.$$

  - The **standard deviation** of Z is

$$\sigma(Z) = \sqrt{V(Z)}.$$

- For two data series X_1, ..., X_N, and Y_1, ..., Y_N:
  - The **sample covariance** is

$$Cov(X,Y) = \frac{\Sigma_{i=1}^N (X_i - \overline{X})(Y_i - \overline{Y})}{N - 1}.$$

  - The **sample correlation** is

$$r(X,Y) = \frac{Cov(X,Y)}{s(X)s(Y)}.$$

- The **return on a portfolio** with N stocks is

$$R = \sum_{n=1}^{N} w_n R_n,$$

where w_n represents the fraction of the portfolio value invested in stock n.

- The **expected return on a portfolio** with N stocks is

$$E(R) = \sum_{n=1}^{N} w_n E(R_n),$$

where w_n represents the fraction of the portfolio value invested in stock n and $E(R_n)$ is the expected return of stock n.

- The **variance of a portfolio** with **two** stocks is

$$V(R) = w_1^2 V(R_1) + w_2^2 V(R_2) + 2 w_1 w_2 Cov(R_1, R_2).$$

- The **variance of a portfolio** with N stocks is

$$V(R) = \sum_{n=1}^{N} w_n^2 V(R_n) + 2 \sum_{n<m} w_n w_m Cov(R_n, R_m).$$

- The **Buck for the Bang Ratio** is

$$\frac{E(R_n) - R_f}{2Cov(R_n, R_{TP})}.$$

- The CAPM **regression equation** is

$$R_n - R_f = \alpha_n + \beta_n (R_M - R_f) + \epsilon_n.$$

- The CAPM **beta** is

$$\beta_n = \frac{Cov(R_n, R_M)}{V(R_M)}.$$

- The **Security Market Line** of the CAPM is

$$E(R_n) - R_f = \beta_n (E(R_M) - R_f).$$

- The pricing equation of a **multi-factor model** is of the form

$$E(R_n) - R_f = \sum_{k=1}^{K} \beta_{n,k} RP_k,$$

where RP_k is the risk premium of factor k and $\beta_{n,k}$ is the factor loading of asset n on factor k.

- The **forward** price is

$$F_{0,T} = (S_0 - PV_0)(1 + r_T)^T.$$

- The **put-call parity** is

$$C = P + S - \frac{X}{(1 + r_T)^T}.$$

- The **risk-neutral probability** of an up move in the price of the derivative's underlying asset is

$$q = \frac{(1 + r) - d}{u - d}.$$

- The **abnormal return** is

$$\alpha = E(R) - [R_f + \beta(E(R_M) - R_f)].$$

Notes:

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