MATH 1402A **Practice Term test #4 – solution** **17 November 2013**

Total marks: XX marks

Non-programmable calculators are permitted. This test is closed book.
<u>Supply your answers on this sheet</u>, but TA's have extra paper if you need it.

PLEASE PRINT ______________________________ ____________

First name Last name Student number

Please show your work where appropriate!

1. Solve the following system: $A = \left[\begin{array}{ccc|c} 0.5 & -0.4 & -0.2 & 500 \\ -0.2 & 0.7 & -0.1 & 400 \\ -0.1 & -0.1 & 0.7 & 200 \end{array}\right]$

$R_1' = 10R_1$
$R_2' = 10R_2$
$R_3' = 10R_3$

$$\left[\begin{array}{ccc|c} 5 & -4 & -2 & 5000 \\ -2 & 7 & -1 & 4000 \\ -1 & -1 & 7 & 2000 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3 \text{ and } R_1' = -R_1} \left[\begin{array}{ccc|c} 1 & 1 & -7 & -2000 \\ -2 & 7 & -1 & 4000 \\ -5 & 4 & 2 & -5000 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & -2000 \\ 0 & 9 & -15 & 0 \\ 0 & 9 & -33 & -15000 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & -7 & -2000 \\ 0 & 9 & -15 & 0 \\ 0 & 0 & -18 & -15000 \end{array}\right] \sim$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -7 & -2000 \\ 0 & 9 & -15 & 0 \\ 0 & 0 & 1 & \frac{-15000}{-18} \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 1 & -7 & -2000 \\ 0 & 9 & -15 & 0 \\ 0 & 0 & 1 & \frac{2500}{3} \end{array}\right]$$

$\frac{17500}{3} - \frac{6000}{3} = \frac{11500}{3}$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & (11500/3) \\ 0 & 9 & 0 & (12500) \\ 0 & 0 & 1 & \left(\frac{2500}{3}\right) \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 0 & (11500/3) \\ 0 & 1 & 0 & (12500/9) \\ 0 & 0 & 1 & (2500/3) \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 0 & (22000/9) \\ 0 & 1 & 0 & (12500/9) \\ 0 & 0 & 1 & (2500/3) \end{array}\right]$$

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2. Consider the following augmented matrix representing an SLE. $\left[\begin{array}{cccc|c} 2 & 0 & 7 & 3 & 6 \\ 4 & -5 & 2 & -2 & -5 \\ -1 & 13 & -4 & -8 & 3 \end{array}\right]$

a. <u>Complete the following</u>: this augmented matrix is the representation of a system of linear equations for which there are _3_ equations and _4_ unknowns.

b. <u>Complete the following</u>: in this matrix, the entry '0' is entry _1, 2_.

c. <u>Complete the following</u>: in this matrix, the entry '–8' is entry _3, 4_.

3. Consider the following system of 3 equations and 3 unknowns:

$$\boxed{\begin{array}{c} x-1=y+2z \\ 2x+y+3=2y+3z \\ -7+2y-9z=2 \end{array}}$$

a. Re-write the SLE in standard form.

$$\begin{aligned} x-y-2z&=1 \\ 2x-y-3z&=-3 \\ 2y-9z&=9 \end{aligned}$$

b. Write the augmented matrix of the system.

$$\left[\begin{array}{ccc|c} 1 & -1 & -2 & 1 \\ 2 & -1 & -3 & -3 \\ 0 & 2 & -9 & 9 \end{array}\right]$$

4. Given the following matrix, rewrite an equivalent matrix obtained after performing the 2 specified E.R.O.'s

$$\left[\begin{array}{ccc|c} 1 & 6 & -7 & -6 \\ 2 & -5 & -4 & 2 \\ -1 & 13 & -8 & 10 \end{array}\right] \xrightarrow[R_3'=R_3+R_1]{R_2'=R_2-2R_1} \left[\begin{array}{ccc|c} 1 & 6 & -7 & -6 \\ 0 & -17 & 10 & 14 \\ 0 & 19 & -15 & 4 \end{array}\right]$$

5. Determine the Total Cost TC if the Marginal Cost is given by $MC = 10q^4 + 6q^2 - 10$ and that TC=8450 when q=5.

$$TC = \int MC\,dq = \int 10q^4 + 6q^2 - 10\,dq = 2q^5 + 2q^3 - 10q + C$$

$$\Rightarrow TC = 8450 = 2(5)^5 + 2(5)^3 - 10(5) + C$$

$$8450 = 6250 + 250 - 50 + C$$

$$\Rightarrow C = 2000$$

$$\therefore\ TC = 2q^5 + 2q^3 - 10q + 2000$$

6. Evaluate the following integrals:

a. $\int \frac{3x}{4x-7}\,dx$	b. $\int_{-1}^{2} \sqrt{8x+9}\,dx$

a. $\int \frac{3x}{4x-7}\,dx \Rightarrow u = g(x) = 4x-7 \Rightarrow x = \frac{1}{4}(u+7)$

$du = g'(x)dx = 4\,dx$

$$\int \frac{3\left(\frac{1}{4}(u+7)\right)}{u}\,\frac{1}{4}du = \frac{3}{16}\int \frac{u+7}{u}\,du = \frac{3}{16}\int 1 + \frac{7}{u}\,du =$$

$$\ldots = \frac{3}{16}\left[u + 7\ln|u|\right] + C_1$$

$$\ldots = \frac{3}{16}\left[4x-7 + 7\ln|4x-7|\right] + C_1 \leftarrow \text{Can leave as is...}$$

b. $\int_{-1}^{2} \sqrt{8x+9}\,dx$

$u = g(x) = 8x+9$

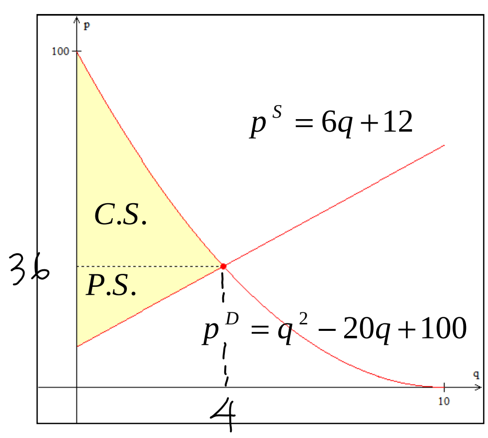
$\frac{1}{8}du = dx$

$u_1 = g(-1) = 1$

$u_2 = g(2) = 25$

$$\int_{1}^{25} \sqrt{u}\left(\frac{1}{8}du\right) = \frac{1}{8}\cdot\frac{2}{3}\left[u^{3/2}\right]_1^{25} = \frac{1}{12}\cdot\left(5^3 - 1\right) = \boxed{\frac{31}{3} = 10\tfrac{1}{3}}$$

7. Determine the **Consumer's Surplus** for the following case:



$p^D = p^S \Rightarrow q^2 - 20q + 100 = 6q + 12$

$q^2 - 26q + 88 = 0 \Rightarrow q = 4$ ✓, $q = 22$ ✗

$\therefore q_0 = 4 \Rightarrow p_0 = 6q_0 + 12 = 36$

$$\therefore C.S. = \int_0^4 p^D - p_0\,dq = \int_0^4 q^2 - 20q + 64\,dq$$

$$= \left[\frac{1}{3}q^3 - 10q^2 + 64q\right]_0^4 = \underbrace{\frac{64}{3} - 160 + \underbrace{64(4)}_{256}}_{96 = \frac{288}{3}} = \boxed{117\tfrac{1}{3}}\quad \frac{352}{3}$$

8. Determine the Total Revenue *TR* if the Marginal Revenue is given by $MR = 14 - 7q$. Assume that *TR*=0 when *q*=0. Use the result obtained for *TR* to determine the inverse demand function (i.e. assume that *p* corresponds to p^D).

$$TR = \int MR\,dq = \int 14 - 7q\,dq = 14q$$

But $(q = 0 \Rightarrow TR = 0) \Rightarrow C_1 = 0$

$$\therefore TR = 14q - \frac{7}{2}q^2 = \left(14 - \frac{7}{2}q\right)q = pq = p^D q$$

$$\therefore p^D = 14 - \frac{7}{2}q$$