

***SOIL MECHANICS II***  
***CIVL 311***  
**COURSE NOTES**  
**2012**

***Module 5***  
***Bearing Capacity***  
***of Shallow Foundations***



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# **Module 5**

## **Bearing Capacity**

### ***Overall Learning Objectives***

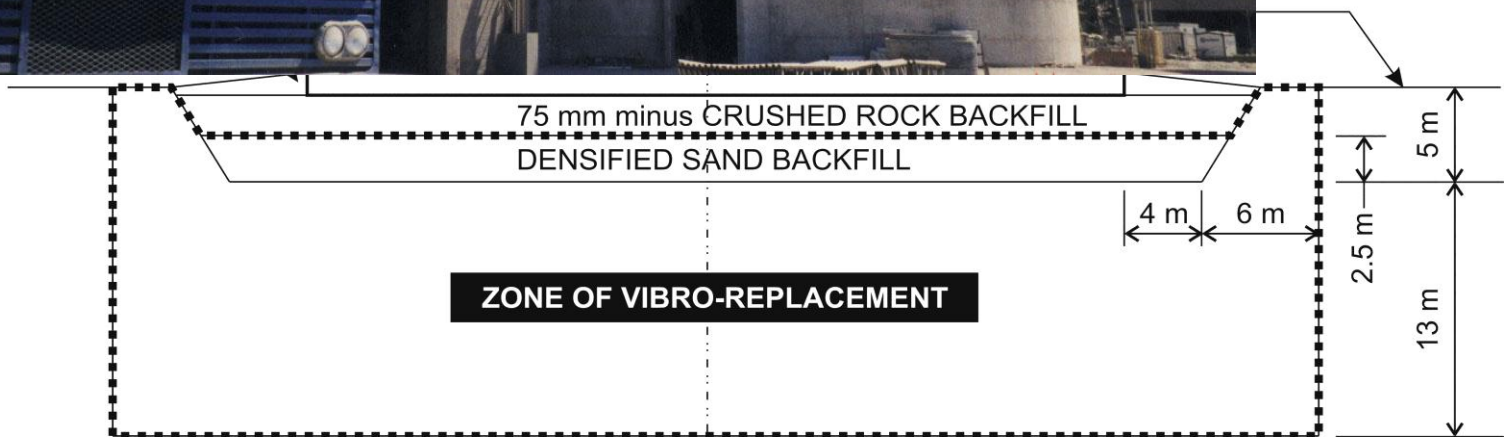
- Introduction and Definition of Key Terms  
(Budhu Text - Book 2 pp. 200-201)
- Bearing Capacity of Shallow Foundations  
(Budhu Text - Book 2 pp. 203-218)

# Big Qualicum River Bridge Vancouver Island Highway



# Lafarge Canada Cement Plant

**500 kPa  
loading, 45 m  
diameter silo**

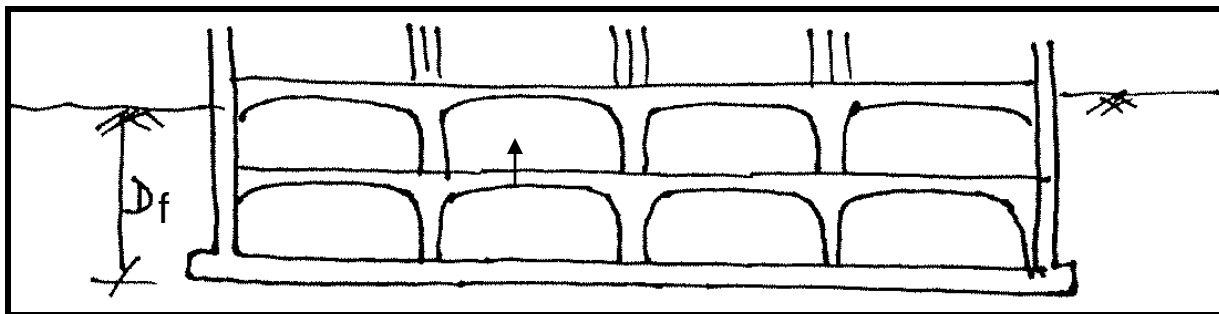
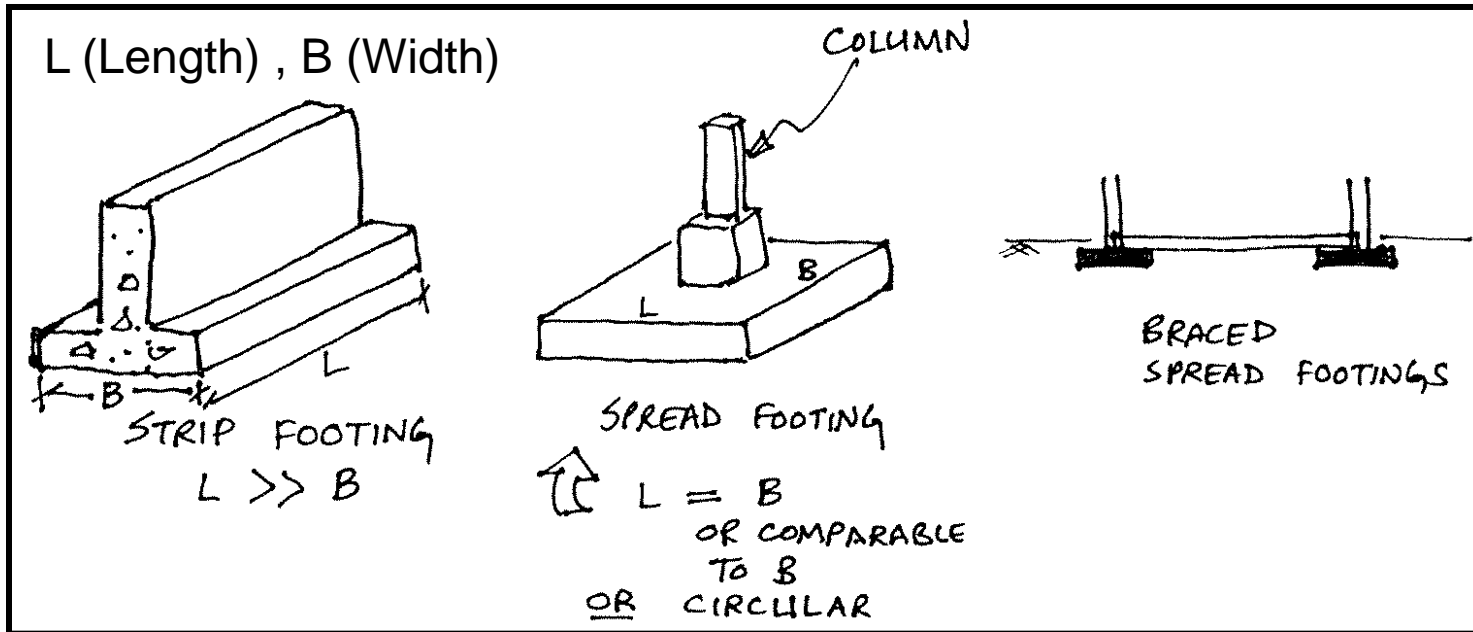


SECTION Y-Y (see Figure 2 for section location)

# Vancouver International Airport Shell Aerocentre



# Shallow Foundations

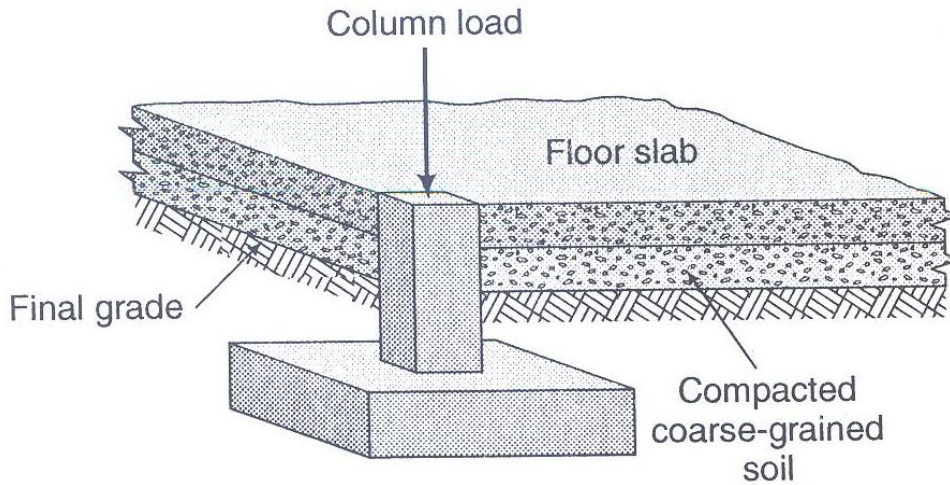
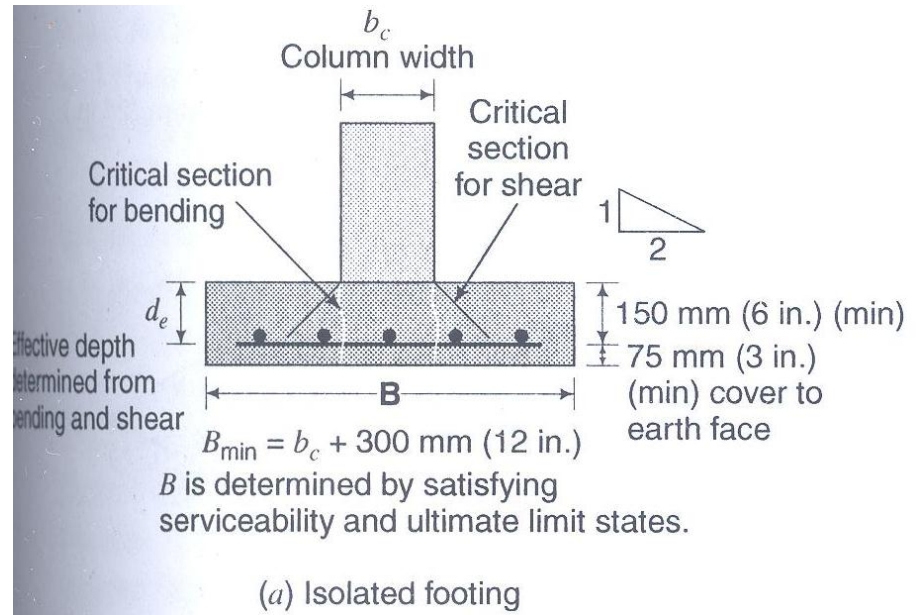


(L x B) Raft Foundation

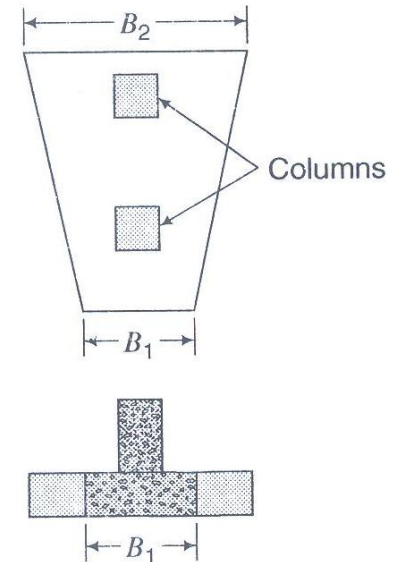
$D_f$  (Depth of footing below ground surface)

→  $(D_f/B) \leq 2.5$  for shallow foundations

# Shallow Foundations

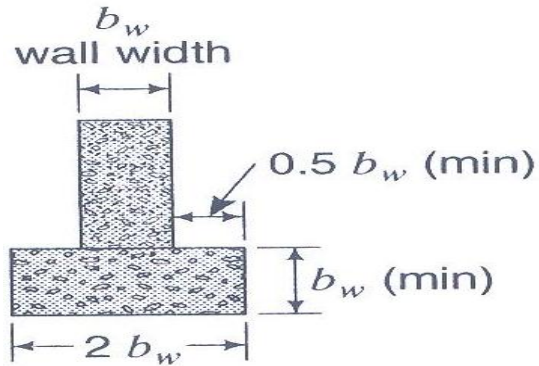


(b) Isolated footing and floor slab



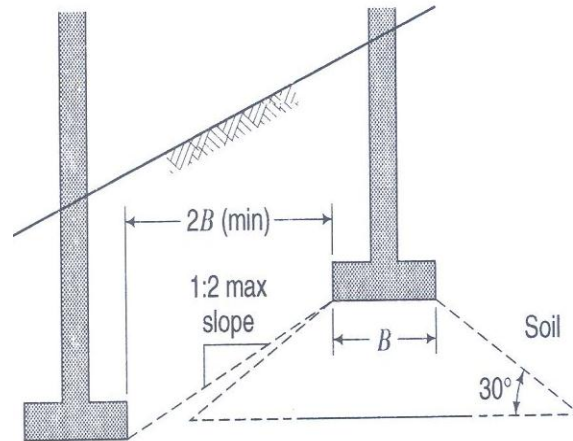
(c) Combined footing

# Shallow Foundations

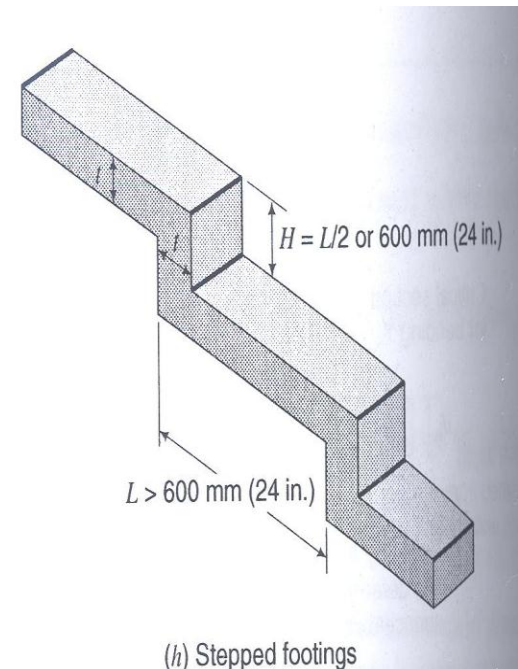


(d) Strip footing

→  $B/L \leq 10$  for strip footings



(g) Location of footings on a slope

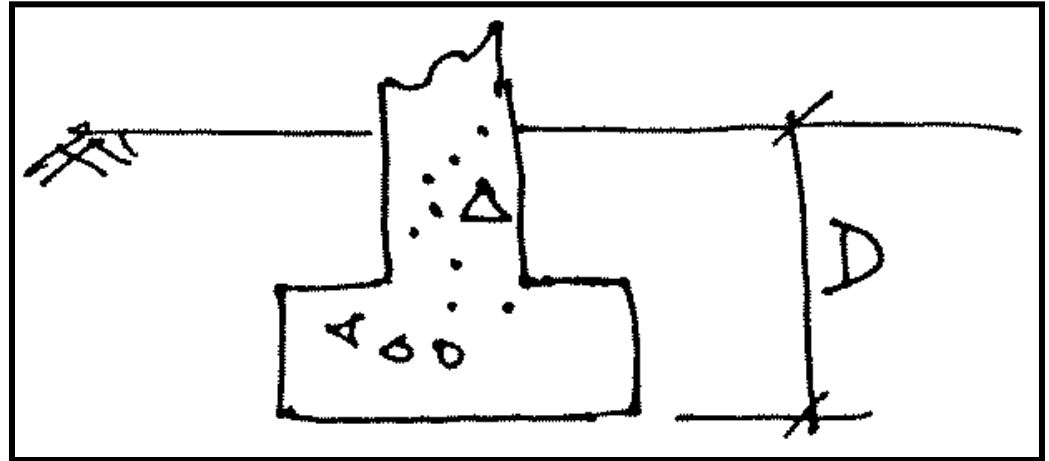


(h) Stepped footings

# Depth of Footing below Ground Surface (D)

Typically,

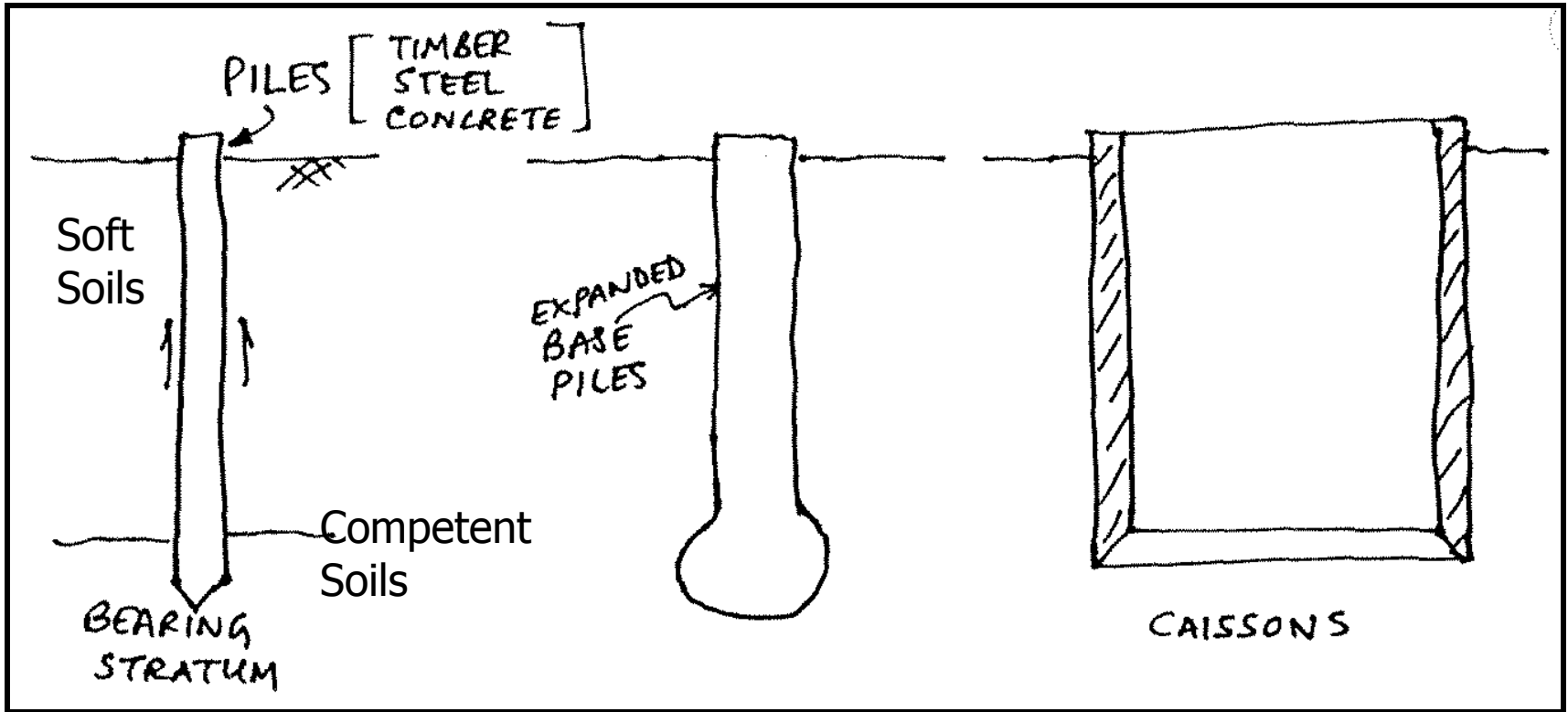
$D_{\text{Min}} \sim 0.5\text{m} - 0.75\text{m}$



**D reqd. To get below:**

- Top soil
- Depth of frost penetration
- Potential depth of erosion
- Seasonal moisture variations
- D also contributes to increasing bearing capacity.

# Deep Foundations



# Selection of a Foundation Type

## Governing Considerations

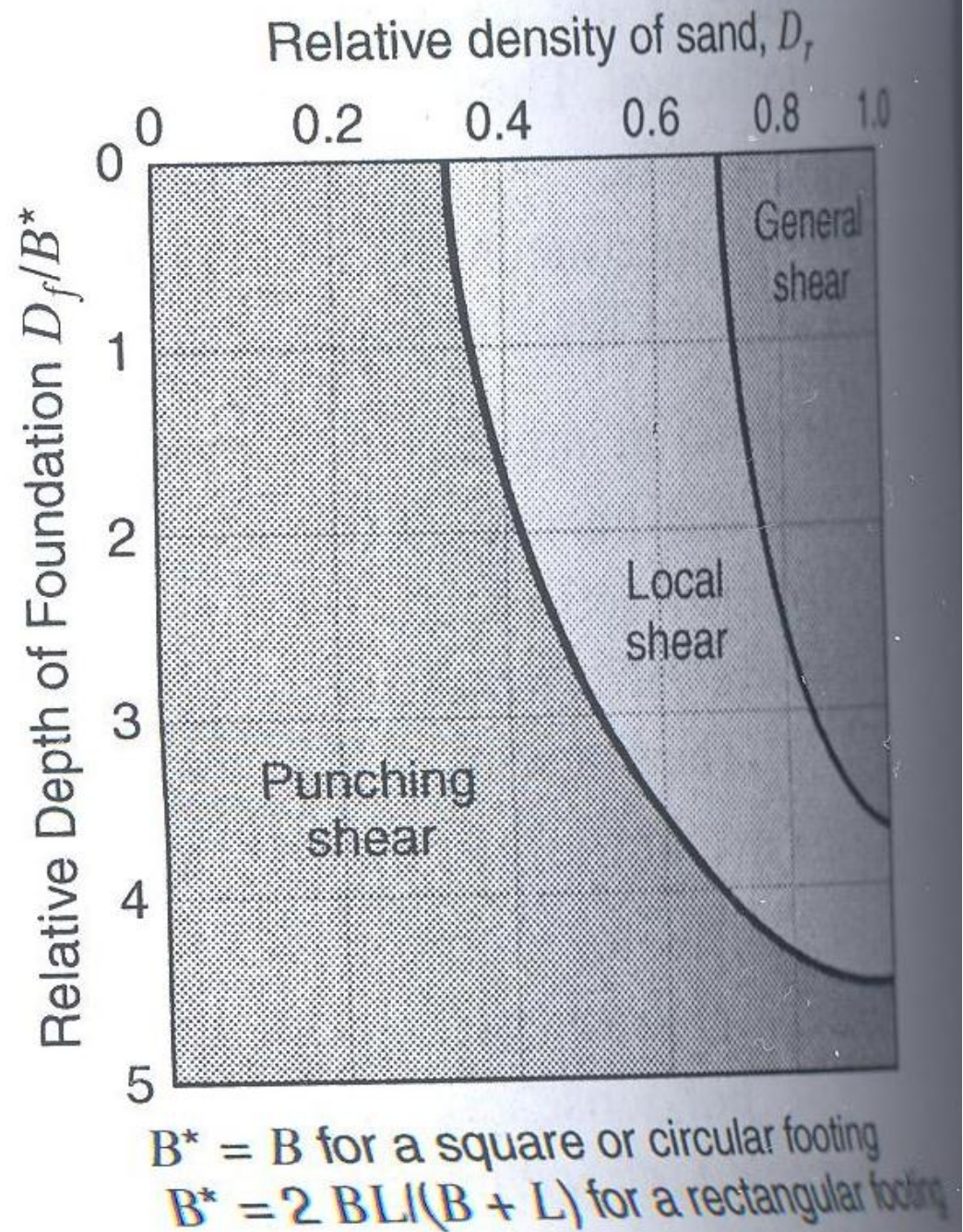
- Geological conditions;
- Loading conditions;
- Construction methods;
- Design Codes & Regulations
- Environmental Conditions;
- Impact on neighbouring structures
- Construction times and costs



# Pipe Lateral Load Test



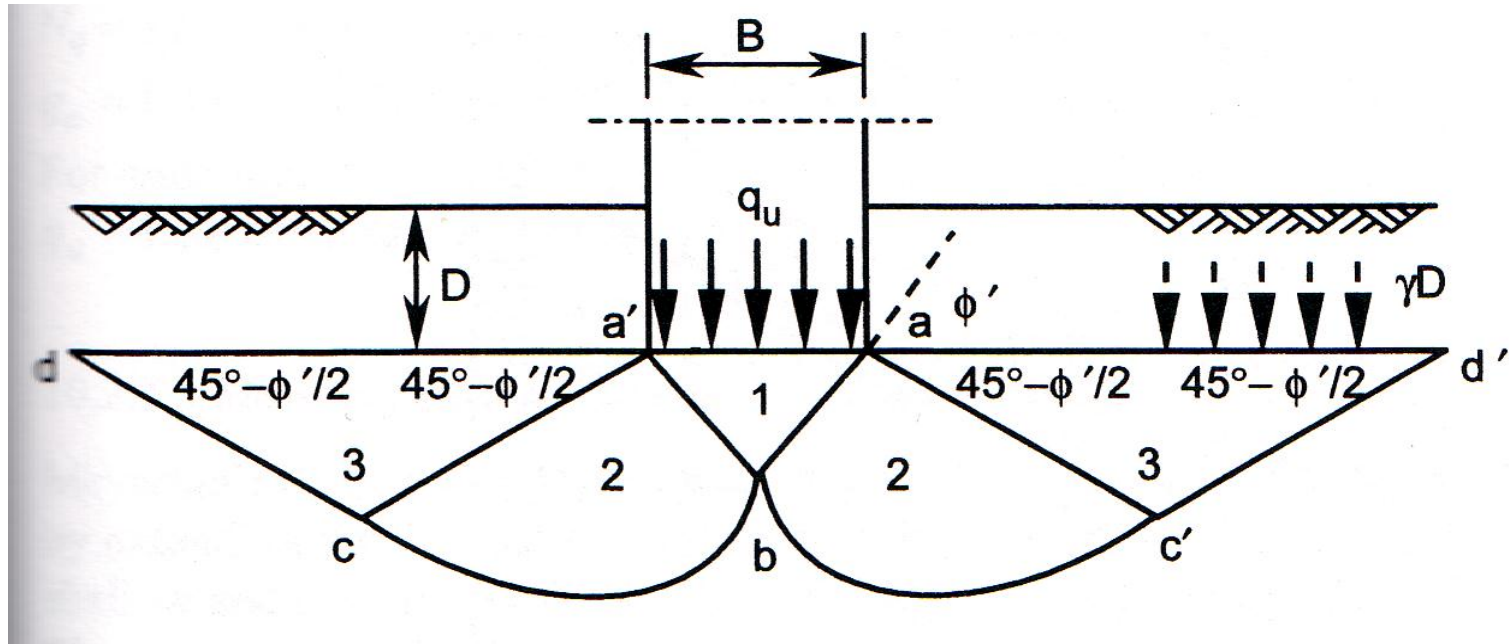
# Failure Mechanisms



# Bearing Capacity - Strip Footings

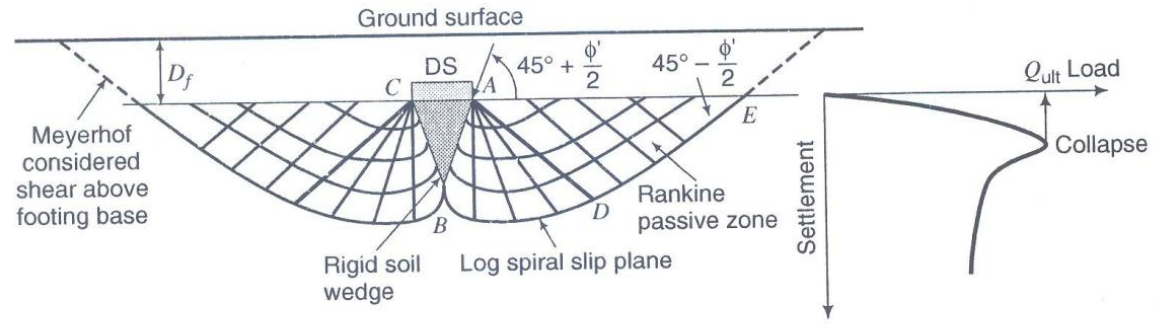
Long Footing – Width  $B$  – Essentially A 2-D Problem

$$L \rightarrow \infty$$



# Bearing Capacity - Strip Footings

Theoretical analysis  
for  $L \rightarrow \infty$  :



(a) General shear failure

Resistance due to the  
presence of soil above  
footing level

Resistance due to  
frictional soil weight  
below footing level

$$q_{ult} = q_u + \gamma D_f = \gamma D_f (N_q) + \frac{1}{2} \gamma B N_\gamma$$

$$q_u = \gamma D_f (N_q - 1) + \frac{1}{2} \gamma B N_\gamma$$

- $q_{ult}$  - Ultimate gross bearing capacity
- $q_u$  - Ultimate net bearing capacity
- $N_q, N_\gamma$  - Bearing capacity factors
- $\gamma$  - Effective unit weight of soil
- $D_f$  - Depth of footing below ground surface
- $B$  - Width of footing

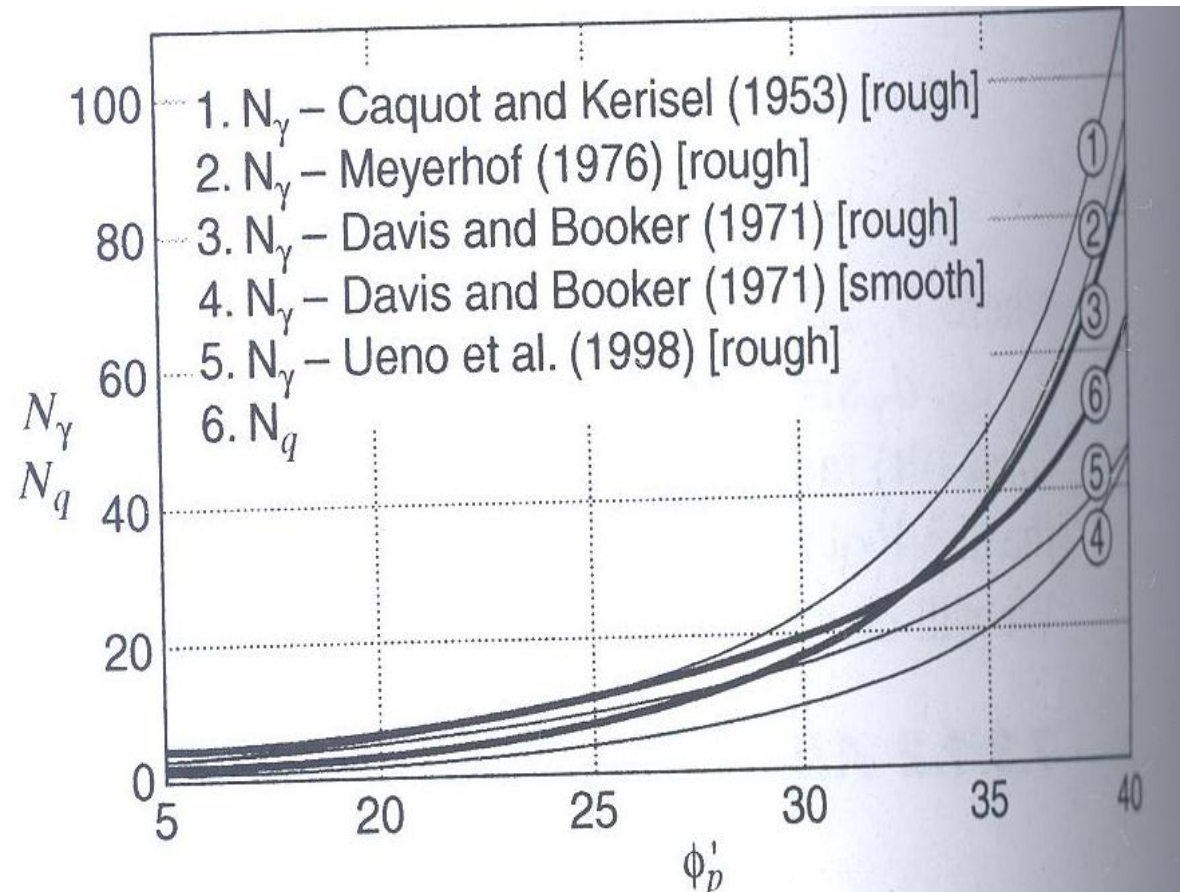
Note:  $N_q, N_\gamma$  - Essentially dependent on  $\phi'_p$

# Bearing Capacity - Strip Footings

$$q_u = \gamma D_f (N_q - 1) + \frac{1}{2} \gamma B N_\gamma$$

Several theoretical analyses have led to slightly different

$N_q$  and  $N_\gamma$  : see text book



# Bearing Capacity - Strip Footings

Total stress analysis  
(Undrained  
condition) :

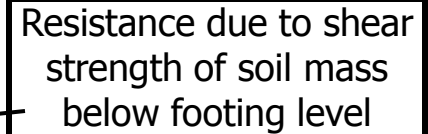
$$q_u = N_c s_u$$

$$N_c = 5.14$$

$$q_u = N_c s_u = 5.14 s_u$$

$$q_{ult} = q_u + \gamma D_f = 5.14 s_u + \gamma D_f$$

Resistance due to shear  
strength of soil mass  
below footing level



- $q_{ult}$  - Ultimate gross bearing capacity
- $q_u$  - Ultimate net bearing capacity
- $N_c$  - Bearing capacity factor (total stress)
- $\gamma$  - Effective unit weight of soil
- $D_f$  - Depth of footing below ground surface
- $s_u$  - Undrained Shear Strength of soil

# Allowable Bearing Capacity Strip Footings

$$q_a = (q_u / FOS) + \gamma D_f$$

- $q_a$  - Allowable bearing capacity
- $q_{ult}$  - Ultimate gross bearing capacity
- FOS - Factor of Safety
- $D_f$  - Depth of footing below ground surface

# Accounting for Geometric Variations in Footing Shape/Location and Loadings, Soil Compressibility, and Groundwater

## Effective Stress Analysis (ESA)

$$q_u = \gamma D_f (N_q - 1)(s_q d_q i_q b_q g_q r_q w_q) + 0.5 \gamma B' N_\gamma (s_\gamma d_\gamma i_\gamma b_\gamma g_\gamma r_\gamma w_\gamma)$$

## Total Stress Analysis (TSA)

$$q_u = 5.14 s_u (s_c d_c i_c b_c g_c r_c)$$

Note:  $B'$  = Effective Footing Width  
(discussed later)

Definition of the factors to account for the different variations :

$s_q, s_\gamma, s_c$  - Shape of Footing

$d_q, d_\gamma, d_c$  - Depth of Footing Embedment

$i_q, i_\gamma, i_c$  - Inclination of Load on Footing

$b_q, b_\gamma, b_c$  - Inclination of the Base of Footing

$g_q, g_\gamma, g_c$  - Inclination of the Ground Surface at the Footing Location

$r_q, r_\gamma, r_c$  - Soil Compressibility

$w_q, w_\gamma$  - Groundwater level

Commonly encountered situations need to account for:  
(i) footing shapes; (ii) footing depth; and (iii) groundwater

## Effective Footing Dimensions

$$B' = B - 2e_B$$

$$L' = L - 2e_L$$

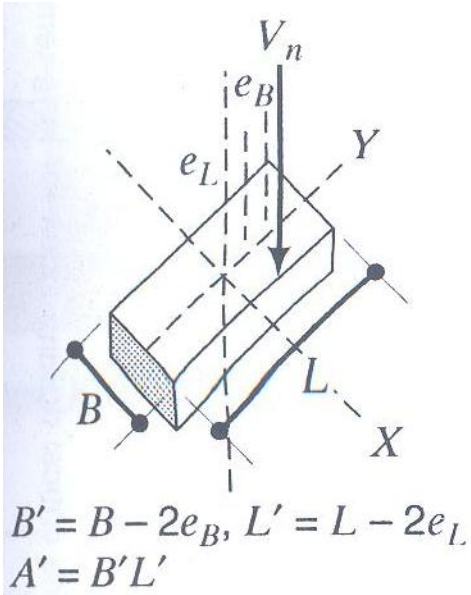
Definition of the factors to account for the different variations :

$s_q, s_\gamma, s_c$  - Shape of Footing

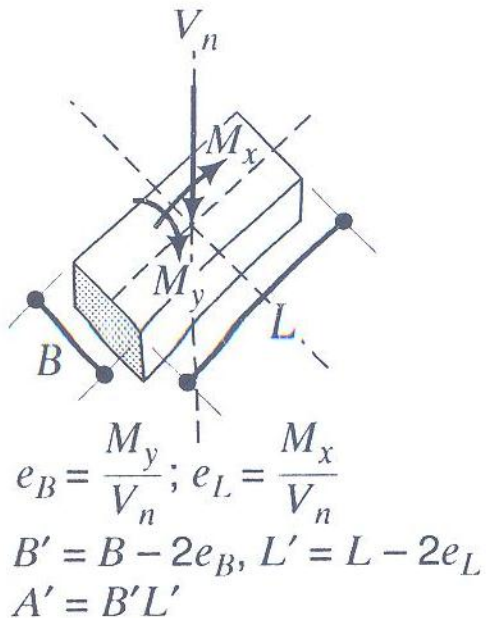
$d_q, d_\gamma, d_c$  - Depth of Footing Embedment

$w_q, w_\gamma$  - Groundwater level

# Eccentrically Loaded Footings and Effective Footing Sizes

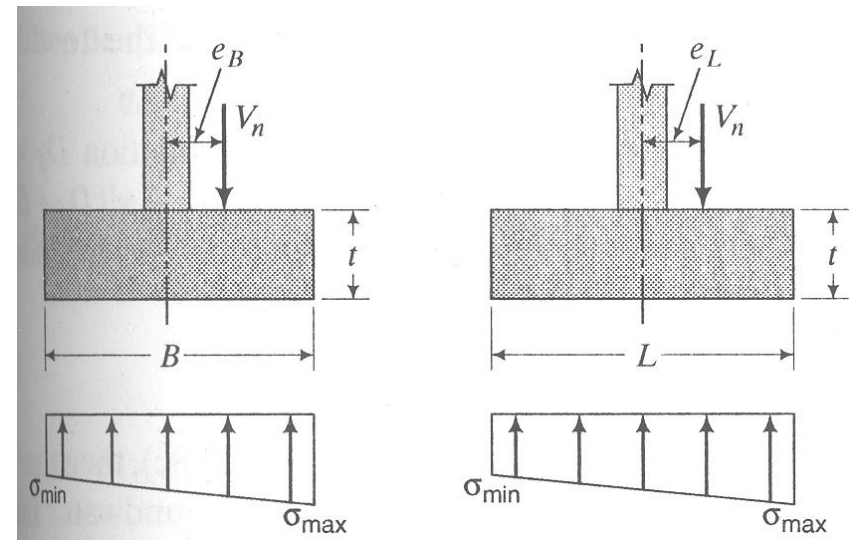


When resultant load is not coincident with the centroid of the foundation, the foundation dimension need to be theoretically adjusted to align the load center with the centroid.



See figures illustrating the effective footing dimensions  $B'$  and  $L'$ .

# Stress Distribution Beneath Eccentrically Loaded Footings



If  $e_B = (B/6)$ , or  $e_L = (L/6)$  then  $\sigma_{\min} = 0$ .  
 $\sigma_{\min} < 0$ , means footing would “lift off”.

Try and avoid this situation and design such that  
 $e_B < (B/6)$  AND  $e_L < (L/6)$ .

This is called “middle third rule”. The resultant  
 force should be within the middle third of the  
 footing to avoid zones where the footing will  
 not transmit loads to the soil.

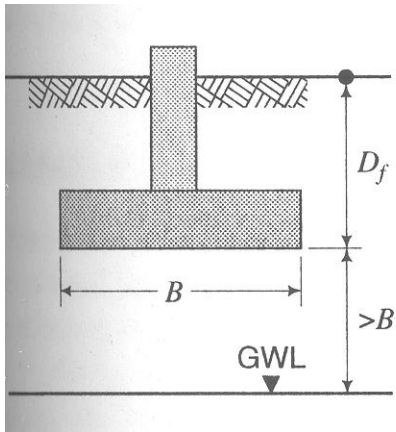
$$\sigma_{\max} = \frac{V_n}{A} \left( 1 + \frac{6e_B}{B} \right)$$

$$\sigma_{\min} = \frac{V_n}{A} \left( 1 - \frac{6e_B}{B} \right)$$

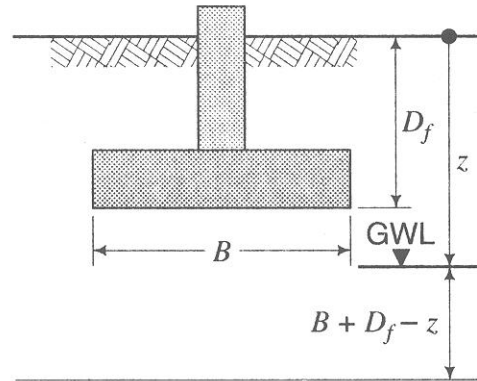
$$\sigma_{\max} = \frac{V_n}{A} \left( 1 + \frac{6e_L}{L} \right)$$

$$\sigma_{\min} = \frac{V_n}{A} \left( 1 - \frac{6e_L}{L} \right)$$

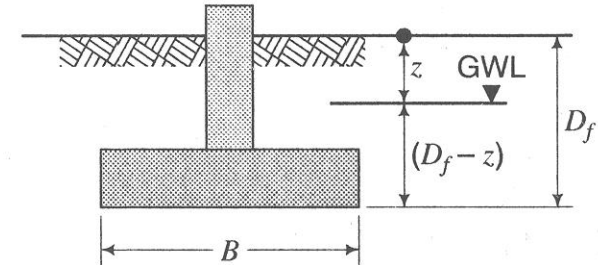
# Correction for Groundwater: for ESA



**Case 1**



**Case 2**



**Case 3**

Case 1:  $z > (B + D_f)$

$$w_q = 1; w_\gamma = 1$$

Case 2:  $(B + D_f) > z > D_f$

$$w_q = 1; w_\gamma = \frac{(z - D_f)}{B} + \frac{\gamma'}{\gamma_{sat}} \left( 1 + \frac{D_f}{B} - \frac{z}{B} \right)$$

Case 3:  $z < D_f$

$$w_q = \frac{z}{D_f} + \frac{\gamma'}{\gamma_{sat}} \left( 1 - \frac{z}{D_f} \right); w_\gamma = \frac{\gamma'}{\gamma_{sat}}$$

# Equations for $N_q$ and $N_\gamma$

$$N_q = e^{\pi \tan \phi'_p} \tan^2 \left( 45^\circ + \frac{\phi'_p}{2} \right); \phi'_p \text{ in degrees}$$

Caquot and Kerisel (1953)<sup>2</sup>:  $N_\gamma = 2(N_q + 1) \tan \phi'_p$ ;  $\phi'_p$  in degrees

Meyerhof (1976):  $N_\gamma = (N_q - 1) \tan(1.4\phi'_p)$ ;  $\phi'_p$  in degrees

Davis and Booker (1971)<sup>3</sup> :

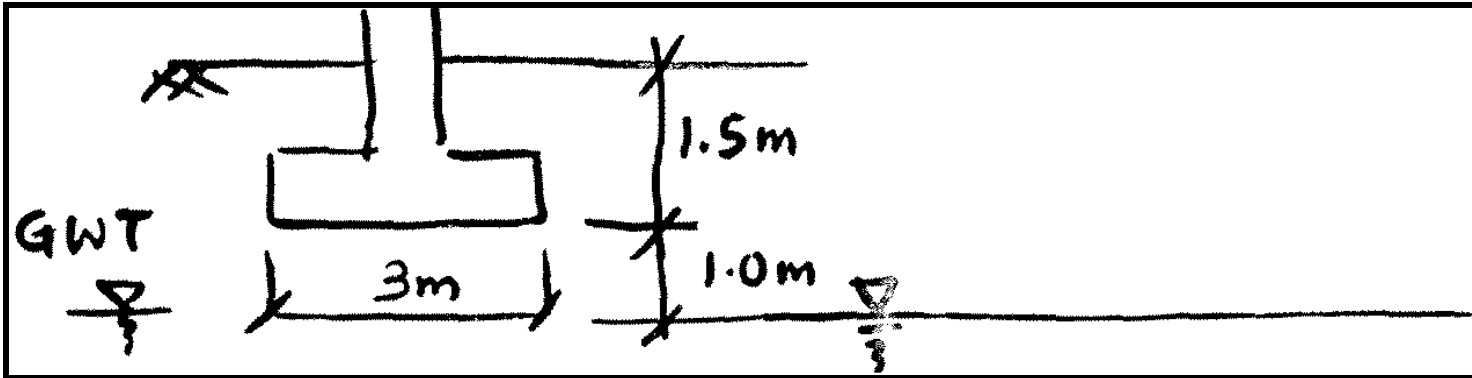
smooth foundation,  $N_\gamma = 0.0663 \exp(9.3\phi'_p)$ ;  $\phi'_p$  in radians

rough foundation,  $N_\gamma = 0.1054 \exp(9.6\phi'_p)$ ;  $\phi'_p$  in radians

Ueno et al. (1998)<sup>4</sup> :

rough foundation,  $N_\gamma = 0.477 \exp(6.52\phi'_p)$ ;  $\phi'_p$  in radians

## Example (1)



### Square Footing :

Density above GWT =  $\gamma_{\text{moist}} = 16.0 \text{ kN/m}^3$ ;  $\phi' = 35^\circ$  ;  $c' = 0$

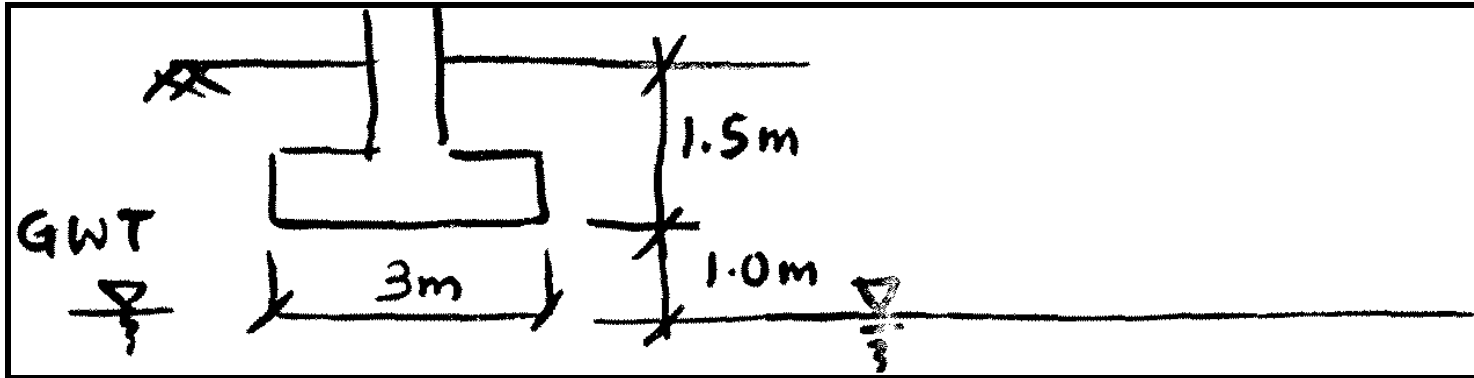
Density below GWT =  $\gamma_{\text{sat}} = 19.0 \text{ kN/m}^3$ ;  $\phi' = 35^\circ$  ;  $c' = 0$

Find  $q_u$ ,  $q_{\text{ult}}$ , and  $q_a$

### Effective Stress Analysis (ESA)

$$q_u = \gamma D_f (N_q - 1)(s_q d_q w_q) + 0.5 \gamma B' N_\gamma (s_\gamma d_\gamma w_\gamma)$$

## Example (1)



$$q_u = \gamma D_f (N_q - 1)(s_q d_q w_q) + 0.5 \gamma B' N_\gamma (s_\gamma d_\gamma w_\gamma)$$

## Example (1) (con't)

$$N_q = e^{\pi \tan \phi'_p} \tan^2 \left( 45^\circ + \frac{\phi'_p}{2} \right); \phi'_p \text{ in degrees}$$

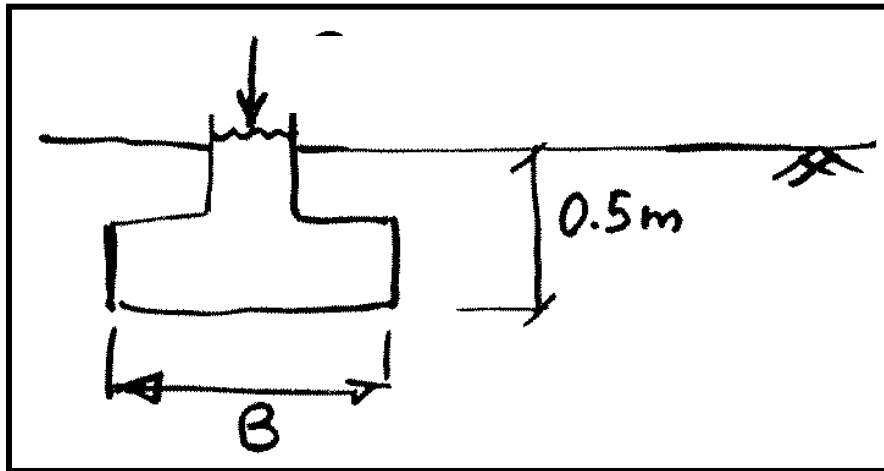
## **Example (1) (con't)**

## Example (1) (con't)

$$q_u = \gamma D_f (N_q - 1)(s_q d_q w_q) + 0.5 \gamma B' N_\gamma (s_\gamma d_\gamma w_\gamma)$$

## **Example (1) (con't)**

## Example (2) - Square Footing on clay.



$$\gamma = 18.5 \text{ kN/m}^3$$

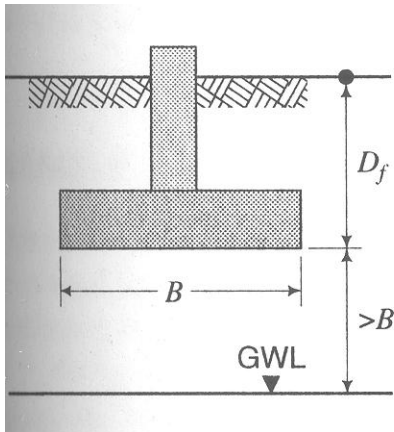
$$s_u = 60 \text{ kPa}$$

Total Stress Analysis (TSA)

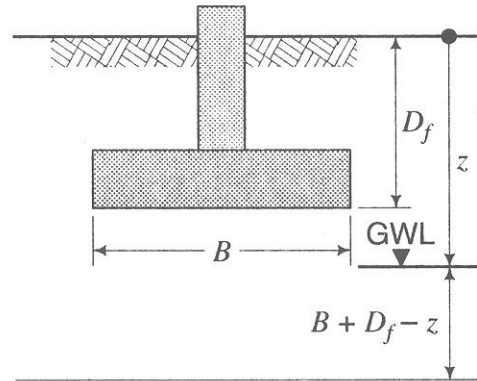
$$q_u = 5.14 s_u (s_c d_c)$$



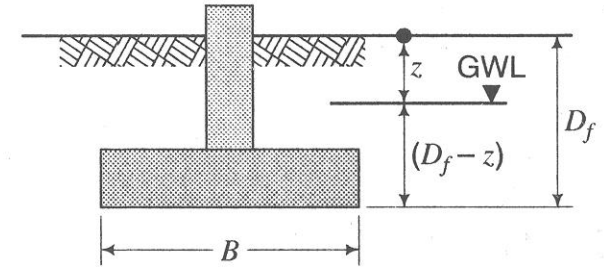
# Correction for Groundwater: for ESA



**Case 1**



**Case 2**



**Case 3**

Case 1:  $z > (B + D_f)$        $w_q = 1; w_\gamma = 1$

Note:  $z$  = Depth to Groundwater Table

Case 2:  $(B + D_f) > z > D_f$        $w_q = 1; w_\gamma = \frac{(z - D_f)}{B} + \frac{\gamma'}{\gamma_{sat}} \left(1 + \frac{D_f}{B} - \frac{z}{B}\right)$

Case 3:  $z < D_f$        $w_q = \frac{z}{D_f} + \frac{\gamma'}{\gamma_{sat}} \left(1 - \frac{z}{D_f}\right); w_\gamma = \frac{\gamma'}{\gamma_{sat}}$