An eConcordia Examination

Solutions

Department of Mathematics and Statistics

Course: Math 208EC　　　　Date: August 2011

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Question 1

A) The slope, $m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{(4245.00 - 3855.00)}{(60 - 50)} = 39.0$

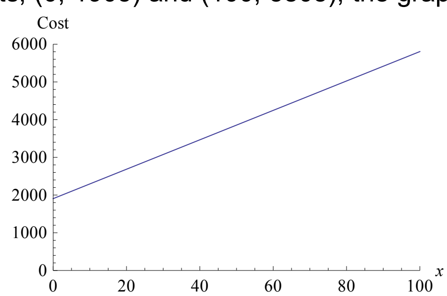
Point/Slope form: $y = m(x - x_1) + y_1$

$$y = 39\,(x - 50) + 3855.00$$

$$y = 39\,x + 1905$$

Answer: $C(x) = 39\,x + 1905$

B) Using the points, (0, 1905) and (100, 5805), the graph for the daily cost:



C) The slope is the marginal cost. It is the cost of producing each additional tennis racket.
The y-intercept is the fixed costs. It is the cost that will be incurred when no rackets are made.

Question 2

A) $7^{x^2} = 7^{2x+3}$

$$x^2 = 2x + 3$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

Solution : $x = 3$ or $x = -1$

B) $4^{5x - x^2} = 2^{-12}$

$$\left(2^2\right)^{5x - x^2} = 2^{-12}$$

$$10\,x - 2\,x^2 = -12$$

$$x^2 - 5\,x - 6 = 0$$

$$(x - 6)(x + 1) = 0$$

Solution : $x = 6$ or $x = -1$

C) $\log_b x = \frac{3}{2}\log_b 4 - \frac{2}{3}\log_b 8 + 2\log_b 2$

$\log_b x = \log_b 4^{3/2} - \log_b 8^{2/3} + \log_b 2^2$

$\log_b x = \log_b 8 - \log_b 4 + \log_b 4$

$\log_b x = \log_b 8$

Solution: x = 8

D) $\log_b(x+2) + \log_b x = \log_b 24$

$\log_b(x+2)\,x = \log_b 24$

$(x+2)\,x = 24$

$x^2 + 2x - 24 = 0$

$(x+6)(x-4) = 0$

Solution: x = 4 Note: x = -6 is not a solution.

E) $\log_{10}(x+6) - \log_{10}(x-3) = 1$

$\log_b \frac{(x+6)}{(x-3)} = \log_{10} 10$

$\frac{(x+6)}{(x-3)} = 10$

$x + 6 = 10x - 30$

$36 = 9x$

Solution: x = 4

Question 3

A) Find $f(1) + ... + f(225)$ if $f(x) = 3x - 7$

This is an arithmetic series with $a_1 = -4$; $a_n = a_{225} = 3(225) - 7 = 668$; and $n = 225$

$S_n = \frac{n}{2}(a_1 + a_n)$ $S_{225} = \frac{225}{2}(-4 + 668) = 74\,700$

Answer: The sum is 74,700

B) Find $g(1) + ... + g(121)$ if $g(x) = \left(\frac{1}{3}\right)^x$

This is a geometric series with $a_1 = \frac{1}{3}$; $r = \frac{1}{3}$; and $n = 121$

$S_n = \frac{a_1(r^n - 1)}{r - 1}$ $S_{121} = \frac{(1/3)\left((1/3)^{121} - 1\right)}{(1/3 - 1)}$

Here, the term $(1/3)^{121}$ is very small so $\left((1/3)^{121} - 1\right) = -1$ approximately

$S = \frac{-(1/3)}{(1/3 - 1)} = \frac{-(1/3)}{-(2/3)} = \frac{1}{2}$

Answer: The sum is 1/2

Question 4

A) Sinking fund with PMT = 2000; interest is 5 % compounded monthly; FV = 75 000

$$FV = PMT\,\frac{[(1+i)^n - 1]}{i}$$

$$75\,000 = 2000 \times \frac{\left[\left(1+\frac{0.05}{12}\right)^n - 1\right]}{(0.05/12)}$$

$$0.15625 = \left(1 + \frac{0.05}{12}\right)^n - 1$$

$$1.15625 = (1.0041667)^n$$

$$\ln(1.15625) = \ln(1.0041667)^n = n \times \ln(1.0041667)$$

$$\frac{\ln(1.15625)}{\ln(1.0041667)} = n = 34.92$$

Answer: The account will be worth $75,000 in approximately 35 months.

B) The account will be worth $75,000. There were 35 payments of $2000 each for a total of $35 \times 2000 = 70\,000$ deposited.

The amount of interest earned is $75\,000 - 70\,000 = 5000$

Answer: $5,000 of interest was earned.

Question 5

A) Mortgage. PV = 280 000; interest is 3.4% compounded monthly so $i = \frac{0.034}{12}$;

term is 25 years, so $n = 12 \times 25 = 300$ payments.

$$PV = PMT \times \frac{[1-(1+i)^{-n}]}{i}$$

$$280\,000 = PMT \times \frac{\left[1-\left(1+\frac{0.034}{12}\right)^{-300}\right]}{\left(\frac{0.034}{12}\right)}$$

$$PMT = 280\,000 \times \frac{\left(\frac{0.034}{12}\right)}{\left[1-\left(1+\frac{0.034}{12}\right)^{-300}\right]} = 1386.77$$

Answer: The monthly payment will be $1386.77

B) After 15 years, there will still be 10 more years of payments (120 payments) to make. The outstanding balance after 15 years will be the present value of these 120 payments.

$$PV = PMT \times \frac{[1-(1+i)^{-n}]}{i}$$

$$PV = 1386.77 \times \frac{\left[1-\left(1+\frac{0.034}{12}\right)^{-120}\right]}{\left(\frac{0.034}{12}\right)} = 140\,905.98$$

Answer: After 15 years, the outstanding balance will be $140,905.98.

C) After 20 years there will be 5 more years (60 payments) left.

$$PV = PMT \times \frac{[1-(1+i)^{-n}]}{i}$$

$$PV = 1386.77 \times \frac{\left[1-\left(1+\frac{0.034}{12}\right)^{-60}\right]}{\left(\frac{0.034}{12}\right)} = 76418.71$$

Answer: After 20 years, the outstanding balance will be $76,418.71.

Question 6

The augmented matrix is: $\begin{pmatrix} 1 & 2 & 3 & 5 \\ 3 & -1 & -2 & 10 \\ 2 & 4 & -1 & 9 \end{pmatrix}$ The reduced row echelon form is: $\begin{pmatrix} 1 & 0 & 0 & \frac{176}{49} \\ 0 & 1 & 0 & \frac{24}{49} \\ 0 & 0 & 1 & \frac{1}{7} \end{pmatrix}$

Solution: $x = \frac{176}{49}$, $y = \frac{24}{49}$, $z = \frac{1}{7}$

Question 7

The Technology Matrix $M = \begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{pmatrix}$ $D = \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix}$

To solve the matrix equation, $(I - M)\, X = D$

$$(I - M) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} 0.2 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.1 \\ 0.2 & 0.1 & 0.1 \end{pmatrix} = \begin{pmatrix} 0.8 & -0.4 & -0.3 \\ -0.2 & 0.9 & -0.1 \\ -0.2 & -0.1 & 0.9 \end{pmatrix}$$

The augmented matrix is: $\begin{pmatrix} 0.8 & -0.4 & -0.3 & 10 \\ -0.2 & 0.9 & -0.1 & 15 \\ -0.2 & -0.1 & 0.9 & 20 \end{pmatrix} \rightarrow$Row operations $\begin{pmatrix} 1 & 0 & 0 & 40.1 \\ 0 & 1 & 0 & 29.4 \\ 0 & 0 & 1 & 34.4 \end{pmatrix}$

Answer: The outputs are, x_1=40.1 B$, $x_2 = 29.4$ B$, $x_3 = 34.4$ B$

Alternate solution is to find the inverse of $(I - M)$.

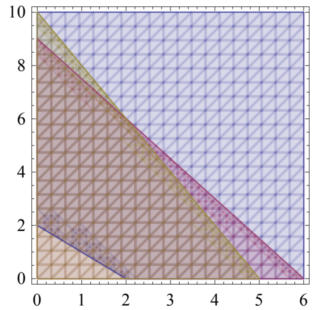
$$(I - M)^{-1}\, D = \begin{pmatrix} 1.6 & 0.78 & 0.62 \\ 0.4 & 1.32 & 0.28 \\ 0.4 & 0.32 & 1.28 \end{pmatrix} \begin{pmatrix} 10 \\ 15 \\ 20 \end{pmatrix} = \begin{pmatrix} 40.1 \\ 29.4 \\ 34.4 \end{pmatrix}$$

Question 8

A graph of the inequalities shows the feasible region for the equations.

1. $x + y \geq 2$ 2. $6x + 4y \leq 36$ 3. $4x + 2y \leq 20$

Solving equations 2 & 3 gives the intersection point, (2, 6). All other corner points occur on one of the axes.



Points (x, y)	P (x, y) 2 x + y
(0, 2)	2
(2, 0)	4
(0, 9)	9
(2, 6)	10
(5, 0)	10

The function, P has a minimum value of 2, which occurs at the point (0, 2).
The function has a maximum of 10 at two of the corner points, (2, 6) and (5, 0) which means that for all points on the line segment between these two points,P will also have the maximum value of 10.

Question 9

A) There are 8 stores in Quebec, 12 in Ontario and 10 in Manitoba, for a total of 30 stores.
The number of ways that 10 of these stores can be closed is given by: ${}_{30}C_{10} = 30\,045\,015$ ways

B) There are ${}_{8}C_{2} = 28$ ways to choose 2 stores to close in Quebec.
There are ${}_{12}C_{5} = 792$ ways to choose 5 stores to close in Ontario.
There are ${}_{10}C_{3} = 120$ ways to choose 3 stores to close in Manitoba.
The total number of ways to close these stores is: $28 \times 792 \times 120 = 2\,661\,120$ ways.

Question 10

In a shipment of 100 watches, there are 6 defective watches. The shipment is rejected if a sample of 10 watches has one or more defectives. This means that the shipment is accepted only if there are no defective watches at all.
There are ${}_{94}C_{10}$ ways to select 10 working watches. There are ${}_{100}C_{10}$ ways to select a sample of 10 watches.

The probability that the shipment will be accepted is $P(\text{accept}) = \frac{{}_{94}C_{10}}{{}_{100}C_{10}} = 0.5223$

The probability that the shipment will be rejected is $P(\text{reject}) = 1 - P(\text{accept}) = (1 - 0.5223) = 0.4777$
Answer: The probability that the shipment will be rejected is 0.4777 or 47.77%

Alternate solution: The probability of selecting a working watch changes each time a watch is selected. Multiplying all of these probabilities for the 10 selections gives:

$$P(\text{accept}) = \frac{94}{100} \times \frac{93}{99} \times \frac{92}{98} \times \frac{91}{97} \times \frac{90}{96} \times \frac{89}{95} \times \frac{88}{94} \times \frac{87}{93} \times \frac{86}{92} \times \frac{85}{91} = 0.5223$$