ECON 4021
ASSIGNMENT 3

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1. Problem 4 (Chapter 11, page 390) The representative firm's optimization problem is to choose current and future employment levels, and the investment level to maximize the present value of profits

$$\max_{N,N',I}\left\{zF(K,N) - wN - I + \frac{1}{1+r}\Big(z'F(K',N') - w'N' + P_{K'}(1-\delta)K'\Big)\right\} \tag{0.1}$$

subject to the capital accumulation process[1]

$$K' = (1-\delta)K + I, \quad 0 < \delta < 1 \tag{0.2}$$

• Note that the value of the capital stock in the future period, left over after production, is determined by the relative price of capital that prevails in the future period, $P_{K'}$. In the baseline case, we assumed this price to equal 1.

Using the capital accumulation process (0.2) to substitute out K' in (0.1) and considering only the first order condition with respect to investment we get

$$-1 + \frac{1}{1+r}\Big(\underbrace{z'F_1}_{MP_{K'}} + (1-\delta)P_{K'}\Big) = 0 \tag{0.3}$$

which we can write as

$$\boxed{MP_{K'} + (1-\delta)P_{K'} - 1 = r} \tag{0.4}$$

The left hand side of (0.4) represents the marginal benefit of one unit of investment. The right hand side of (0.4) represents the marginal cost of one unit of investment. The representative firm chooses the optimal investment level that equates the marginal benefit to marginal costs.

When $P_{K'} = 1$ we get the baseline optimal investment rule

$$MP_{K'} - \delta = r \tag{0.5}$$

[1] I use the notation δ or d interchangeably to denote the depreciation rate.

How does (0.4) differ from (0.5)? To see this add and subtract δ on the left and side of (0.4) and rewrite it as

$$MP_{K'} - \delta + (1-\delta)[P_{K'} - 1] = r \tag{0.6}$$

From (0.6) it is clear that for a given r, the marginal benefit of investment can be greater or lower relative to the baseline case in (0.5) if $P_{K'} > 1$ or $P_{K'} < 1$, respectively. Hence, the optimal investment schedule can be to the right or left of the baseline case, respectively. When $P_{K'} = 1$, (0.6) is the same as (0.5).

If we interpret $P_{K'}$ as the stock price of the firm, then (0.6) suggests that a stock price boom should be associated with a rise in investment. That is, we should observe investment and stock price movements to be positively correlated in the data.

2. Suppose that the government temporarily increases spending from G_1 to G_2. At the same time, future total factor productivity, z', decreases. Using the *real intertemporal model with investment*, determine the equilibrium effects on all the endogenous variables of the model. The figures must be clearly labelled.

IMPACT EFFECTS:

Note: When discussing IMPACT EFFECTS*, we figure out what is happening in the economy for any given the real interest rate, r, and the real wage, w.*

- Suppose the economy is in a competitive equilibrium with the following levels of the current period endogenous variables $\{N_1, w_1, Y_1, r_1, C_1, I_1\}$. A temporary increase in government spending from G_1 to G_2 will increase the demand for current output and hence shift the output demand curve to the right, horizontally, by an amount $G_2 - G_1$, for any given level of the real interest rate, r. Since the government's intertemporal budget constraint must hold (meaning that the present value of government spending must equal the present value of the lump sum taxes), the increase in spending must be financed by an increase in the present value of taxes by the same amount. That is, the representative consumer's lifetime wealth must decrease by $G_2 - G_1$. This reduction in lifetime wealth will cause a negative wealth effect and the representative consumer will decrease current consumption and leisure for a given r as they are both normal goods. The reduction in consumption reflects the *crowding out* effect of the temporary increase in G. The reduction in current leisure means that the current labour supply will increase at any given level of w and r from $N_1(r_1)$ to $N_2(r_1)$. As a result, output supply curve will shift to the right from Y_1^s to Y_2^s. The rightward shift in the output supply would be less than the rightward shift in output demand as the increase in government spending is temporary so the negative lifetime wealth effect is likely to be small.

- A decrease in future productivity, z', will decrease the marginal product of future capital, $MP_{K'}$. According to the investment rule, at the current real interest rate r_1, $MP_{K'} - d < r$, where d is the depreciation rate. The inequality implies a disincentive to invest. The decrease in the optimal level of investment means that current investment demand will decrease from $I_1^d(r_1)$ to $I_2^d(r_1)$. Consequently, output demand curve will shift to the left. The decrease in z' will also decrease future real wage w' and hence real income. This will induce a wealth effect on labour supply, causing it to shift to the right. Consequently, the output supply curve will shift to the right as well. The wealth effect on current consumption will also contribute to shifting the output demand curve to the left even further. *Note that the wealth effect of a change in z' is ignored in the labour supply and output supply curves shown in the textbook (pages 378-380).*

Table 1: Summary of IMPACT EFFECTS

	N^d	N^s	Y^d	Y^s
$\uparrow$ G	no shift	rightward shift	rightward shift	rightward shift
$\downarrow z'$	no shift	rightward shift	leftward shift	rightward shift

EQUILIBRIUM EFFECTS: In the new competitive equilibrium, will the real interest rate, r_2, and the real wage, w_2, be higher, lower, or the same as the old equilibrium? Will the new equilibrium output, Y_2, be higher or lower? Will the change in the real interest rate offset or reinforce the initial slump in investment? Will the change in the real interest rate offset or reinforce the wealth effect on consumption?

From the theoretical point of view these are all possible depending up the magnitude of the shifts of the output demand and output supply curves. Once you pick a particular case, you can determine the position of the labour supply curve associated with the new equilibrium real interest rate and immediately determine the equilibrium real wage, w_2. Then you can discuss the potential effects on consumption and investment.

3. Suppose that the representative firm produces output only from capital. Current output is given by $Y = zK^\alpha$, and future output is given by $Y' = z'\left(K'\right)^\alpha$, where $0 < \alpha < 1$. Determine the optimal level of investment for this firm, and show how investment depends on the real interest rate, future total factor productivity, the depreciation rate, and α. Explain your results in words. You may use clearly labelled diagrams to illustrate your explanation.

ECON 4021 A3 Suggested Solutions (1)

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- current period profit: $\pi = zK^{\alpha} - I$
- future period profit: $\pi' = z'(K')^{\alpha} + (1-d)K'$

  where $K' = (1-d)K + I$

- $$\max_{I} \left\{ zK^{\alpha} - I + \frac{1}{1+r}\left(z'(K')^{\alpha} + (1-d)K' \right) \right\}$$

  s.t. $K' = (1-d)K + I$

$$\Leftrightarrow \max_{I} \left\{ zK^{\alpha} - I + \frac{1}{1+r}\left(z'\left((1-d)K+I\right)^{\alpha} + (1-d)\left((1-d)K+I\right) \right) \right\}$$

F.O.C:

$$-1 + \frac{z'\alpha\left((1-d)K + I^{*}\right)^{\alpha-1} + 1 - d}{1+r} = 0 \quad (1)$$

[see slide 22 in Lecture 9 & 10]

$$(1) \Rightarrow z'\alpha\left((1-d)K + I^{*}\right)^{\alpha-1} + \cancel{1} - d = \cancel{1} + r$$

$$\left((1-d)K + I^{*}\right)^{\alpha-1} = \frac{r+d}{z'\alpha}$$

$$(1-d)K + I^{*} = \left[\frac{r+d}{z'\alpha}\right]^{\frac{1}{\alpha-1}}$$

$$\Rightarrow \boxed{I^{*} = \left[\frac{z'\alpha}{r+d}\right]^{\frac{1}{1-\alpha}} - (1-d)K} \quad (2)$$

optimal level of investment

From equation (2) (2)

- An increase in the real interest rate r will lower the optimal investment level. The reason is that an increase in r means the firm values future profit less as the discount factor $\frac{1}{1+r}$ is smaller.

- An increase in z' (future TFP) raises the optimal investment level as future marginal product is higher. This increase in z' raises the marginal benefit of investment relative to the cost.

- An increase in d, the depreciation rate, has two offsetting effects on I^*.
  (i) an ↑ in d means a smaller K' and hence a larger marginal product of future capital, $MP_{K'}$. This effect tends to increase the optimal I^*
  (ii) an ↑ in d also means a smaller amount of scrap capital at the end of the future period with lowers the value of the discounted profits, hence a disincentive to investment. This effect tends to lower I^*.

- A ~~p~~ increase in α raises the numerator and the coefficient of the first term in equation (2). Thus optimal I^* increases. The reason is that an increase in α raises the marginal product of future capital, hence increasing the optimal I^*, given all other variables and parameters.