PHY 1322
I MIDTERM
Winter2010

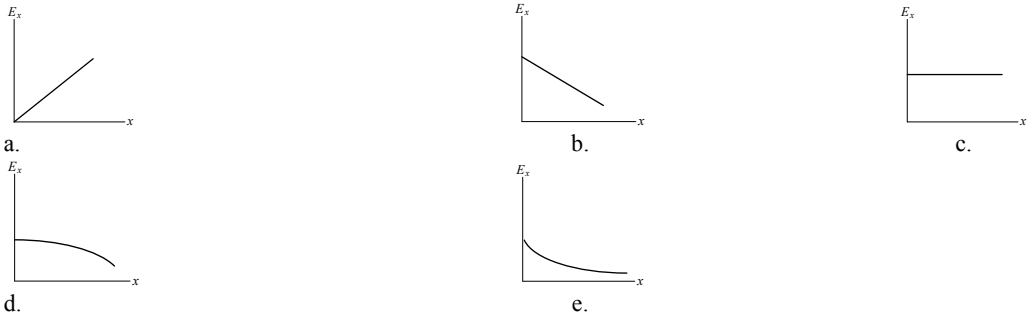
Closed book test
Duration: 100 min

Part I Multiple choice >answer all best 6 count towards 48 % of the test grade

1. What happens to the net electric flux through the surface of a sphere when its radius is doubled?
a. It is doubled. b. It is cut in half. c. It increases by a factor of three.
d. It increases by a factor of 4. e. It remains the same.

2. When a conductor is in electrostatic equilibrium, the electric field inside the conductor is
a. $k_e \frac{q}{r^2}$. b. zero. c. $\frac{q}{4\pi\varepsilon_o}$. d. $\frac{q_2}{r^2}$. e. $\frac{\varepsilon_o}{q}$.

3. A plane of charge of surface charge density σ lies in the *y*-*z* plane. The *x*-axis has its origin at the center of the plane. The correct graph of the electric field E_x versus distance *x* from the plane is



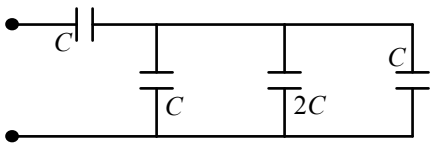
a. b. c.

d. e.

ANS:c

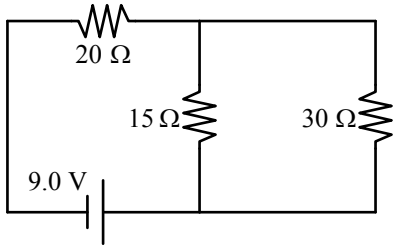
4. A point charge *Q* is placed on the *x*-axis at $x = 2.0$ m. A second point charge –*Q* is placed at $x = 3.0$ m. If $Q = 40\ \mu$C, what is the magnitude of the electrostatic force in N on a $30\ \mu$C charge placed at the origin?
($k_e = 8.99 \times 10^9$ Nm^2/C^2)
a. 7.2 b. 3.9 c. 1.5 d. 14 e. 8.1

5. Determine the equivalent capacitance in pF for the network shown when $C = 15$ pF.
a. 20
b. 16
c. 12
d. 24
e. 75



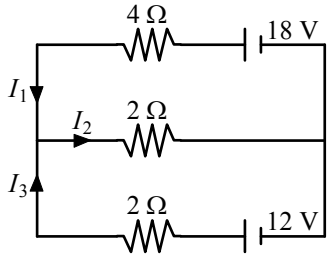
6. What is the current in A in the 15 Ω resistor?

a. 0.20
b. 0.30
c. 0.10
d. 0.26
e. 0.60



7. Given the circuit shown below, find I_1.

a. 1.2 A
b. 1.8 A
c. 2.4 A
d. 4.2 A
e. 4.8 A



PART II
In the exam booklet provide full solution to 5 out of following 6 problems
(each problem is worth 10%)

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1. Four charges of equal magnitude q =+1nC and are placed in the corners o a square of side a (a= 10cm) Find the resulting potential at the centre of the square

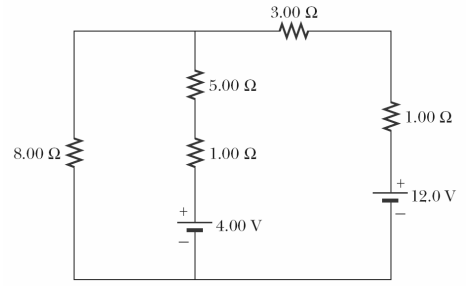
2 a) The charge of 100nC is placed in the geometrical centre of a regular tetrahedron of side a=4cm. Find the flux through one wall of the tetrahedron! (7p)

b) Charge q is placed on the tip of the cone of height L and radius of the base r
Find the expression for the flux through the base when L>>r (3P)

3 A thin wire has been bend into a circle of radius r=1m with a small 2cm gap in it. The system is charged with q=1C .
a) Find the Electric field at the centre of the circle (5)
b) Find the electric potential at the centre of the circle (5)

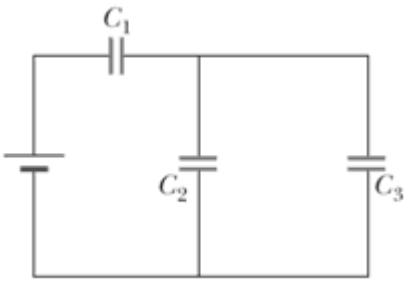
4 Find the potential at the centre of a charged box cage (L=H=W=1m) made out of 12 charged rods of equal and uniform charge Q on each of them (Q=100nF)

5 Determine the current in each branch of the circuit shown



6 Three capacitors are connected to a battery as shown in Figure P20.44. Their capacitances are $C_1 = 3C$, $C_2 = C$, and $C_3 = 5C$. (a) What is the equivalent capacitance of this set of capacitors? (b) State the ranking of the capacitors according to the charge they store, from largest to smallest. c) Rank the capacitors according to the potential differences across them, from largest to smallest. (d) If now C_3 is increased, what happens to the charge stored by each of the capacitors?

e) Obtain the expression for the three capacitors in series



NOTE: $k_e = 8.99 \times 10^9 \ \mathrm{N\,m^2/C^2}$

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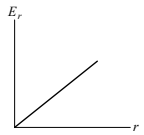
Part I Multiple choice >answer all best 6 count towards 48 % of the test grade

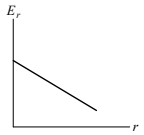
1. What happens to the net electric flux through the surface of a sphere when its radius is tripled doubled?
a. It remains the same.
b. It is tripled .
c. It is cut by factor of three .
d. It increases by a factor of nine .
e. It is cut by factor a factor of nine

2. When a conductor is in electrostatic equilibrium, the electric field inside the conductor is
a. $k_e \frac{q}{r^2}$. b. $\frac{q}{4\pi\varepsilon_o}$ c. zero. d. $\frac{q_2}{r^2}$. e. $\frac{\varepsilon_o}{q}$.

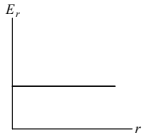
3. A radial line is drawn outward from the center of a solid conducting sphere. Charge +Q is placed on the sphere. The correct graph of E_r versus r is



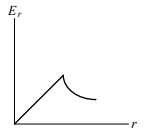
a.



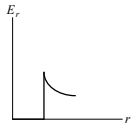
b.



c.



d.

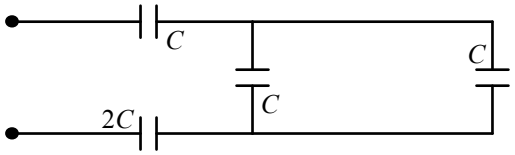


e.

ANS: e

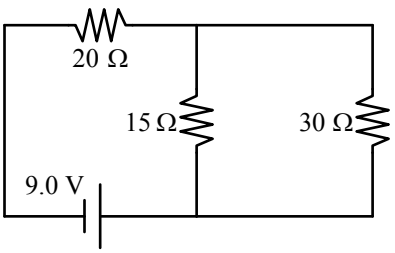
4. A point charge Q is placed on the x-axis at $x = -2.0$ m. A second point charge, $-Q$ is placed at $x = 1.0$ m. If $Q = 60\ \mu$C, what is the magnitude of the electrostatic force in N on a 40 μC charge placed at the origin? ($k_e = 8.99 \times 10^9$ N m^2/C^2)
a. 16 b. 27 c. 32 d. 11 e. 3.0

5. Determine the equivalent capacitance in pF for the network shown when $C = 12$ pF.

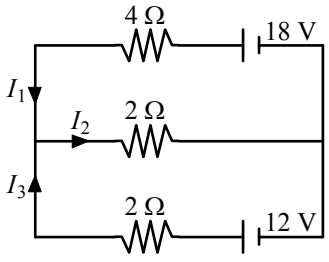


a. 48 b. 12 c. 24 d. 6.0 e. 59

6. What is the current in A in the 30 Ω resistor?
a. 0.20 b. 0.30
c. 0.10 d. 0.26
e. 0.60



7. Given the circuit shown below, find I_2.
a. 1.2 A
b. 1.8 A
c. 2.4 A
d. 4.2 A
e. 4.8 A



PART II

In the exam booklet provide full solution to 5 out of following 6 problems (each problem is worth 10%)

1. Four charges of equal magnitude q = + 2nC and are placed in the corners o a square of side a (a= 20cm) Find the resulting potential at the centre of the square

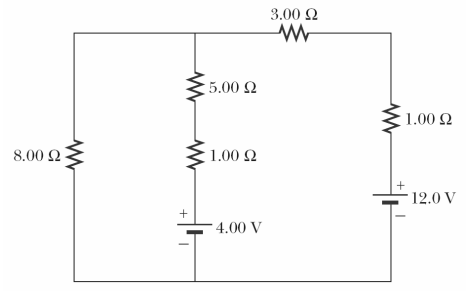
2 a) The charge of 50nC is placed in the geometrical centre of a regular tetrahedron of side a=3cm. Find the flux through one wall of the tetrahedron (7p)

b) Charge q is placed on the tip of the cone of height h and radius of the base R
Find the expression for the flux through the base when L>>R (3P)

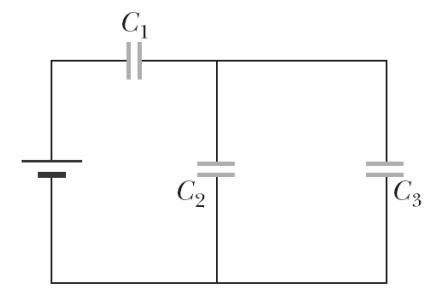
4 A thin wire has been bend into a circle of radius r=1m with a small 1cm gap in it. This system is charged with q=1C .
a) Find the electric potential at the centre of the circle (5)
b) Find the Electric field at the centre of the circle (5)

4 Find the potential at the centre of a charged box cage (L=H=W=1m) made out of 12 charged rods of equal and uniform charge Q on each of them (Q=200nF)

5 Determine the current in each branch of the circuit shown



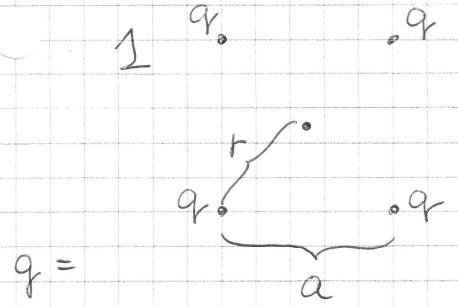
6 Three capacitors are connected to a battery as shown . Their capacitances are $C_1 = 2C$, $C_2 = 3C$, and $C_3 = C$.
(a) What is the equivalent capacitance of this set of capacitors?
(b) State the ranking of the capacitors according to the charge they store, from largest to smallest.
(c) Rank the capacitors according to the potential differences across them, from largest to smallest.
(d) If now C_3 is increased, what happens to the charge stored by each of the capacitors?
(e) Obtain the expression for the three capacitors connected in parallel



NOTE: $k_e = 8.99 \times 10^9 \ \mathrm{N\,m^2/C^2}$

Solutions to long problems

1



$V_1 = k_e \frac{q}{r}$ ← potential resulting from a single charge

$q =$

$a =$

$V_4 = 4 V_1$ ← all charges are the same and at equal distances from the centre.

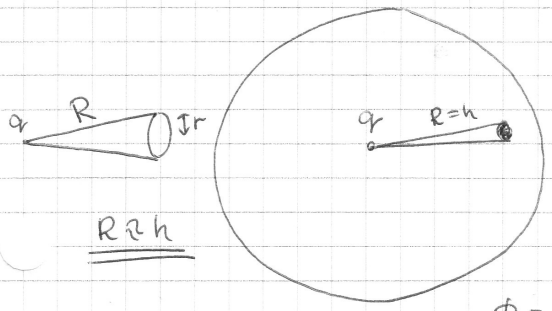
$$V_4 = 4\ k_e \frac{q}{r} = 4\ k_e \frac{q}{\frac{\sqrt{2}}{2} a}$$

$$V_4 = 4 \frac{2}{\sqrt{2}} k_e \frac{q}{a} = \frac{8}{\sqrt{2}} \frac{k_e q}{a}$$

ANS

2 a) $\Phi_{\text{single wall}} = \frac{1}{4} \Phi_{\text{total}} = \frac{1}{4} \frac{q}{\varepsilon_0}$

b) $\Phi_{\text{SPHERE}} = \frac{q_e}{\varepsilon_0}$



$R \geq h$

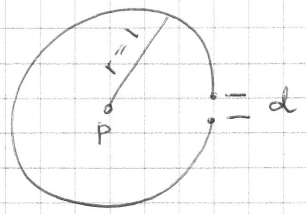
We build a sphere of the radius $R = h$

Flux through the base of a cone is a fraction of the flux through the whole sphere!

$$\Phi = \frac{\pi r^2}{4 \pi R^2} \frac{q}{\varepsilon_0} = \frac{r^2 q}{4 R^2 \varepsilon_0}$$

3

$$\lambda = \frac{q}{l} = \frac{q}{(2\pi r - d)} =$$

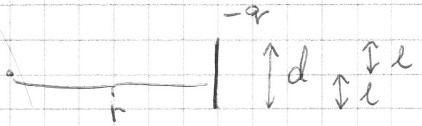


$E_p = ?$

We can close the gap in this circle to simplify our problem.

$$\bigcirc \equiv \bigcirc + \bullet\,(-q')$$

$$\vec{E}_p = \underbrace{\vec{E}''_{full}}_{0} + \underbrace{\vec{E}''_{gap}}$$



$$E_x = 2k_e \lambda \frac{1}{y\sqrt{l^2+y^2}} \leftarrow \boxed{\text{from the summary table}}$$

in our case $y = r$

$2l = d$

Ans a) $E_x = 2k_e \lambda \frac{1}{r\sqrt{\left(\frac{d}{2}\right)^2 + r^2}} =$

Ans b)

$$V_p = \underbrace{V_{\text{full circle}}}_{\#} - \underbrace{V_{\text{gap}}}_{''}$$

$$\#\ 0$$

$$V_{\text{full circle}} = 2\pi k_e \lambda = k_e \frac{q}{r}$$

$$V_{\text{gap}} = 2 k_e \lambda \ln \frac{\sqrt{\left(\frac{d}{2}\right)^2 + r^2}}{r}$$

Ans b) $V_p = k \frac{q}{r} - 2 k_e \lambda \ln \frac{\sqrt{\frac{d}{2} + r^2}}{r}$

4 The cage is build of 12 identical rods

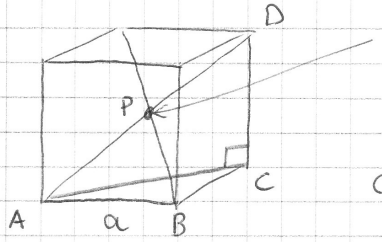
therefore $V_{total} = 12\, V_{single\ rod}$

to use our formula

$$V = 2k_e\lambda \ln\left(\frac{\sqrt{l^2+y^2}+l}{y}\right)$$

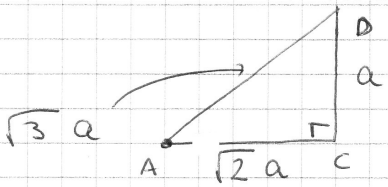
we need to establish y - distance from the point P to the centre of the rod.

It is a geometry problem!



point P is on intersection of two diagonals of this cube

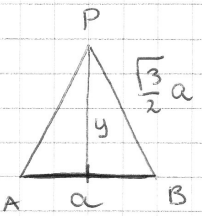
consider triangle A C D



$|AD| = \sqrt{3}\,a$

$|AP| = \frac{1}{2}\sqrt{3}\,a$

Now consider triangle A B P



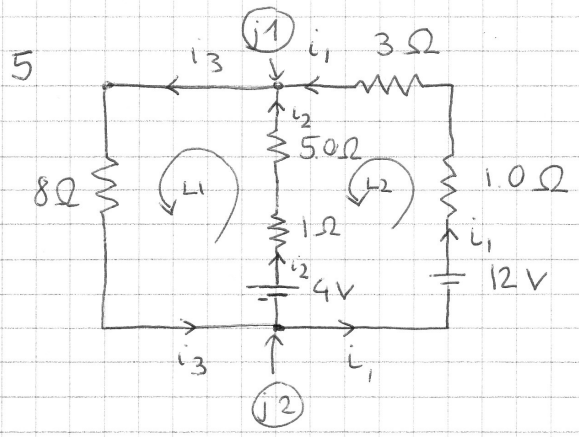
$$y^2 = -\left(\frac{1}{2}a\right)^2 + \left(\frac{\sqrt{3}}{2}a\right)^2$$

$$y^2 = -\frac{1}{4}a^2 + \frac{3}{4}a^2 = \frac{1}{2}a^2$$

$$y = \frac{\sqrt{2}}{2}a$$

$$V = 12 \cdot 2k_e\lambda \ln \frac{\sqrt{\frac{1}{2}a^2+\frac{1}{4}d^2}+\frac{1}{2}d}{\frac{\sqrt{3}}{2}a}$$

5



j1 junction 1

$i_1 + i_2 = i_3$ (1)

L1 : $-4 - 1i_2 - 5i_2 - 8i_3 = 0$

L2 : $-12 - 1i_1 - 3i_1 + 5i_2 + 1i_2 + 4 = 0$

$$\left.\begin{array}{r} i_1 + i_2 = i_3 \\ 4 + 6i_2 + 8i_3 = 0 \\ -8 - 4i_1 + 6i_2 = 0 \end{array}\right\} \quad \left.\begin{array}{r} i_1 + i_2 = i_3 \\ 4 + 6i_2 + 8i_1 + 8i_2 = 0 \\ -8 - 4i_1 + 6i_2 = 0 \end{array}\right\}$$

$$\left.\begin{array}{r} 4 + 14i_2 + 8i_1 = 0 \\ -8 + 6i_2 - 4i_1 = 0 \;/(\times 2) \end{array}\right\} \Rightarrow \left.\begin{array}{r} 4 + 14i_2 + 8i_1 = 0 \\ -16 + 12i_2 - 8i_1 = 0 \end{array}\right\}$$

add $-12 + 26i_2 + 0 = 0$

$26i_2 = 12$

$i_2 = \frac{6}{13}$

$4 + 14\frac{6}{13} + 8i_1 = 0$

$\frac{4\times 13 + 14\times 6}{13} + 8i_1 = 0$

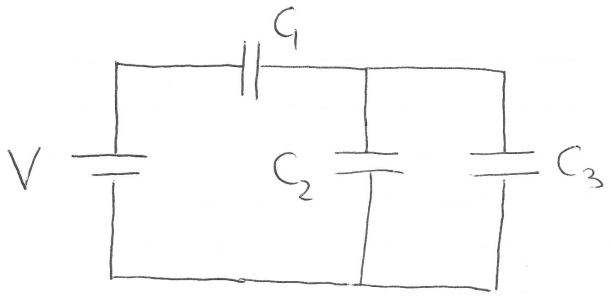
$$i_2 = \frac{6}{13} \qquad i_1 = -\frac{1}{8}\,\frac{116}{13} = -\frac{29}{26}$$

$$i_3 = i_1 + i_2 = \frac{6}{13} - \frac{29}{26} = \frac{12-29}{26} = -\frac{17}{26}$$

ANS $\quad i_1 = -\frac{29}{26} \qquad i_2 = \frac{12}{26} \qquad i_3 = -\frac{17}{26}$

negative signs mean that the currents flow in the direction opposite to the one originally presumed

6



$C_1 = 3C$
$C_2 = C$
$C_3 = 5C$

C_3 and C_2 are connected in parallel

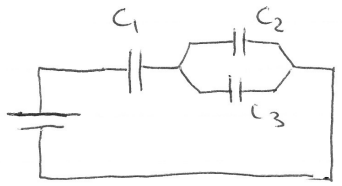
$$C_E' = C_2 + C_3 = 6C$$

C_1 is connected with $6C$ in series

$$C_{EQ}^{-1} = \left(\frac{1}{3C} + \frac{1}{6C}\right) = \left(\frac{2+1}{6C}\right)$$

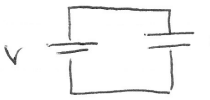
$$C_{EQ} = \frac{6}{3}C = 2C$$

b)



We will find V_1, V_2, V_3 and Q_1, Q_2, Q_3 on each capacitor!

Charge on the whole circuit is given by



$$C_{EQ} = \frac{Q_{tot}}{V}$$

$$Q_{tot} = C_{EQ} V = 2VC$$

this will be the charge on C_1 !

Q_1 on C_1 is $Q_1 = 2VC$

V_1 on C_1 is given be $C_1 = \frac{Q_1}{V_1} \rightarrow V_1 = \frac{Q_1}{C_1}$

$$V_1 = \frac{2CV}{3C} = \frac{2}{3}V$$

V_2 on C_2 and C_3 is identical (parallel connection

so

$$V_2 = V_3 = \frac{1}{3}V$$

Q_2 on C_2 is $Q_2 = C_2 V_2 = C \cdot \frac{1}{3}V = \frac{1}{3}CV$

Q_3 on C_3 is $Q_3 = C_3 V_3 = 5C \cdot \frac{1}{3}V = \frac{5}{3}CV$

AWS

$$\begin{aligned} C_1 &= 3C \rightarrow Q_1 = 2VC \; ; \; V_1 = \tfrac{2}{3}V \\ C_2 &= C \rightarrow Q_2 = \tfrac{1}{3}VC \; ; \; V_2 = \tfrac{1}{3}V \\ C_3 &= 5C \rightarrow Q_3 = \tfrac{5}{3}VC \; ; \; V_3 = \tfrac{1}{3}V \end{aligned}$$

VERSION B:

$C_1 = 2C$
$C_2 = 3C$
$C_3 = C$

$C_E^{\ 1} = C_2 + C_3 = 4C$

$C_{EQ}^{-1} = \frac{1}{4C} + \frac{1}{2C} = \frac{1}{4C} + \frac{2}{4C} = \frac{3}{4C}$

$C_{EQ} = \frac{4}{3}C$

$\frac{Q_{tot}}{V} = C_{EQ} \Big| \rightarrow Q_{tot} = C_{EQ} V = \frac{4}{3}CV$

on C_1 $\quad Q_1 = \frac{4}{3}CV \rightarrow V_1 = \frac{\frac{4}{3}CV}{2C} = \frac{2}{3}V$

on C_2 $\quad V_2 = \frac{1}{3}V \rightarrow Q_2 = (3C)\frac{1}{3}V = CV$

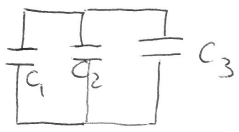
on C_3 $\quad V_3 = \frac{1}{3}V \rightarrow Q_3 = (C)\frac{1}{3}V = \frac{1}{3}CV$

ANS:

$C_1 = 2C$	$V_1 = \frac{2}{3}V$	$Q_1 = \frac{4}{3}CV$
$C_2 = 3C$	$V_2 = \frac{1}{3}V$	$Q_2 = CV$
$C_3 = C$	$V_3 = \frac{1}{3}V$	$Q_3 = \frac{1}{3}CV$

d) If C_3 is increased the charge on it will increase while the charge on the C_2 will decrease, charge on C_1 will not be affected by this change!

e) parallel



C_1 V_1 Q_1
C_2 V_2 Q_2
C_3 V_3 Q_3

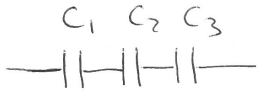
$$C_{EQ} = \frac{Q_{tot}}{V_{tot}} \qquad Q_1 + Q_2 + Q_3 = Q_{tot}$$

$$V_1 = V_2 = V_3 = V_{tot}$$

$$C_{EQ} = \frac{Q_1 + Q_2 + Q_3}{V} = \frac{Q_1}{V} + \frac{Q_2}{V} + \frac{Q_3}{V} = C_1 + C_2 + C_3$$

$$\underline{C_{EQ} = C_1 + C_2 + C_3}$$

series



C_1 V_1 Q_1
C_2 V_2 Q_2
C_3 V_3 Q_3

$$C_{EQ} = \frac{Q_{tot}}{V_{tot}} = \frac{Q}{V_1 + V_2 + V_3} \qquad Q_1 = Q_2 = Q_3 = Q_{tot} = Q$$

$$V_1 + V_2 + V_3 = V_{tot}$$

$$\frac{1}{C_{EQ}} = \frac{V_1 + V_2 + V_3}{Q} = \frac{V_1}{Q} + \frac{V_2}{Q} + \frac{V_3}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\underline{\frac{1}{C_{EQ}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$