PHYS 1003 – MECHANICS AND THERMODYNAMICS

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FORMAL LAB: Spring Constant

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PURPOSE

The purpose of this lab is to find the spring constant through static and dynamic methods using a little equation manipulation, a spring and some precisely measured mass. Then using a comparison test to see how consistent the two values are and whether they are within error.

APPARATUS

- One spring.
- One set of masses, 10g, 20g, and 60g.
- One precise weigh scale to measure the masses.
- One meter stick.
- One clock for timing, in this case: An iphone timer.

PROCEDURE & THEORY

Theory:

PART A: STATIC

The theory of the static method in this lab is that, given the force of gravity and the distance a spring stretches when a precise amount of mass is added to it you can find a relationship which will result in the force the spring exerts times the distance moved. This value, according to Hooke's law of F = -kx, will actually be equal to the static spring constant.

PART B: DYNAMIC

The theory of the dynamic method in this lab is that, given the period of oscillation and the precise amount of mass added to the spring, you can find a relationship which results with a value of mass over the period squared. This is then multiplied by a distance of oscillation. This is also known as distance over time squared which is the same units found within acceleration multiplied with a mass. This value gives us the dynamic force of the spring which in step gives us the dynamic spring constant.

Part A:

- The apparatus was setup and the zero position of the spring was taken, including the pan and its weight.
- Measurements were taken at 100g intervals between 100 and 600g.
- A graph was drawn and its slope calculated. The error on the slope was also calculated and recorded.
- Using gravity and the slope from the graph, the static spring constant was calculated and the error on k_s (static constant) also calculated.

Part B:

- For the dynamic method, the period of oscillation were observed in 5 trials for each increment of 20g between 600 and 700g.
- A T^2 vs M graph was plotted.
- The errors on mass, time, average time, period, and period squared were all calculated.
- The slope and the error on the slope were calculated and recorded.
- The dynamic spring constant, k_d, and its error were calculated and recorded.
- The comparison test was used to observe that the results were consistent and within error.

RESULTS

Refer to the appendices for results.

DISCUSSION

The values for the static and dynamic spring constants were expected to be relatively the same. A constant, by definition, does not change whether the system is still or in motion. The spring constant of a specific spring will always remain the same, unless there is damage to the spring. However there could be a small discrepancy between the static and dynamic constants. This discrepancy is assumed to be a result of the spring stretching out during the more rigorous testing and not constricting back to its original state, losing some of its elasticity. The static method should be more accurate because it requires a less stressing test; as a result there is less room for spring damage or permanent stretching. The precisions of both tests could be greatly removed by using a much higher quality of spring that has a no history of stretching and not returning to its previous state. It could also be greatly improved using motion sensors and a slow motion camera to get extremely accurate values. At the culmination of this lab the comparison test was carried out and the values were shown to be only slightly different; they passed the comparison test with 0.258 < 2. This means they were very consistent and within error. The error here could be determined to be due to human reaction time errors.

CONCLUSIONS

This lab proved to be a very useful method in determining the spring constant of a particular spring using only precisely measured mass. The static and dynamic spring constants were calculated and determined to be very consistent and far within error. This makes the lab a very strong method for determining a spring constant.

STATIC METHOD: PART A

TABLE 1:

Starting position: +11.6cm

Mass (g)	Starting Position (cm)	Ending Position (cm)	ΔX (cm)
100g	11.6	18.1	6.5
200g	18.1	24.6	6.5
300g	24.6	30.1	5.5
400g	30.1	37.4	7.3
500g	37.4	43.8	6.4
600g	43.8	50.2	6.4

$Slope = 0.06440\ \frac{cm}{g}$ Taken from the weighted regression table.

$$= 0.06440\ \frac{\cancel{cm}}{\cancel{g}} * \frac{1m}{100\cancel{cm}} * \frac{1000\cancel{g}}{1kg}$$

$$= 0.6440\ \frac{m}{kg}$$

$$k_s = \frac{9.81\ N}{6.44E^{-1}\ m/kg}$$

$$= \frac{9.81}{0.644}\ \frac{N}{m}$$

$$= 15.23\ \frac{N}{m}$$

$\sigma k_s = \frac{9.81\ N}{0.644^2} * 6.57E^{-5}$ σm = $6.57E^{-5}$ Taken from the weighted regression table.

$$\sigma k_s = 1.55E^{-3}$$

DYNAMIC METHOD: PART B

TABLE 2:

m +/- σm (g)	t +/- σt (s)	t_{av} +/- $σt_{av}$ (s)	T +/- σT (s)	T^2 +/- $σT^2$ (s^2)
600 +/- 1	12.70 +/- 0.4 13.25 +/- 0.4 12.93 +/- 0.4 12.72 +/- 0.4 13.02 +/- 0.4	12.94 +/- 0.11	1.294 +/- 0.011	1.674436 +/- 0.028468
620 +/- 1	13.33 +/- 0.4 13.32 +/- 0.4 13.25 +/- 0.4 13.17 +/- 0.4 13.63 +/- 0.4	13.34 +/- 0.092	1.334 +/- 0.0092	1.779556 +/- 0.0245456
640 +/- 1	13.17 +/- 0.4 13.20 +/- 0.4 13.13 +/- 0.4 13.13 +/- 0.4 13.12 +/- 0.4	13.15 +/- 0.016	1.315 +/- 0.0016	1.729225 +/- 0.004208
660 +/- 1	13.69 +/- 0.4 13.39 +/- 0.4 13.68 +/- 0.4 13.67 +/- 0.4 13.12 +/- 0.4	13.576 +/- 0.048	1.3576 +/- 0.0048	1.84307776 +/- 0.01303296
680 +/- 1	13.80 +/- 0.4 13.60 +/- 0.4 13.84 +/- 0.4 13.67 +/- 0.4 13.65 +/- 0.4	13.712 +/- 0.048	1.3712 +/- 0.0048	1.8818944 +/- 0.01316352
700 +/- 1	13.77 +/- 0.4 13.71 +/- 0.4 14.02 +/- 0.4 13.98 +/- 0.4 13.95 +/- 0.4	13.886 +/- 0.062	1.3886 +/- 0.0062	1.92820996 +/- 0.0172864

$$md = 2.664\ \frac{s^2}{kg}$$

$$k_d = \frac{4\pi^2}{md}$$

Where md = dynamic slope

$$k_d = 14.82\ \frac{N}{m}$$

$$\sigma k_d = \frac{4\pi^2}{(md)^2 * \sigma md}$$

$$\sigma k_d = 0.3656$$

Comparison Test:

$$\frac{|k_s - k_d|}{\sqrt{\sigma^2 k_d + \sigma^2 k_s}} < 2$$

$$\frac{|15.23 - 14.82|}{\sqrt{0.3656^2 + 1.554^2}} < 2$$

$$\frac{0.41}{1.5965} < 2$$

$$0.2568 < 2$$

∴ The values are consistent and within the error range!!

Mean Calculation:

$$k_{wm} = \frac{\left(\frac{k_{dynamic}}{\sigma^2{}_{dynamic}} + \frac{k_{static}}{\sigma^2{}_{static}}\right)}{\left(\frac{1}{\sigma^2{}_{dynamic}} + \frac{1}{\sigma^2{}_{static}}\right)}$$

$$k_{wm} = \frac{\left(\frac{14.82}{1.554^2} + \frac{15.23}{0.3656^2}\right)}{\left(\frac{1}{1.554^2} + \frac{1}{0.3656^2}\right)}$$

$$k_{wm} = 15.2085$$

Figure 1

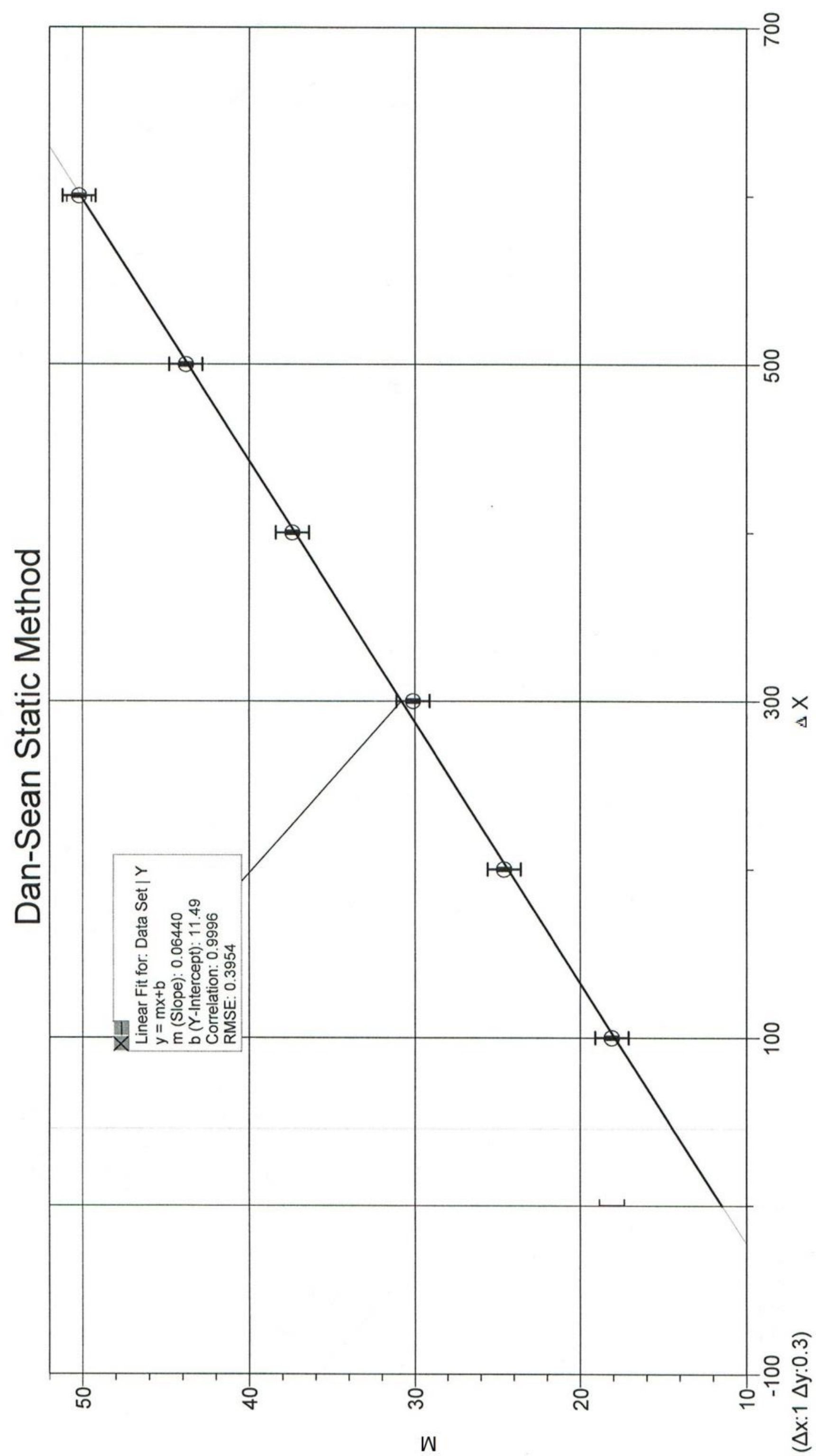

Dan-Sean Static Method
Linear Fit for: Data Set | Y
y = mx+b
m (Slope): 0.06440
b (Y-Intercept): 11.49
Correlation: 0.9996
RMSE: 0.3954
M
10
20
30
40
50
-100
100
300
500
700
Δ X
(Δx:1 Δy:0.3)

TABLE 3:

Data Set: Static		
	X	Y
1	100	18.1
2	200	24.6
3	300	30.1
4	400	37.4
5	500	43.8
6	600	50.2

TABLE 4:

Data Set: Dynamic			
	Mass (g)	**T^2 (s^2)**	**error**
1	600	1.674	0.0284
2	620	1.779	0.0245
3	640	1.729	0.0042
4	660	1.843	0.0130
5	680	1.876	0.0131
6	700	1.928	0.0172

Figure 2

x	σ_x	y	σ_y	Weight w	wx	wy	wxy	wx^2	wy^2	Δx = x-	Δy=y-y	ΔxΔy	Δx²	Δy²	Pred y	y-pred	(y-prec	n	6
100	1.00E-03	18.1	0.1	9.9995	1000	181	2E+05	1E+06	1E+06	-250	-15.9	3983	62500	253.9	17.93	163.1	26588	Slope m	0.0644000
200	1.00E-03	24.6	0.1	9.9995	2000	246	5E+05	4E+06	4E+06	-150	-9.43	1415	22500	88.99	24.37	221.6	49113	Intercept b	11.493333
300	1.00E-03	30.1	0.1	9.9995	3000	301	9E+05	9E+06	9E+06	-50	-3.93	196.7	2500	15.47	30.81	270.2	72993	Correlation r^2	0.9991391
400	1.00E-03	37.4	0.1	9.9995	4000	374	1E+06	2E+07	2E+07	50	3.367	168.3	2500	11.33	37.25	336.7	1E+05	RMSE	394.23991
500	1.00E-03	43.8	0.1	9.9995	5000	438	2E+06	2E+07	2E+07	150	9.767	1465	22500	95.39	43.69	394.3	2E+05	Error on slope	0.0094251
600	1.00E-03	50.2	0.1	9.9995	6000	502	3E+06	4E+07	4E+07	250	16.17	4042	62500	261.4	50.13	451.8	2E+05	Error on Intercept	36.703573
∑x		∑y					∑xy	∑x²	∑y²										
2100	0.006	204.2	0.6	59.997	20999	2042	8E+06	9E+07	9E+07	0	0	11270	2E+05	726.4	204.2	1838	6E+05	Dan-Sean Static Method	

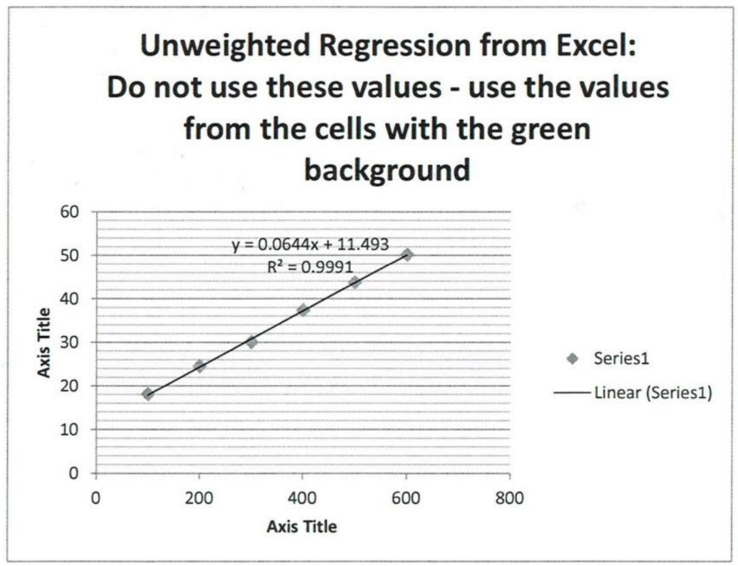


Figure 3

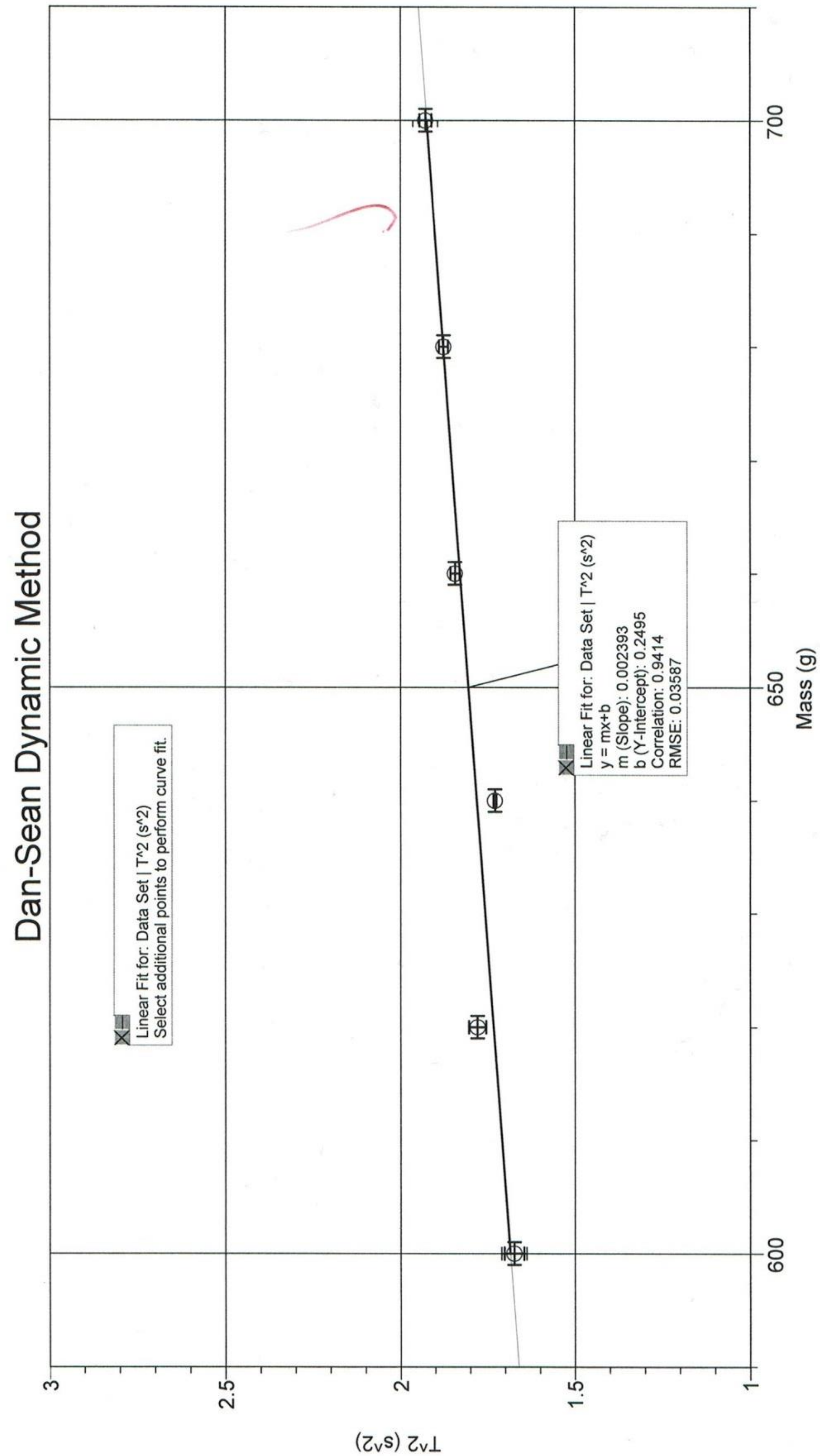
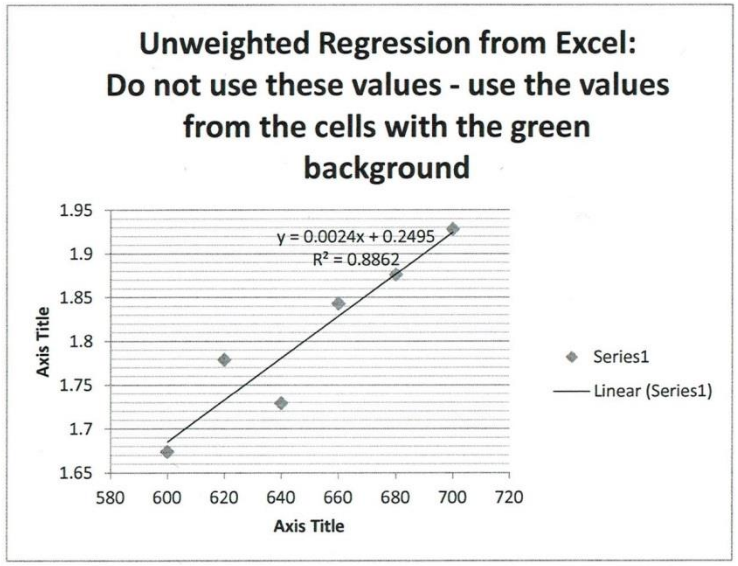

Dan-Sean Dynamic Method
Linear Fit for: Data Set | T^2 (s^2)
Select additional points to perform curve fit.
Linear Fit for: Data Set | T^2 (s^2)
y = mx+b
m (Slope): 0.002393
b (Y-Intercept): 0.2495
Correlation: 0.9414
RMSE: 0.03587
T^2 (s^2)
3
2.5
2
1.5
1
600
650
700
Mass (g)

Figure 4

x	σ_x	y	σ_y	Weight w	wx	wy	wxy	wx^2	wy^2	Δx = x-	Δy=y-y	ΔxΔy	Δx²	Δy²	Pred y	y-pred	(y-prec	n	6
600	1.00E-03	1.674	0.0284	35.189	21114	58.91	1E+06	4E+08	4E+08	-50	-0.13	6.542	2500	0.017	1.672	57.24	3276	Slope m	0.0026635
620	1.00E-03	1.779	0.0245	40.782	25285	72.55	2E+06	6E+08	6E+08	-30	-0.03	0.775	900	7E-04	1.725	70.83	5016	Intercept b	0.073685
640	1.00E-03	1.729	0.0092	108.06	69158	186.8	1E+07	5E+09	5E+09	-10	-0.08	0.758	100	0.006	1.778	185.1	34246	Correlation r^2	0.886207
660	1.00E-03	1.843	0.013	76.696	50620	141.4	7E+06	3E+09	3E+09	10	0.038	0.382	100	0.001	1.832	139.5	19466	RMSE	153.257
680	1.00E-03	1.876	0.0131	76.114	51758	142.8	7E+06	3E+09	3E+09	30	0.071	2.135	900	0.005	1.885	140.9	19854	Error on slope	6.57E-05
700	1.00E-03	1.928	0.0172	58.042	40629	111.9	5E+06	2E+09	2E+09	50	0.123	6.158	2500	0.015	1.938	110	12093	Error on Intercept	3.027761
∑x		∑y					∑xy	∑x²	∑y²										
3900	0.006	10.829	0.1054	394.88	3E+05	714.3	4E+07	1E+10	1E+10	0	-0	16.75	7000	0.045	10.83	703.5	93951	Dan-Sean Dynamic Method	



Theory Equations:

PART A: STATIC

$$F = -kx$$

$$(\Delta M)g = k_s(\Delta x)$$

$$Mg = k_s(x - x_0)$$

$$x = \frac{g}{k_s}M + x_0$$

$$k_x = \frac{g}{x}M + 0$$

PART B: DYNAMAIC

$$x = A\cos\omega t$$

$$\omega = \sqrt{\frac{k_d}{M}} = 2\pi f$$

$$T = 2\pi\sqrt{\frac{M}{k_d}}$$

$$T^2 = \frac{4\pi^2}{k_d}M + \frac{4\pi^2}{k_d}(\frac{M_s}{3} + M_h)$$

$$k_d = \frac{4\pi^2}{T^2}M + \frac{4\pi^2}{T^2}(\frac{M_s}{3} + M_h)$$

ACKNOWLEDGEMENTS

I would like to acknowledge my partner Sean Hargrave in this report. Without his help and dedication during the lab we would not have gotten such good results as were found. I would also like to acknowledge my TA, Paul for explaining so much to us about how the 'little things' worked within the lab. Without his help, I would not be able to produce a report such as this.

REFERENCES

For many of the equations used within this report I must reference the slides posted on CU Learn and the lab manual.

❖ Robinson, Andrew. "Phys 1003 Introductory Mechanics and Thermodynamics Laboratory Manual" Fall 2013 Ed. Carleton University, 2013.