MATH 1402A **Term test #1 – solution** **28 September 2012**

Maximum score: **30** marks

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Non-programmable calculators are permitted. This test is closed book.

Supply your answers on this sheet, but TA's have extra paper if you need it.

PLEASE PRINT ______________________________ ____________

First name Last name Student number

Please show your work where appropriate!

1. [2+3+3 marks] Determine the domain of the following functions, given the rule:

   a. $y = f(x) = x^2 + 5xy + y^2 - 2y^3 \Rightarrow$ Polynomial...

   $\therefore \text{dom}(f) = \mathbb{R}^2$

   b. $y = f(x) = (7 - 14x)\cdot\sqrt{2x+4}$

   $2x + 4 \geq 0 \Rightarrow x \geq -2$

   $\therefore \text{dom}(f) = \{x \in \mathbb{R} \mid x \geq -2\}$

   or $\text{dom}(f) = [2, \infty)$

   c. $z = f(x, y) = e^{-(x+y)} \cdot \ln(30x + 5y - 15)$

   $30x + 5y - 15 > 0$

   $5y > -30x + 15$

   $y > -6x + 3$

   $\therefore \text{dom}(f) = \{(x, y) \in \mathbb{R}^2 \mid y > -6x + 3\}$

2. [1+3] Let $z = f(x, y) = 36x^3 - 24x^2 + 12y + 62$.

   a. Determine $f(-1,3) = 36(-1)^3 - 24(-1)^2 + 12(3) + 62 = -36 - 24 + 36 + 62$

   $= 38$

   b. Determine the rule $y = g(x)$ of the function obtained when considering an iso–z section (i.e. constant z) where $z = z_0 = -10$.

   $z_0 = -10 = 36x^3 - 24x^2 + 12y + 62$

   $12y = -36x^3 + 24x^2 - 62 - 10$

   $y = -3x^3 + 2x^2 - 6 \; (\equiv g(x))$

3. [3] $z = f(x, y) = \underbrace{(x^2 + y^2)}_{u}\underbrace{\sqrt{x+y}}_{v}$ Determine $\dfrac{\partial z}{\partial x}$. Do not simplify.

$u = x^2 + y^2$	$v = \sqrt{x+y}$
$u' = 2x$	$v' = \dfrac{1}{2\sqrt{x+y}}$

$$\frac{\partial z}{\partial x} = uv' + vu' = \boxed{\frac{(x^2+y^2)}{2\sqrt{x+y}} + 2x\sqrt{x+y}}$$

→ good enough

Can develop and re-arrange:

$$\therefore \frac{\partial z}{\partial x} = \frac{x^2 + y^2 + 4x(x+y)}{2\sqrt{x+y}} = \boxed{\frac{5x^2 + 4xy + y^2}{2\sqrt{x+y}}}$$

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4. [3] Given $z = f(x,y) = x^3 + x^2y - xy^2 - y^3$, $\frac{\partial z}{\partial x} = 3x^2 + 2xy - y^2$ and $\frac{\partial z}{\partial y} = x^2 - 2xy - 3y^2$. Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial z}{\partial y}\right) = \frac{\partial}{\partial x}(x^2 - 2xy - 3y^2) = \boxed{2x - 2y}$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial z}{\partial x}\right) = \frac{\partial}{\partial y}(3x^2 + 2xy - y^2) = \boxed{2x - 2y}$$

Q.E.D.

5. [3] Let $Q = f(K,L) = 9K^{\frac{1}{6}} L^{\frac{2}{3}}$ be the rule for a production function. Determine the rule $K = g(L)$ that represents an isoquant for which $Q = Q_0 = 27$.

$$Q_0 = 27 = 9K^{1/6}L^{2/3}$$
$$K^{1/6}L^{2/3} = 3 \Rightarrow K^{1/6} = \frac{3}{L^{2/3}} \Rightarrow K = \frac{3^6}{(L^{2/3})^6} \Rightarrow K = \frac{729}{L^4} \left(\equiv g(L)\right)$$

6. [3] Given $K = g(L) = \frac{360}{L^2}$, determine ***MRS***, the <u>*marginal rate of substitution*</u> $\left|\frac{dK}{dL}\right|$ for $L = 4$.

$$K = \frac{360}{L^2} \Rightarrow K = 360L^{-2} \Rightarrow \frac{dK}{dL} = -720L^{-3} = \frac{-720}{L^3}$$

$$\therefore MRS = \left|\frac{dK}{dL}\right| = \frac{720}{L^3} \Rightarrow MRS\Big|_{L=4} = \frac{720}{4^3} = \boxed{11.25}$$

7. [3] Given $Q = f(K,L) = 25K^{\frac{2}{5}} L^{\frac{4}{5}}$, determine the <u>*marginal product of labour*</u> **(**$MPL = \frac{\partial Q}{\partial L}$**)**, for $K=4$ and $L=512$

$$\frac{\partial Q}{\partial L} = \left(\frac{4}{5}\right)25K^{2/5}L^{-1/5} = 20\sqrt[5]{\frac{K^2}{L}} \Rightarrow \frac{\partial Q}{\partial L}\Big|_{(K,L)=(4,512)} = 20\sqrt[5]{\frac{16}{512}} = 20\cdot\sqrt[5]{\frac{1}{32}} = \boxed{10}$$

8. [3] Let $Q = f(K,L) = 9K^{\frac{1}{6}} L^{\frac{2}{3}}$, (as in exercise 5). Show that *APK* is greater than *MPK.*

$$MPK = \frac{\partial Q}{\partial K} = \frac{9}{6}K^{-5/6}L^{2/3} = \frac{3}{2}\frac{L^{4/6}}{K^{5/6}} = \frac{3}{2}\sqrt[6]{\frac{L^4}{K^5}}$$

and $$APK = \frac{Q}{K} = \frac{9K^{1/6}L^{2/3}}{K} = 9K^{-5/6}L^{4/6} = 9\sqrt[6]{\frac{L^4}{K^5}}$$

$$\therefore APK = 9\sqrt[6]{\frac{L^4}{K^5}} > \frac{3}{2}\sqrt[6]{\frac{L^4}{K^5}} = MPK$$