Assignment 2 Part II

Note: Your submission will consist of two steps. First, use the answer-box to provide a statement to the TA alerting them that a PDF document containing your answers to this assignment's Part II questions has been submitted (document sharing). Second, use the shared-documents tool within MyStatLab to upload your PDF document. Do not forget to include a statement of academic integrity within the statement that you provide in the answer area to part II. Finally, note that you are required to show your work for full credit --- correct numeric answers may earn you little credit unless you show your work. [DUE DATE Feb 18th]

QUESTION 1 [20 marks]

Your sister owns and operates a restaurant called Quiet Waters (QW) and seeks your help on Human Resource issues.

Depending on a number of factors the wait-staff requirements (number of waitresses and/or waiters) at a QW restaurant can be either ***low*** (4 people), or ***moderate*** (7 people), or ***peak*** (12 people).

QW has two wage levels for wait-staff: $20.5/hr for experienced staff; and $10.5/hr for inexperienced staff (less than two years experience).

Suppose that the flagship QW restaurant has a large pool of staff (1/5th of which are experienced), and that each person in this pool has an equal chance of being assigned a shift (regardless of wage level). Based on this information answer the following (**make sure to report units where appropriate**):

(a) What is the probability model for the number of experienced staff assuming a ***low*** staffing level (i.e., 4 wait staff used)? Specify the model and tabulate the possibilities and probabilities associated with that model **[2 mark].**

Binomial. P[X=x] = nCx*px*$q^{(n-x)}$

n = # of wait staff = 4 people

p = probability of experienced works → 1/5 = 0.2

q = probability of inexperienced workers → 1-p = 0.8

x = # of experienced wait staff = 0,1,2,3,4

Therefore the model would be; P[X=0] =4C0(0.2^0)(0.8^4) = 0.4096 chance of experienced worker

P[X=1]=4C1(0.2^1)(0.8^3) = 0.4096 chance of experienced worker

P[X=2]=4C2(0.2^2)(0.8^2) = 0.1536 chance of experienced worker

P[X=3]=4C3(0.2^3)(0.8^1) = 0.0256 chance of experienced worker

P[X=4]=4C4(0.2^4)(0.8^0) = 0.0016 chance of experienced worker

(b) What is the probability model for the total per hour wage assuming a ***low*** staffing level (i.e., 4 wait staff used) **[2 marks]**;

X	P[X=x]	Total Wage
(0*20.5) + (4*10.5) = 42	0.4096	$17.2032
(1*20.5) + (3*10.5) = 52	0.4096	$21.2992
(2*20.5) + (2*10.5) = 62	0.1536	$9.5232
(3*20.5) + (1*10.5) = 72	0.0256	$1.8432
(4*20.5) + (0*10.5) = 82	0.0016	$0.1312

(c) Compute the following summaries for the probability model for total wage described in part (b) above (***low*** case, only) **[3 marks]**

i. **Expected value**

E(x) = 17.2032 + 21.2992 + 9.5232 + 1.8432 + 0.1312
= $50

ii. **Variance, and**

Var(x) = $(42-50)^2(0.4096) + (52-50)^2(0.4096) + (62-50)^2(0.1536) + (72-50)^2(0.0256) + (82-50)^2(0.0016)$
= $64

iii. **Coefficient of variation**;

Therefore CV = E(x)/SD → SD = SQRT(Var(x))
CV = 50/8
= $6.25

(d) Compute the following (total wage) summaries for ***moderate(7)*** and ***peak(12)*** cases as well **[3 marks]**.

Moderate

X	P[X=x]	Total Wage ($)
(0*20.5) + (7*10.5) = 73.5	P[X=0] =7C0$(0.2^0)(0.8^7)$ = 0.2097	15.41295
(1*20.5) + (6*10.5) = 83.5	P[X=1] =7C1$(0.2^1)(0.8^6)$ = 0.367	30.6445
(2*20.5) + (5*10.5) = 93.5	P[X=2] =7C2$(0.2^2)(0.8^5)$ = 0.2753	25.74055
(3*20.5) + (4*10.5) = 103.5	P[X=3] =7C3$(0.2^3)(0.8^4)$ = 0.1147	11.87145
(4*20.5) + (3*10.5) = 113.5	P[X=4] =7C4$(0.2^4)(0.8^3)$ = 0.0287	3.25745
(5*20.5) + (2*10.5) = 123.5	P[X=5] =7C5$(0.2^5)(0.8^2)$ = 0.0043	0.53105
(6*20.5) + (1*10.5) = 133.5	P[X=6] =7C6$(0.2^6)(0.8^1)$ = 0.00036	0.04806
(7*20.5) + (0*10.5) = 143.5	P[X=7] =7C7$(0.2^7)(0.8^0)$ = 0.00000128	0.00018368

i. **Expected value**

E(x) = $87.5062

ii. **Variance,**

Var(x) = $(73.5-87.5)^2(0.2097) + (83.5-87.5)^2(0.367) + (93.5-87.5)^2(0.2753) + (103.5-87.5)^2(0.1147) + (113.5-87.5)^2(0.0287) + (123.5-87.5)^2(0.0043) + (133.5-87.5)^2(0.00036) + (143.5-87.5)^2(0.00000128)$
= 41.1012 + 5.872 + 9.9108 + 29.3632 + 19.4012 + 5.5728 + 0.76176 + 0.004014
= $\$^2$111.987

iii. **Coefficient of variation**;

CV = 87.5062/10.5824
= 8.269

Peak

X	P[X=x]	Total Wage (
(0*20.5) + (12*10.5) = 126	P[X=0] =12C0(0.2^0)(0.8^{12}) = 0.0687	8.6562
(1*20.5) + (11*10.5) = 136	P[X=0] =12C1(0.2^1)(0.8^{11}) = 0.0172	2.3392
(2*20.5) + (10*10.5) = 146	P[X=0] =12C2(0.2^2)(0.8^{10}) = 0.1074	15.6804
(3*20.5) + (9*10.5) = 156	P[X=0] =12C3(0.2^3)(0.8^9) = 0.2362	36.8472
(4*20.5) + (8*10.5) = 166	P[X=0] =12C4(0.2^4)(0.8^8) = 0.1329	22.0614
(5*20.5) + (7*10.5) = 176	P[X=0] =12C5(0.2^5)(0.8^7) = 0.0532	9.3632
(6*20.5) + (6*10.5) = 186	P[X=0] =12C6(0.2^6)(0.8^6) = 0.0155	2.883
(7*20.5) + (5*10.5) = 196	P[X=0] =12C7(0.2^7)(0.8^5) = 0.0033	0.6468
(8*20.5) + (4*10.5) = 206	P[X=0] =12C8(0.2^8)(0.8^4) = 0.0005	0.103
(9*20.5) + (3*10.5) = 216	P[X=0] =12C9(0.2^9)(0.8^3) = 0.00006	0.01296
(10*20.5) + (2*10.5) = 226	P[X=0] =12C10(0.2^{10})(0.8^2) = 0.000004	0.000904
(11*20.5) + (1*10.5) = 236	P[X=0] =12C11(0.2^{11})(0.8^1) = 0.0000002	0.0000472
(12*20.5) + (0*10.5) = 246	P[X=0] =12C12(0.2^{12})(0.8^0) = 0.000000004	0.000000984

i. **Expected value**

E(x) = $98.5943

ii. **Variance,**

Var(x) = (126-98.59)^2(0.06562) + (136-98.59)^2(0.0172) + (146-98.59)^2(0.1074) + (156-98.59)^2(0.2362) + (166-98.59)^2(0.1329) + (176-98.59)^2(0.0532) + (186-98.59)^2(0.0155) + (196-98.59)^2(0.0033) + (206-98.59)^2(0.0005) + (216-98.59)^2(0.00006) + (226-98.59)^2(0.000004) + (236-98.59)^2(0.0000002) + (246-98.59)^2(0.000000004)

= 49.30 + 24.07 + 241.40 + 778.49 + 603.91 + 318.79 + 118.43 + 31.31 + 5.77 + 0.827 + 0.065 + 0.004 + 0.00009

= $\$^2$2172.366

iii. **Coefficient of variation**;

CV = 98.5943/46.6086

= 2.1154

(e) Your sister examined staffing patterns across a large, and equal, number of **low** (4 staff) versus **peak** (12 staff) shifts and noticed something interesting. She has been particularly interested in shifts that have no experienced staff on the floor; referred to as "fragile shifts" because the staff (inexperienced) is less able to react to challenges--- for a low-shift, a fragile-shift refers to the event of having 0 experienced workers out of 4, whereas for a peak-shift a fragile shift refers to the event of having 0 experienced workers out of 12. Your sister notes that fragile shifts happen roughly 6 times (see her graph below) more often on low-shifts than on peak-shifts. Noting that the junior manager is always in charge on "low-shifts" your sister suspects that the junior-manager may be to blame for the excess "fragile-shifts" --- she speculates that experienced staff avoid working when they know the junior manager will be in charge. Provide an alternative explanation, possibly exploiting some calculations and/or information from above **[4 marks]**

(f) How would your answers to question (d) change if the wage rates for wait-staff were increased by 25 percent (multiplied by 1.25). Provide numeric answer for the **moderate** case. *Be sure to address expected value, variance and coefficient of variation.* **[3 marks].**

20.5*1.25 = 25.625 and 10.5*1.25 = 13.125

X	P[X=x]	Total Wage ($)
(0*25.625) + (7*13.125) = 91.875	P[X=0] =7C0(0.2^0)(0.8^7) = 0.2097	19.2661875
(1*25.625) + (6*13.125) = 104.375	P[X=1] =7C1(0.2^1)(0.8^6) = 0.367	38.305625
(2*25.625) + (5*13.125) = 116.875	P[X=2] =7C2(0.2^2)(0.8^5) = 0.2753	32.1756875
(3*25.625) + (4*13.125) = 129.375	P[X=3] =7C3(0.2^3)(0.8^4) = 0.1147	14.8393125
(4*25.625) + (3*13.125) = 141.875	P[X=4] =7C4(0.2^4)(0.8^3) = 0.0287	4.0718125
(5*25.625) + (2*13.125) = 154.375	P[X=5] =7C5(0.2^5)(0.8^2) = 0.0043	0.6638125
(6*25.625) + (1*13.125) = 166.875	P[X=6] =7C6(0.2^6)(0.8^1) = 0.00036	0.600075
(7*25.625) + (0*13.125) = 179.375	P[X=7] =7C7(0.2^7)(0.8^0) = 0.00000128	0.0002296

i. **Expected value**

E(x) = $109.9227/hour

ii. **Variance,**

Var(x) = $(91.875-109.9)^2$(0.2097) + $(104.375-109.9)^2$(0.367) + $(116.875-109.9)^2$(0.2753) + $(129.375-109.9)^2$(0.1147) + $(141.875-109.9)^2$(0.0287) + $(154.375-109.9)^2$(0.0043) + $(166.875-109.9)^2$(0.00036) + $(179.375-109.9)^2$(0.00000128)

= 68.1317 + 11.2029 + 13.3935 + 43.5029 + 29.3429 + 8.5055 + 1.1686 + 0.0618

= $2145.9669

iii. **Coefficient of variation**;

CV = 109.9227/12.0817

= 9.0983

(g) How would your answers to question (d) change if you were instructed to include the manager's hourly wage (manager has a fixed hourly rate of $32/hour).
Provide numeric answer for the **moderate** case. *Be sure to address expected value, variance and coefficient of variation.* **[3 marks]. Assuming 1 manager per shift**

X	P[X=x]	Total Wage ($)
(0*25.625) + (7*13.125) + 32 = 123.875	P[X=0] =7C0(0.2^0)(0.8^7) = 0.2097	25.9765875
(1*25.625) + (6*13.125) + 32 = 136.375	P[X=1] =7C1(0.2^1)(0.8^6) = 0.367	50.049625
(2*25.625) + (5*13.125) + 32 = 148.875	P[X=2] =7C2(0.2^2)(0.8^5) = 0.2753	40.9852875
(3*25.625) + (4*13.125) + 32 = 161.375	P[X=3] =7C3(0.2^3)(0.8^4) = 0.1147	18.5097125
(4*25.625) + (3*13.125) + 32 = 173.875	P[X=4] =7C4(0.2^4)(0.8^3) = 0.0287	4.9902125
(5*25.625) + (2*13.125) + 32 = 186.375	P[X=5] =7C5(0.2^5)(0.8^2) = 0.0043	0.8014125
(6*25.625) + (1*13.125) + 32 = 198.875	P[X=6] =7C6(0.2^6)(0.8^1) = 0.00036	0.0715955
(7*25.625) + (0*13.125) + 32 = 211.375	P[X=7] =7C7(0.2^7)(0.8^0) = 0.00000128	0.00027056

i. **Expected value**

$E(x) = \$141.3847/hour$

ii. **Variance,**

$Var(x) = (123.875-141.4)^2(0.2097) + (136.375-141.4)^2(0.367) + (148.875-141.4)^2(0.2753) + (161.375-141.4)^2(0.1147) + (173.875-141.4)^2(0.0287) + (186.375-141.4)^2(0.0043) + (198.875-141.4)^2(0.00036) + (211.375-141.4)^2(0.00000128)$

$= 64.40 + 9.267 + 7.475 + 45.7654 + 30.2678 + 8.6978 + 1.1892 + 0.0063$

$= \$^2 167.0685$

iii. **Coefficient of variation**;

$CV = 141.3847/12.9255$

$= 10.9384$