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**Assignment 4 Part II**

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**Note: Your submission will consist of two steps. First, use the answer-area to provide a statement to the TA alerting them that a PDF document containing your answers to this assignment's Part II questions has been submitted (via document sharing discussed next). Second, use the shared-documents tool within MyStatLab to upload your PDF document. Do not forget to include a statement of academic integrity within the statement that you provide in the answer area to part II. Finally, note that you are required to show your work for full credit --- correct numeric answers may earn you little credit unless you show your work.**

**A4-II marked out of a total of 20 marks. Please note that Mystatlab answer box indicates that A4 has just one question. While it in fact has two questions, Mystatlab will only dedicate one answer-box for the whole assignment. You are responsible for answering both questions in your submitted PDF.**

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**Question 1 [12 Marks]**: Ontario politicians have recently discussed a proposal to raise the tax on Ontario residents who are labeled "super-rich" (earn $500,000/year or more in taxable income). The NDP assume that the probability model for the earnings among the super-rich has an expected value of $750,000/yr and a standard deviation of $800,000/yr.

(a) Consider the event that a randomly chosen person from the super-rich population has earnings in excess of $800,000/yr. How precisely can you determine the probability of this event? Briefly explain your answer [1 marks].
To correctly determine the probability of this event, a probability model is needed to approximate with. Seeing as we are not given a model, we cannot calculate the probability correctly and therefore precisely.

The NDP propose a trial tax-increase program. Under this trial the names of all super-rich Ontario residents are placed in a barrel, and a random sample of 1200 of them is selected (assume there are a total of 6000 super-rich Ontario residents) for taxation at the higher rate. The tax-income earned from this program will depend on the total income earned by the 1200 sampled residents. (**note:** that if the sample consisted of just two individuals, with earnings of say $900,000 and $630,000 then the total income would be $1,530,000)

(b) The NDP recognize that this total income is a random variable and have asked you to determine the following for that random variable (state any assumptions when obtaining an answer) [5 marks]
   i. Expected value:
   Seeing as 750,000 is the expected value for a single person. With the assumption that you need to find the expected value of the total sample size of 1200 in terms of income; the expected value would be equal to:
   n = 1200

   p = 750,000

   E(Y) = n*p

   = 1200*750,000

   = $900,000,000

ii. Standard deviation

Around standard deviation we must first make the assumption that we are sampling without replacement. With the standard deviation given for a single person, 800,000, it is assumed that SD multiplied by 1200 then squared rooted would be the SD for the whole population. However, since the population size is greater than 10% we must apply FPCF.

SD = $\sqrt{n\sigma_x}$FPCF

= $\sqrt{(1200 \times 800,000)}\sqrt{\left(\frac{N-n}{N-1}\right)}$

= 30, 983.87 $\sqrt{\left(\frac{6000-1200}{6000-1}\right)}$

= 30, 983.87 × 0.895

= **27, 730.56**

iii. Inter-quartile range

IQR = Q3 – Q1

Z scores: Q1 → 25% = -0.67
Q3 → 75% = 0.68

Q1 → -0.67 = $\frac{x - 900,000,000}{27,730.56}$

-18,579.48 = x – 900,000,000

899,981,420.52 = x

Q3 → 0.67 = $\frac{x - 900,000,000}{27,730.56}$

18,879.48 = x – 900,000,000

900,018,879.48 = x

IQR = 900,018,879.48 – 899,981,420.52

= $37,458.96

(c) The Progressive Conservatives claim the average of $750,000/year (as the expected value for the earnings among Ontario super-rich) is an exaggeration, and have bet the NDP leader that the average annual earnings (per person) obtained from a sample of 1200 super-rich will be $700,000 or less. Assuming that the NDP are in fact correct, what is the probability that the NDP will lose such a bet? [4 marks]

The equation to be used for this is:

SD $\vec{x}$ = (SDx /√ n)*FDCF

= (800,000/√1200)*0.895

= 23,094.67*0.895

= 20,669.73

$$Z = \frac{700{,}000 - 750{,}000}{SD\ \vec{x}}$$

$$Z = \frac{700{,}000 - 750{,}000}{20{,}669.73}$$

Z = - 2.42

Probability = Pr[≤ 700,00] = 0.0078 (as found on x score table)
Therefore giving the probability of 0.78% they are right.

(d) How would your answer to part (c) be changed if the sample size were doubled? (Solution should recalculate and identify why (if at all) the answer changed.) [2 marks]

SD $\vec{x}$ = (800,000/√2400)*0.895

= 16,329.86*0.895

$$Z = \frac{700{,}000 - 750{,}000}{SD\vec{x}}$$

$$Z = \frac{700{,}000 - 750{,}000}{14{,}615.22}$$

= - 3.42

In z-score, new probability = 0.0003

**QUESTION 2 [8 marks]**: Endure Inc., supplies batteries to cell-phone producers. You have been contracted to test this supplier's claim that its batteries last for an average of 35 hours. As a preliminary test, you have persuaded 5 of your classmates to use the batteries in a standard cell-phone, and have asked them to record how long the batteries last (from fully charged to "no charge"). Based on the literature you can assume that these duration times would follow a normal model. The times recorded by your classmates are as follows: 35,32,34,33,31 (hours)

Calculate the following for this data-set

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(a) The sample mean

To find mean you must add all values and divided by size of sample.

$$\text{Mean} = \frac{35 + 32 + 34 + 33 + 31}{5}$$
$$= 33$$

(b) The sample variance

To find variance you must find the variance of each value in the sample set and add them up and divide by 4.

$$\text{Var}(x) = [(35-33)^2 + (32-33)^2 + (34-33)^2 + (33-33)^2 + (31-33)^2]/(n-1)$$
$$= [4 + 1 + 1 + 0 + 4]/4$$
$$= 10/4$$
$$= 2.5$$

(c) If Endure's claim is correct, what is the probability of obtaining a sample mean estimate as low (or lower) than the one you have found? You must show calculations to support your conclusion.

$Sx = \sqrt{\text{Var}(x)}$
$= \sqrt{2.5}$
$= 1.58$

$SE(\vec{x}) = Sx/\sqrt{n}$
$= 1.58/\sqrt{5}$
$= 0.71$

$df = n-1$
$=5-1$
$=4$

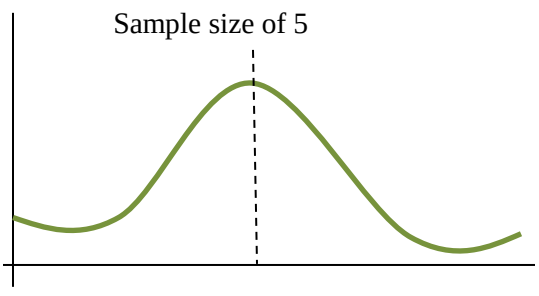
$t = \frac{(\bar{X} - \mu)}{s/\sqrt{n}}$
$= (33-35)/0.71$
$= -2.817$

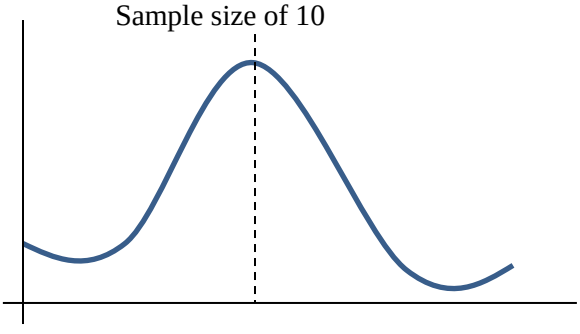
***Pr [$\bar{X} \leq 35$] ≈ 0.025***

Therefore, by looking the value of -2.817 in the t table it is about 0.025; making the probability of the getting sample mean estimate that is as low or lower than the one previously founf is approximately 2.5%

(d) Provide a **rough** sketch the sampling distribution for xbar for this case, and for the case where the size of the sample is twice as large.

As concluded in class as the sample size grows it becomes closer to a normal model otherwise known as a bell curve.



**Personal Ethics Statement**

**Individual Assignment:**

By signing this Statement, I am attesting to the fact that I have reviewed the entirety of my attached work and that I have applied all the appropriate rules of quotation and referencing in use at the Telfer School of Management at the University of Ottawa, as well as adhered to the fraud policies outlined in the Academic Regulations in the University's Undergraduate Studies Calendar. I further attest that I have knowledge of and have respected the "Beware of Plagiarism" brochure found on the Telfer School of Management's doc-depot site.

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| *Helena Devins* | April 5, 2012 |
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