SOIL MECHANICS II
CIVL 311
COURSE NOTES
2012

Module 4
Lateral Earth Pressures

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Module 4
Lateral Earth Pressures
Overall Learning Objectives

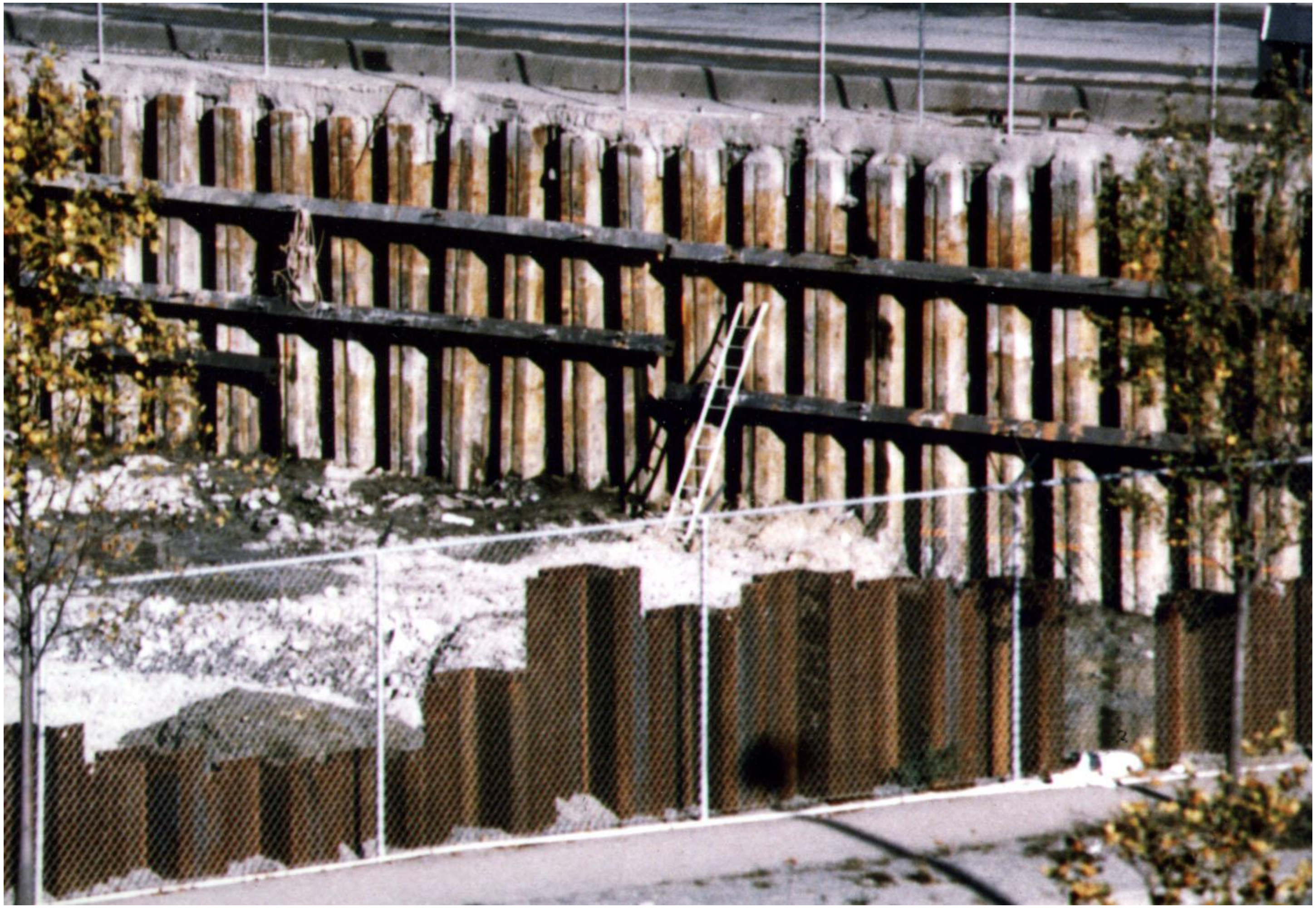
- Basic Concepts (Book 2 pp. 355-359)
- Lateral Stresses from Surface Loads (Book 2 pp. 359)
- Coulomb's Earth Pressure Theory (Book 2 pp. 364)
- Rankine's Lateral Earth Pressure for a Sloping Backfill and a Sloping Wall Face (Book 2 pp. 367)
- Lateral Earth Pressure for a Total Stress Analysis (Book 2 pp. 369-371)

Reading Assignment

- Class Budhu Text - Book 2 Ch. 10 Earth Retaining Structures (pages 355 to 385)

Slopes and Earth Retaining Walls

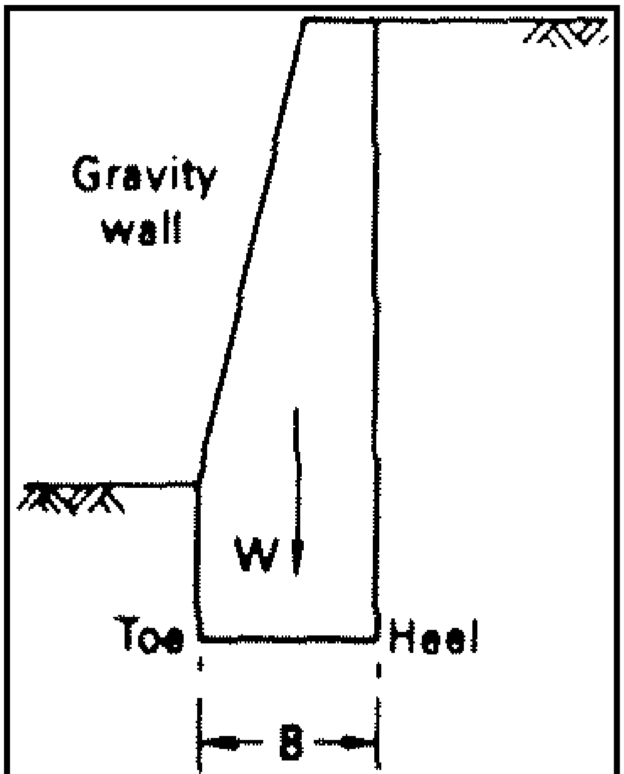
Lateral Earth Pressures

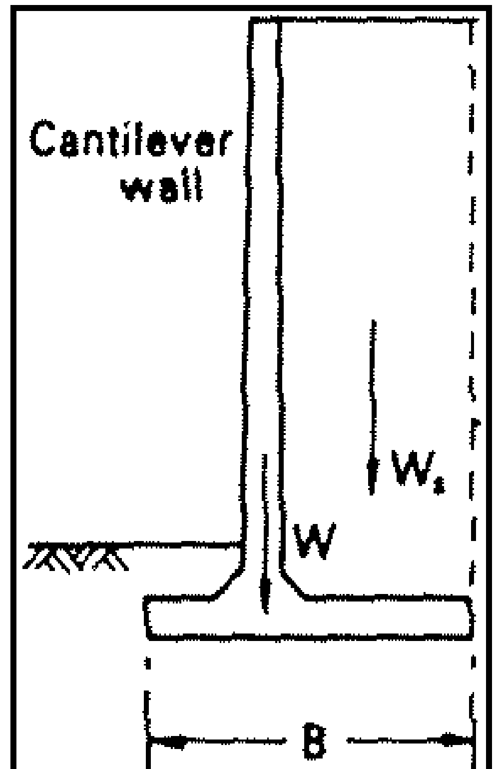


Pressure on Soil Chambers

Typical Problems
Earth retaining structures

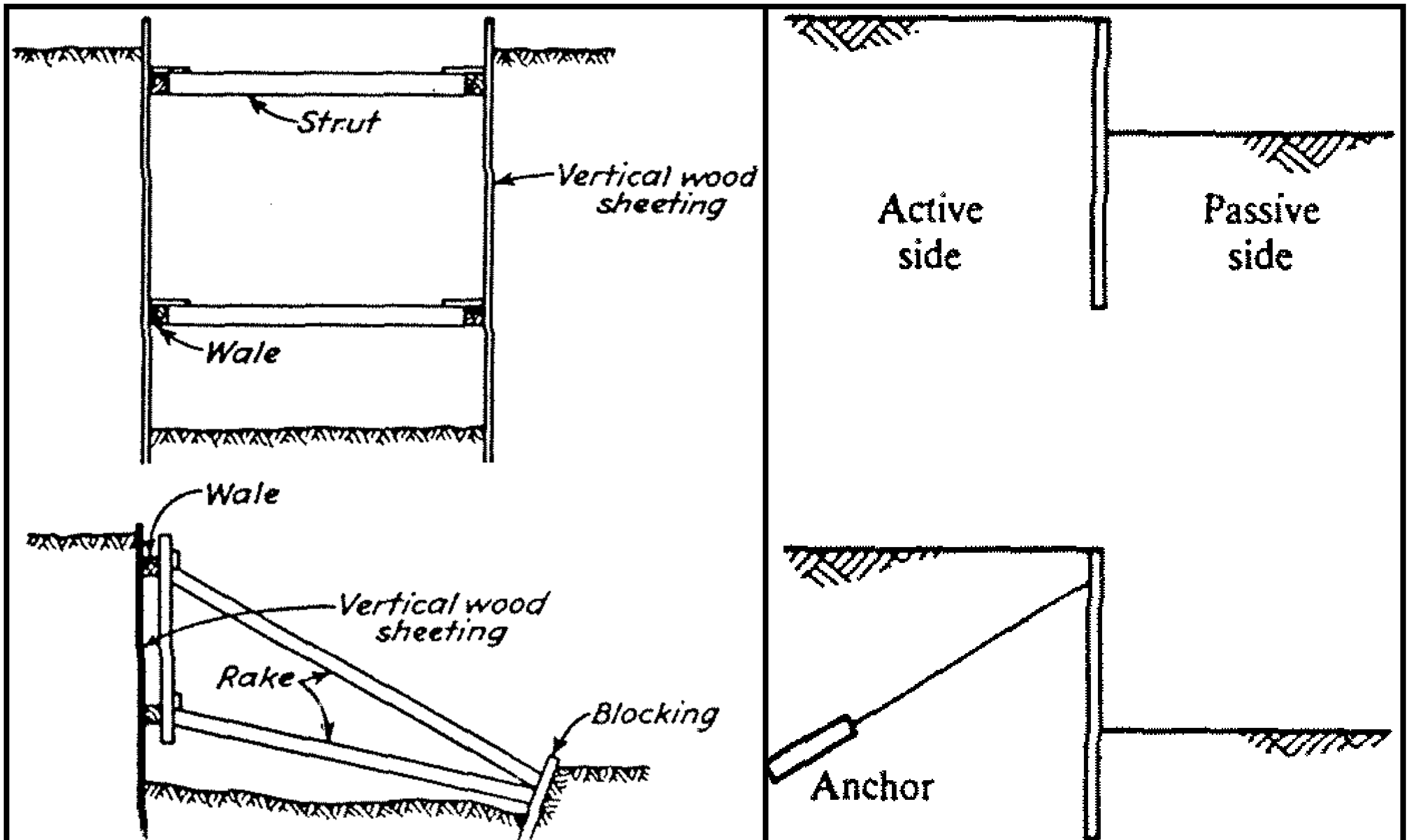
a) Rigid Wall- can translate or rotate only





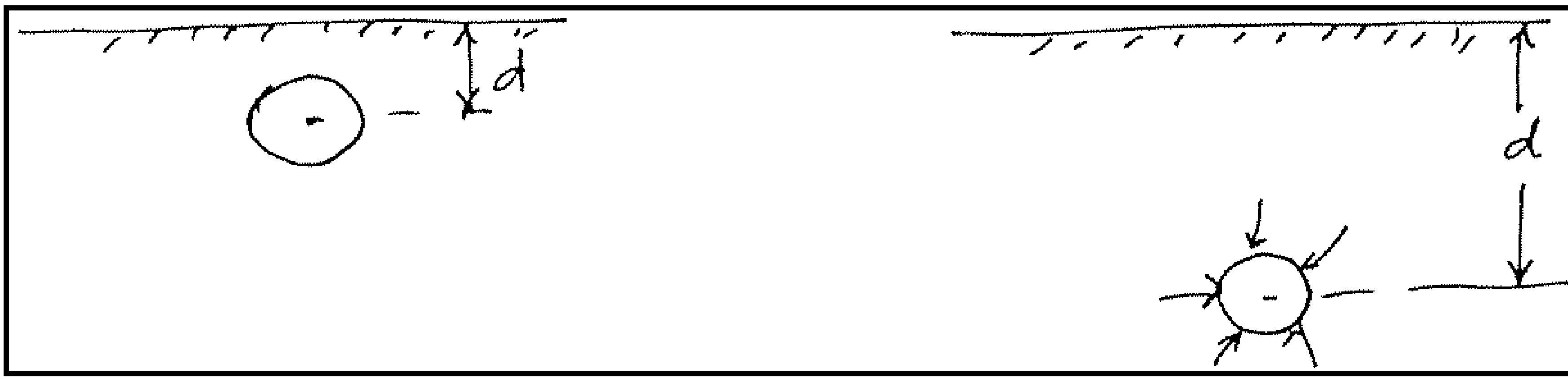
Typical Problems
Earth retaining structures

b) Flexible Walls – can bend (deform) in addition



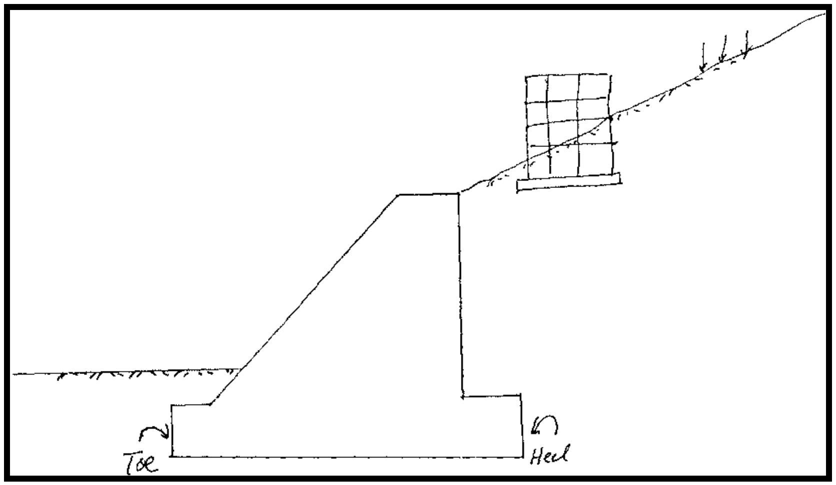
Typical Problems
Earth retaining structures

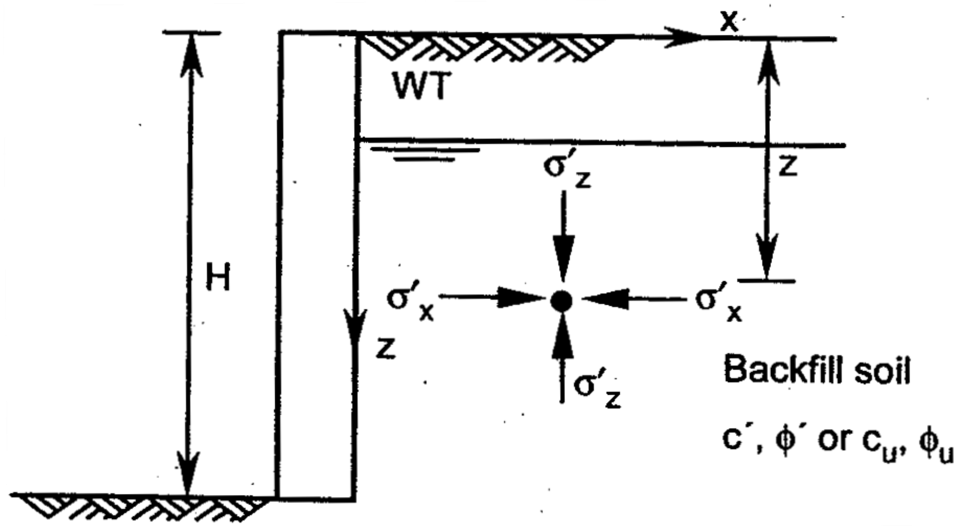
c) Buried Structures



Pressure on buried structure = *f* (flexibility of wall of buried structure and soil behaviour) = i.e. soil structure interaction controls the problem

Forces on gravity retaining structures





Earth pressure on wall is due to:

(i) self weight of soil; and (ii) live load on backfill

Only rigid retaining structures in CIVL 311

Rankine Active & Passive Stress States

Assumptions:

- The earth retaining wall is vertical
- The interface between the wall and soil is frictionless
- The soil surface is horizontal and no shear stress acts on vertical and horizontal boundaries
- The wall is rigid and extends to an infinite depth in a granular, isotropic, homogeneous soil mass,
- The soil is initially loose and initially at an “at-rest” state.

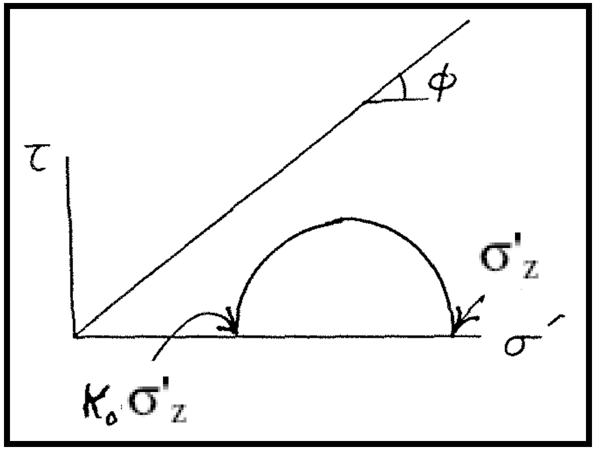
Active earth pressure

1. Initial stress state (geostatic) :
   → Soil in state of elastic equilibrium:

   $\sigma'_Z = \sigma'_1 = \gamma' Z$ (Note: $\gamma' = \gamma_{sat} - \gamma_w$)

   $\sigma'_X = \sigma'_3 = K_0 \gamma' Z$

   where K_0 = Coefficient lateral earth pressure "at rest"

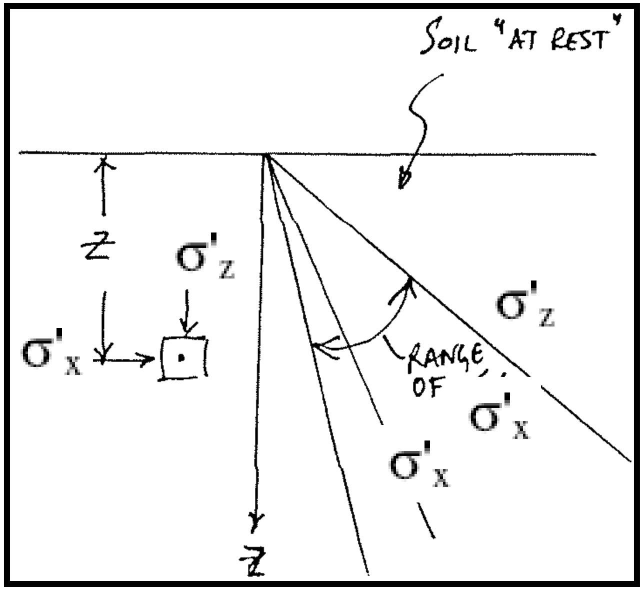


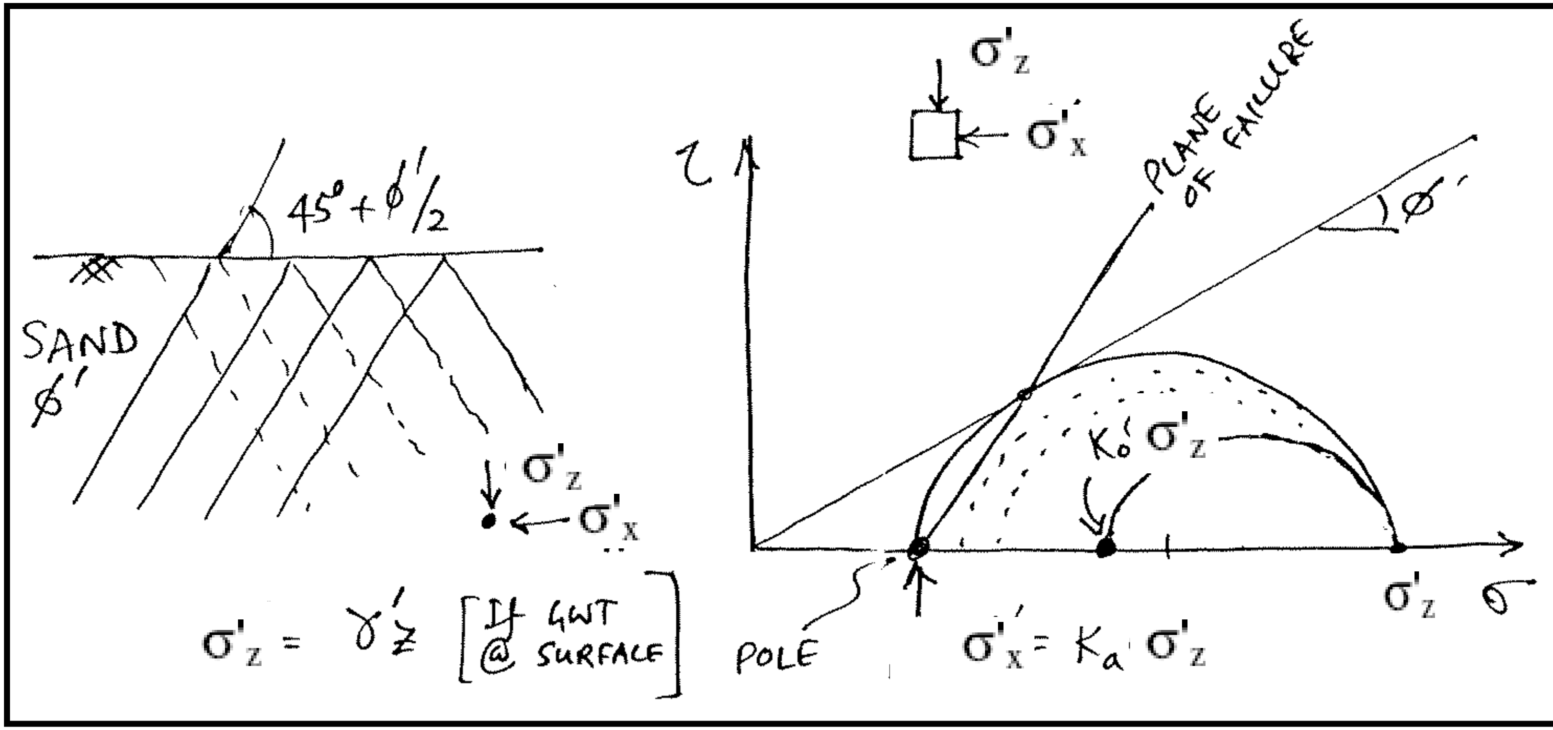
2. Now allow soil mass to relax (stretch) laterally:

BB fixed, AA→A'A'

σ'_Z stays constant; and

σ'_X decreases to a limiting minimum value





With further “stretching” laterally – no change in stresses, but failure (slip) along planes at 45 + ϕ'/2 to horiz. throughout entire soil mass

At failure,

$(\sigma'_3)_f = (\sigma'_X)_f \qquad (\sigma'_1)_f = (\sigma'_Z)_f$

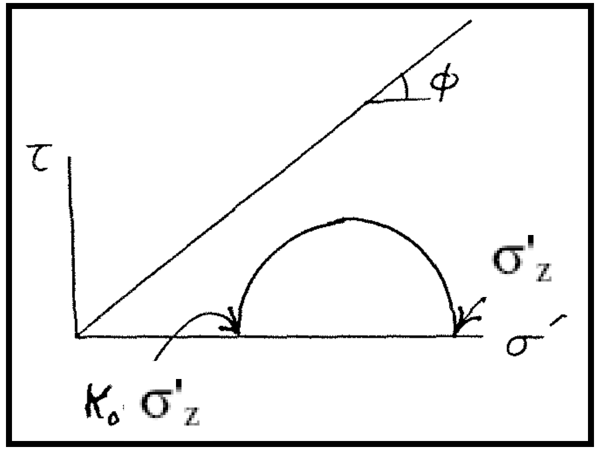
$(\sigma'_3)_f / (\sigma'_1)_f = (\sigma'_x)_f / (\sigma'_z)_f = (1 - \sin\phi') / (1 + \sin\phi') = \tan^2(45 - (\phi'/2)) = K_a$

$\Rightarrow (\sigma'_x)_a = K_a\, \sigma'_z$; K_a = coefficient of active lateral earth pressure

Passive earth pressure

1. Initial stress state (geostatic) :
   → Soil in state of elastic equilibrium:
   $\sigma'_Z = \sigma'_1 = \gamma' Z$
   $\sigma'_X = \sigma'_3 = K_0 \gamma' Z$

   where K_0 = Coefficient lateral earth pressure "at rest"



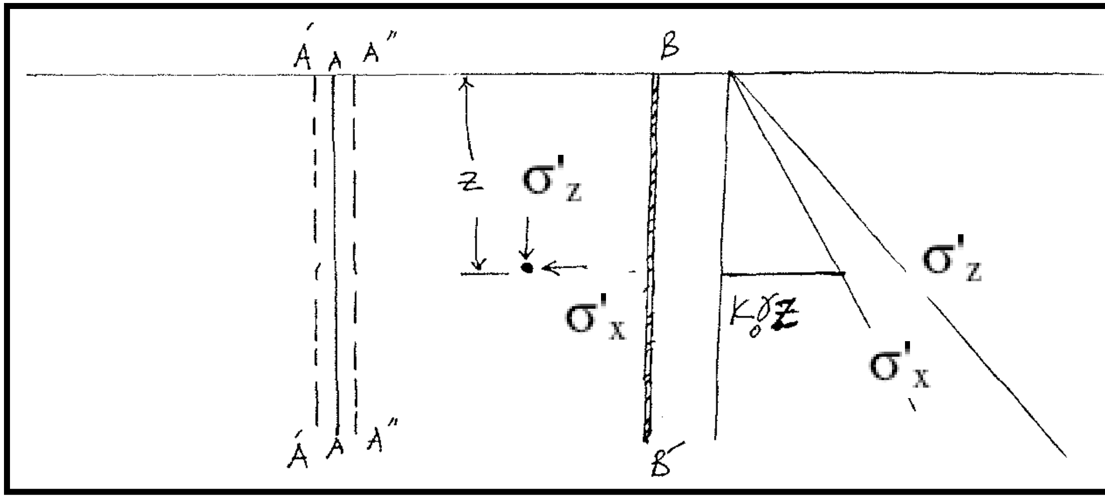
σ'_x

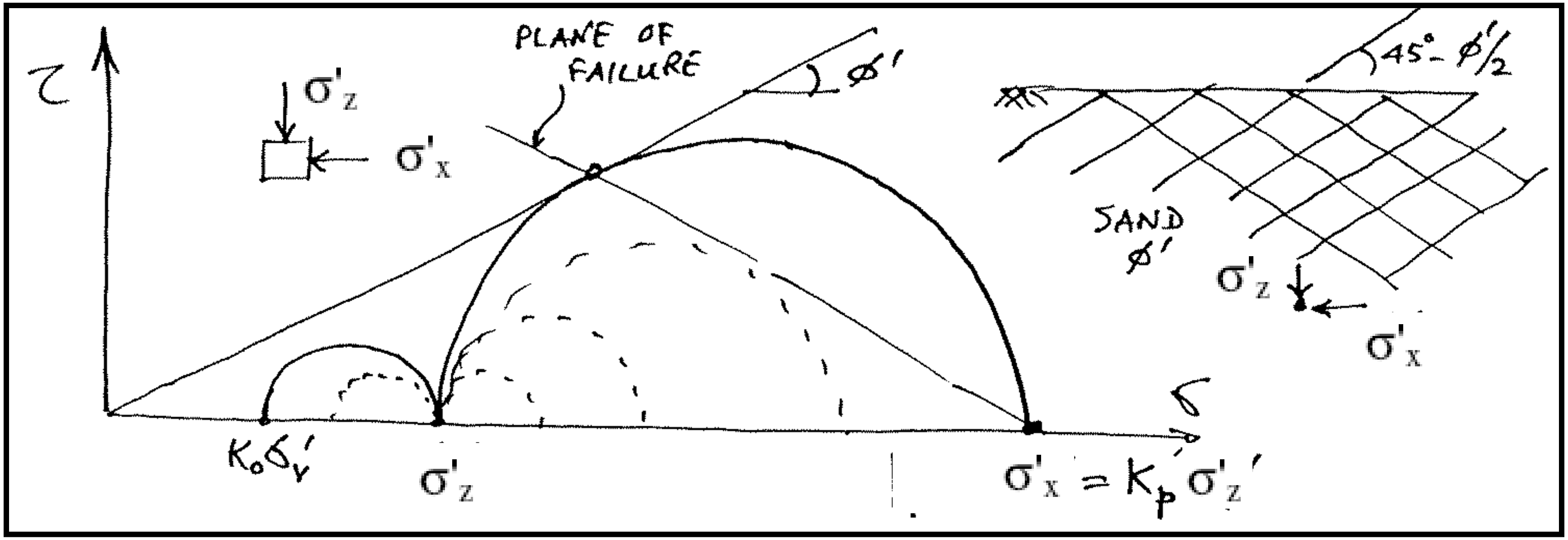
2. Allow soil mass to compress laterally:

BB fixed, AA→A"A'

σ'_z stays constant; and

σ'_x increases to a limiting maximum value





With further “compression” laterally – no change in stresses, but failure (slip) along planes at 45 - $\phi'/2$ to horiz. throughout entire soil mass

At failure,

$(\sigma'_1)_f = (\sigma'_X)_f \qquad (\sigma'_3)_f = (\sigma'_Z)_f$

$(\sigma'_1)_f / (\sigma'_3)_f = (\sigma'_x)_f / (\sigma'_z)_f = (1 + \sin\phi') / (1 - \sin\phi') = \tan^2(45 + (\phi'/2)) = K_p$

$\Rightarrow (\sigma'_x)_p = K_p\, \sigma'_z$; K_p = coefficient of passive lateral earth pressure

$K_p = 1/K_a$

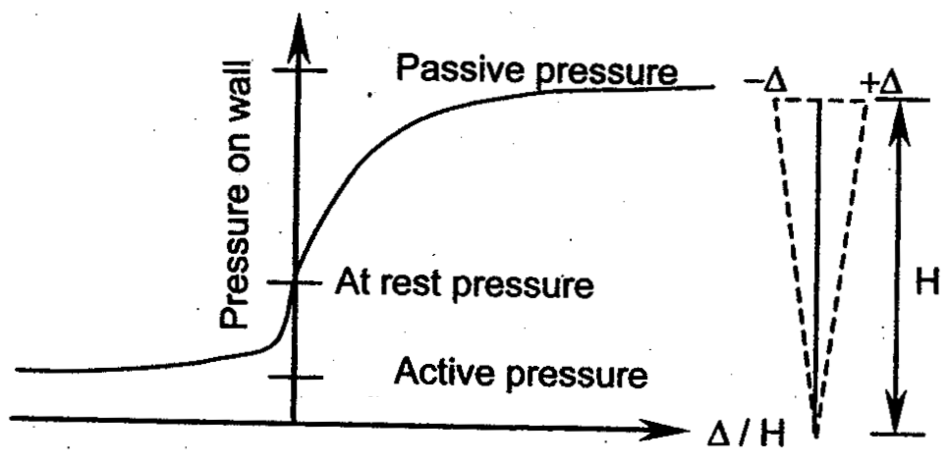
Soil Type	Translation or rotation y/h	
	Active	Passive
Dense cohesionless	0.001	0.020
Loose cohesionless	0.004	0.060
Soft fine-grained soils (undrained condition)	0.020	0.040
Stiff fine-grained soils (undrained condition)	0.010	0.020

as per Canadian Foundation Engineering Manual

If movement insufficient

$K_a < K < K_0$ – Active

$K_0 < K < K_p$ – Passive



e.g., $\phi' = 30^\circ$

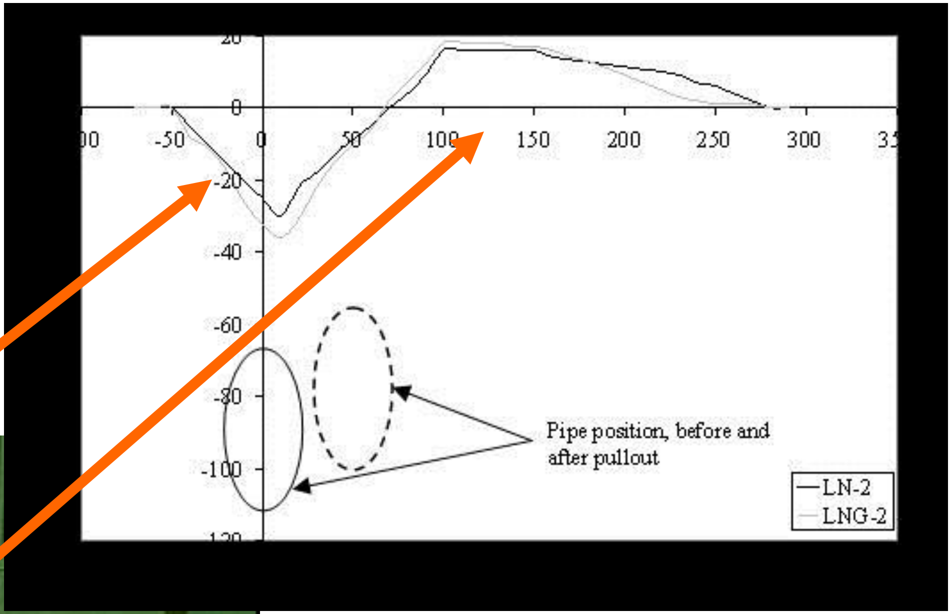
At Rest $\Rightarrow K_o \approx 1 - \sin\phi' \approx 0.5$

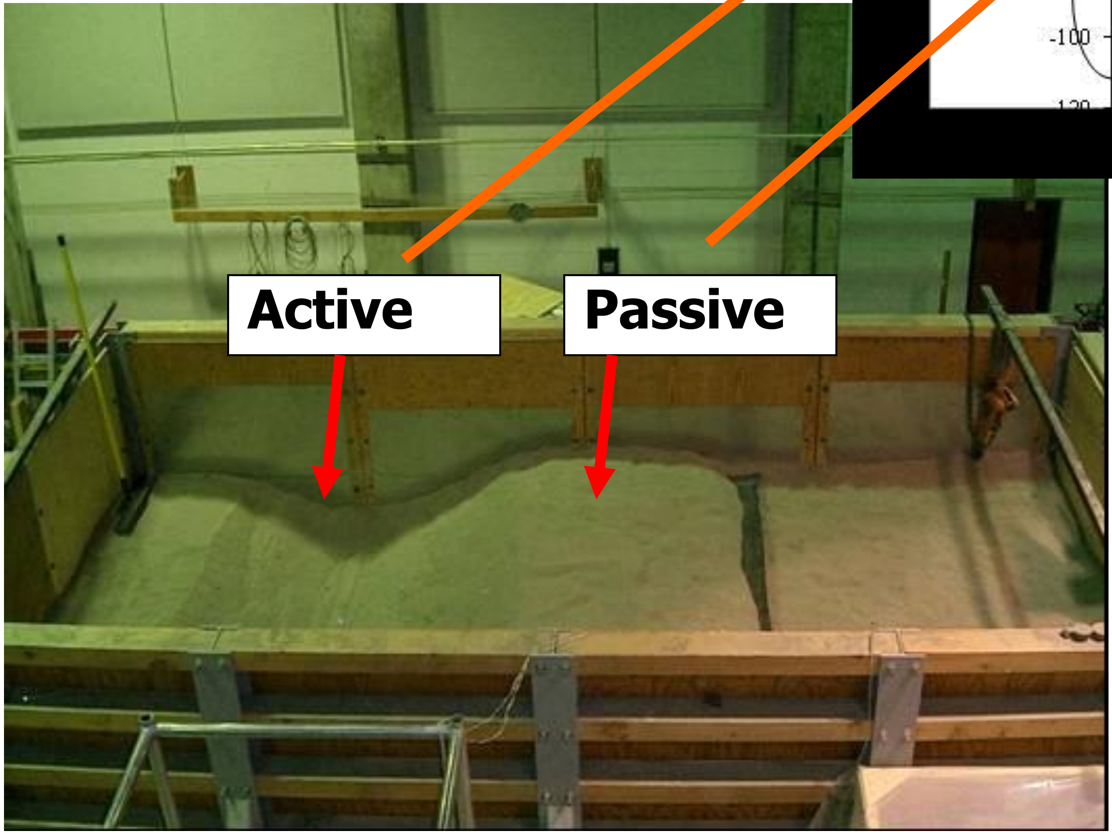
Active $\Rightarrow K_a = (1 - \sin\phi')/(1 + \sin\phi') = 0.33$

Passive $\Rightarrow K_p = (1 + \sin\phi')/(1 - \sin\phi') = 3.0$

$$\underset{\text{min}}{K_a} < K_o < \underset{\text{max}}{K_p}$$

Pipe-soil interaction



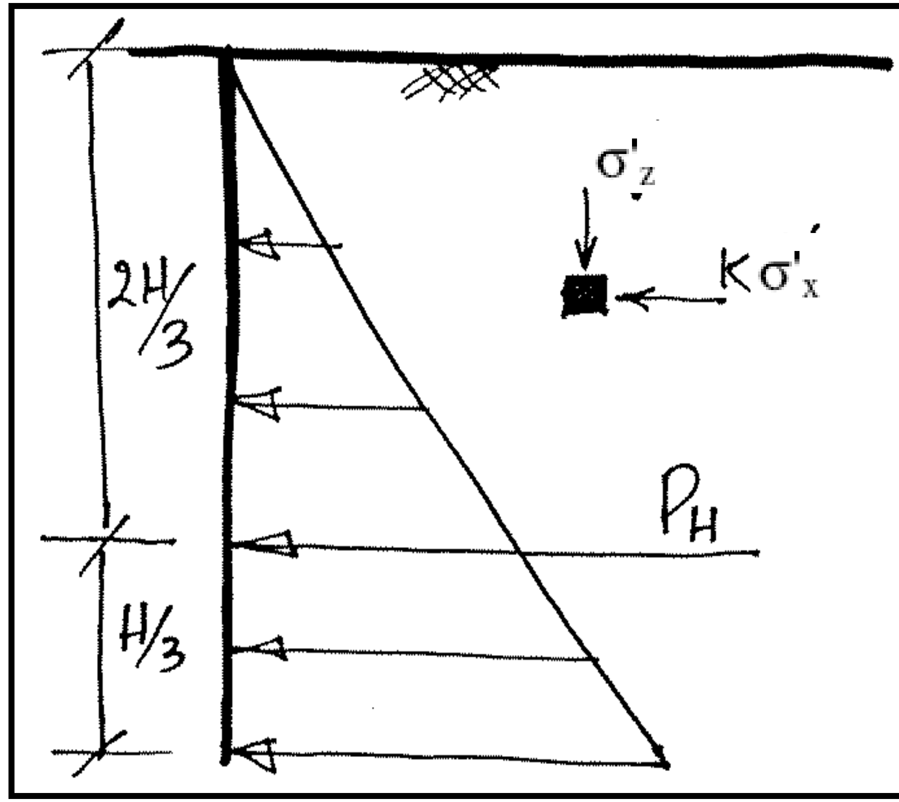


Horizontal Ground Surface with Dry Sand Backfill

(i) due to self weight of soil:

$\sigma'_z = \gamma' z$

$$\sigma'_x \Rightarrow \begin{cases} K_a(\gamma' z) \\ K_o(\gamma' z) \\ K_p(\gamma' z) \end{cases}$$



$$P_H \Rightarrow \begin{cases} \text{Active } P_a = \tfrac{1}{2} K_a \gamma' H^2 \\ \text{Passive } P_p = \tfrac{1}{2} K_p \gamma' H^2 \\ \text{At rest } P_o = \tfrac{1}{2} K_o \gamma' H^2 \end{cases}$$

NOTE: Upper Case 'P' means force on the wall / unit length of wall

Horizontal Ground Surface with Dry Sand Backfill + Surcharge

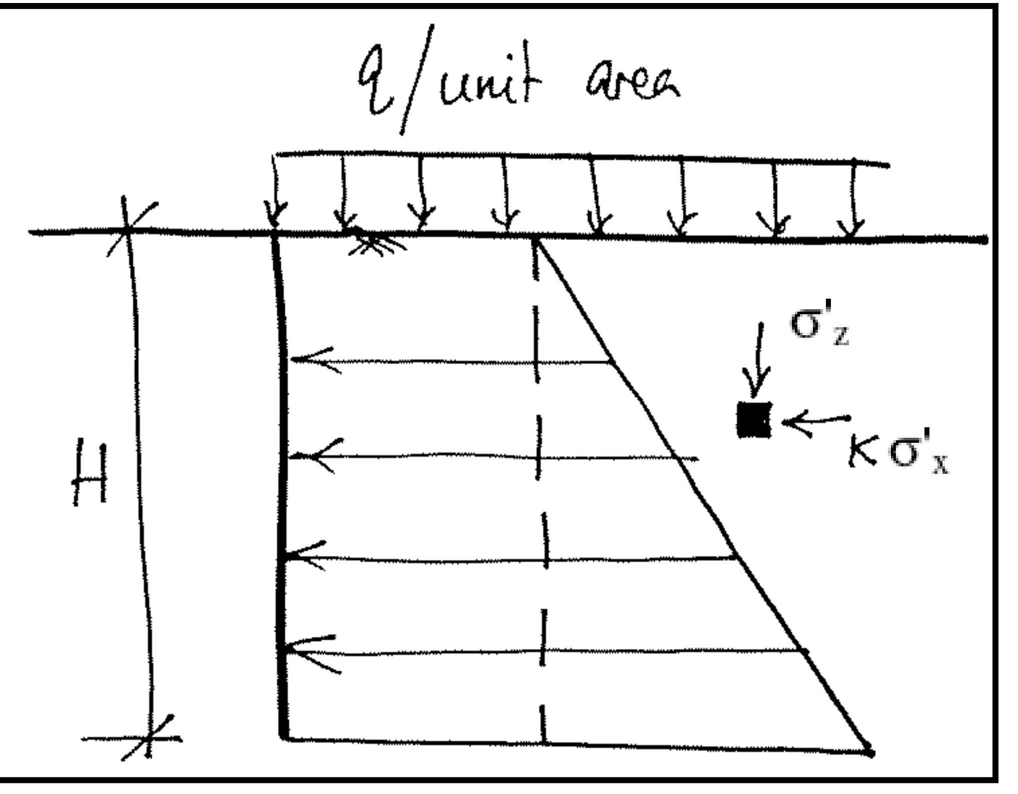
(ii) Self weight +surcharge o
backfill

$\sigma'_z = \gamma' z + q$

$K_a (\gamma' z+q)$

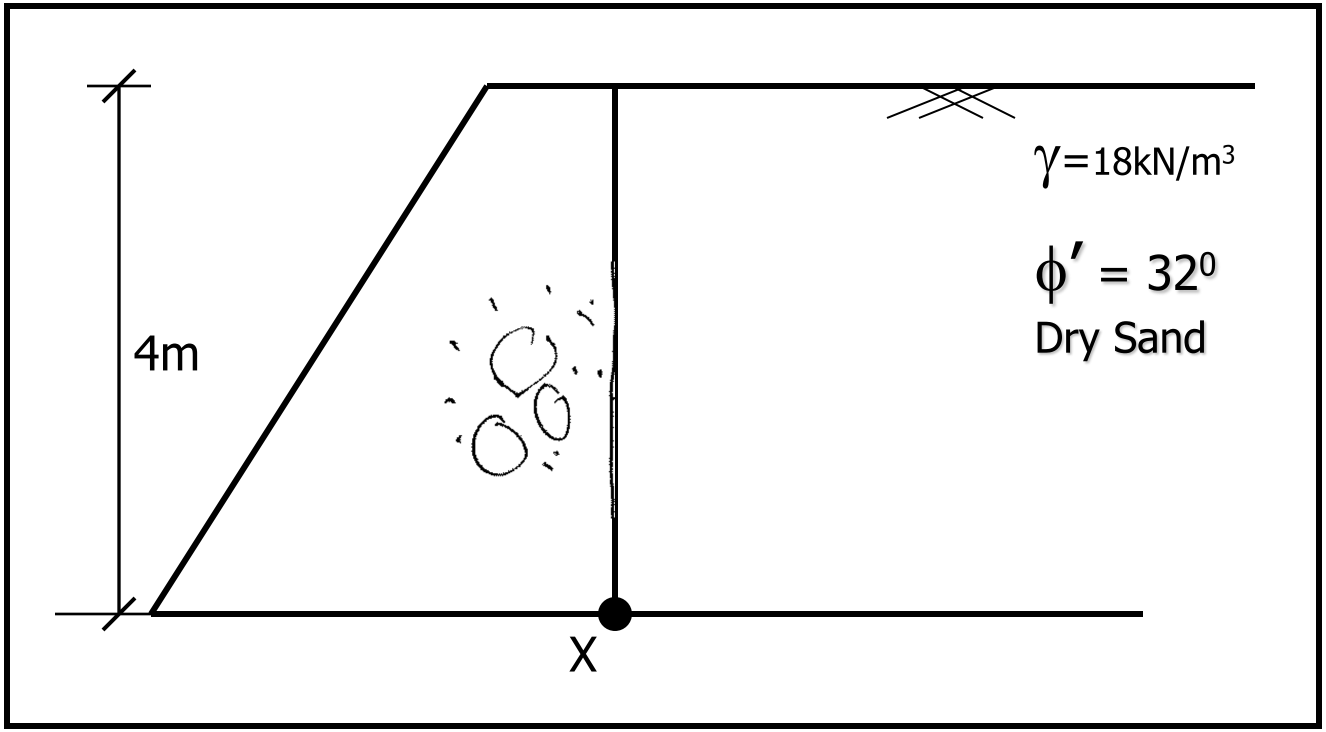
$(\sigma'_x) \Rightarrow K_o (\gamma' z+q)$

$K_p (\gamma' z+q)$



$P_H = ½ K\gamma' H^2 + qKH$ /m length of wall
where $K = K_a$, or K_0 , or K_p

Example: Active earth pressure Dry sand backfill – smooth wall



Find active pressure distribution & total active force

Example (con't):

Rankine Theory:

$$K_a = \frac{1 - \sin 32}{1 + \sin 32}$$

$K_a = 0.31$

$(\sigma_z')_A$ = 18 x 4 = 72 kPa,

therefore, $(\sigma_x')_A$ = 72 x 0.31 = 22.3 kPa

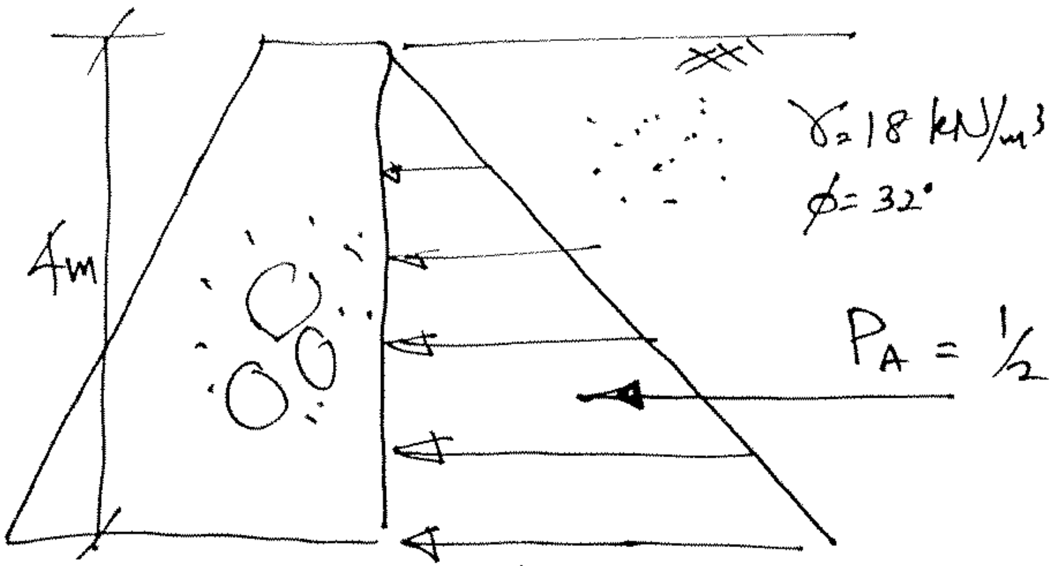
$(P_A)_X$ = ½ $K_a \gamma H^2$ = 0.31/2 x [18 x 4^2]

= 44.6 kN /m length of wall

If K_0 condition was present behind the wall:

$K_0 \sim 1 - \sin 32^0$

$K_0 = 1 - 0.53 = 0.47$



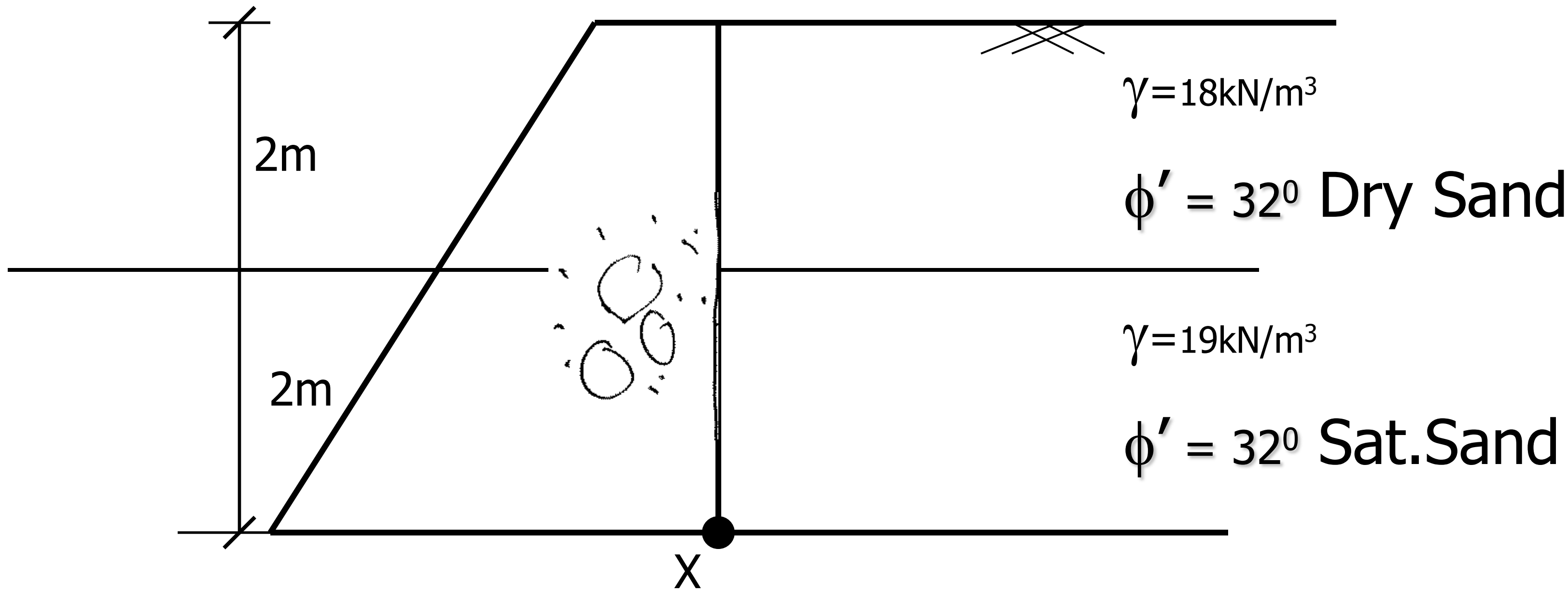
$P_A = \frac{1}{2} K_0 \gamma H^2 = \frac{1}{2} \times 0.47 \times 18 \times 4^2$

$= 67.7$ kN/m length

$K_0 \gamma H = 0.47 \times 18 \times 4$

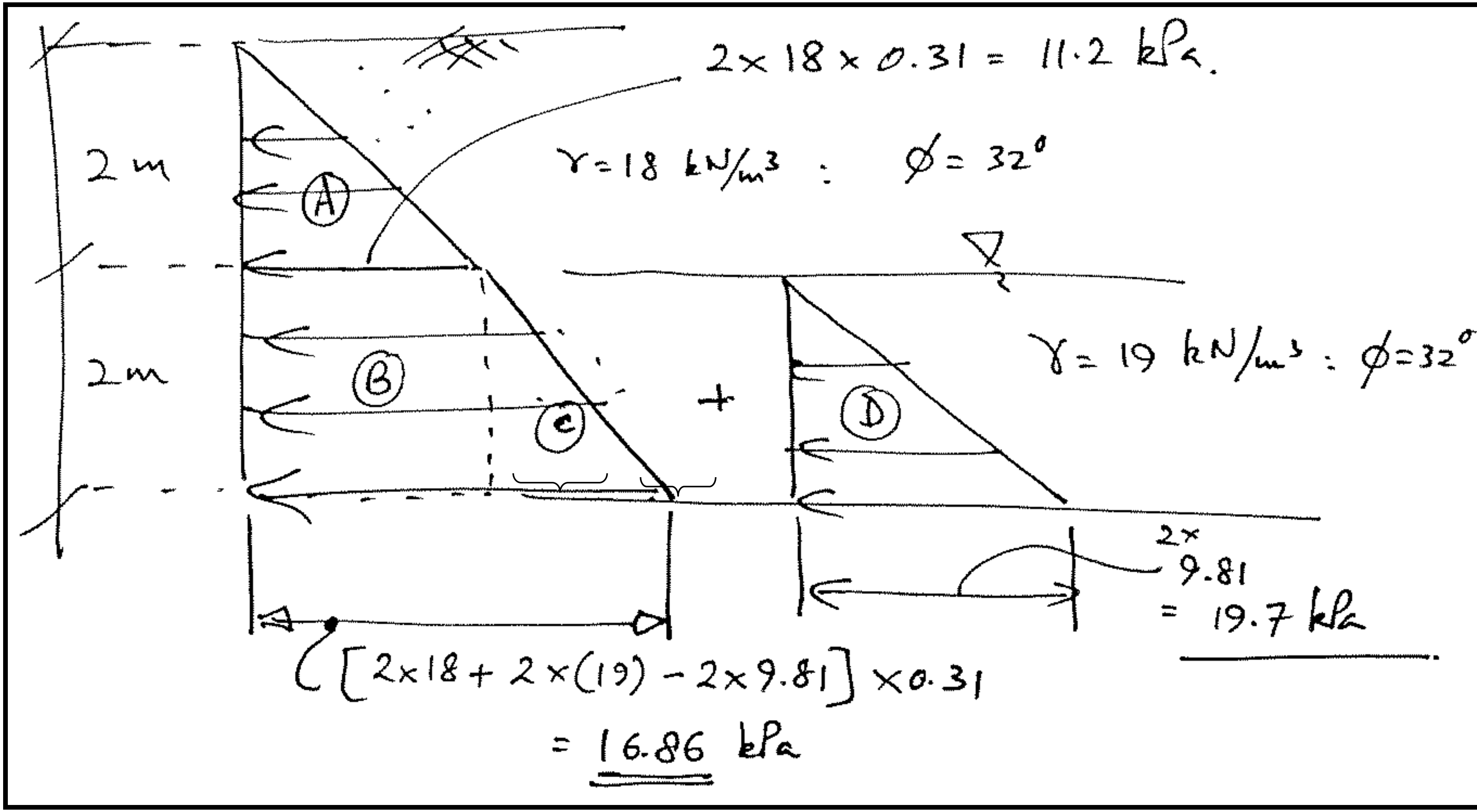
$= 33.8$ kPa

Example: Active earth pressures Sand backfill with groundwater



Find active pressure distribution & total active force

Use Rankine Theory



Total thrust = Area [A + B + C] + Area D

Soil (A + B + C)

Water (Area D)

Coulomb's Earth Pressure Theory

(Vertical, frictionless wall, horizontal backfill)

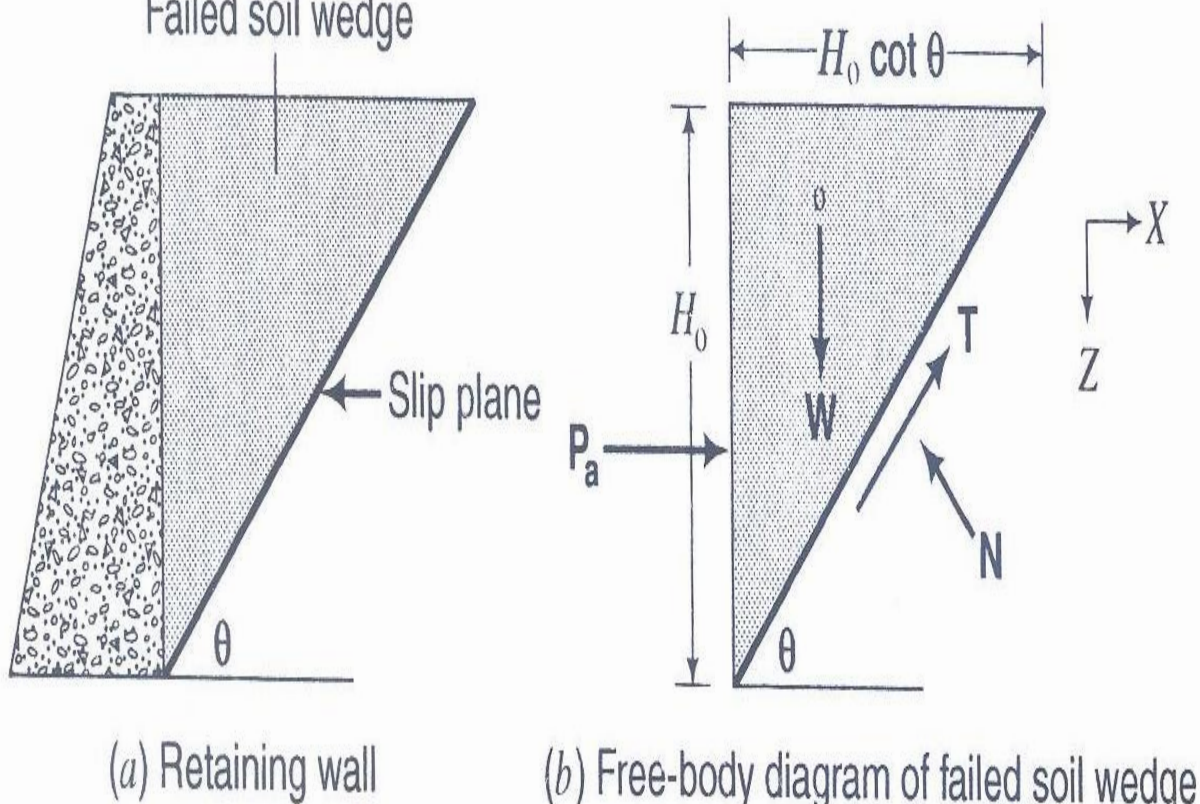


FIGURE 10.8 Coulomb failure wedge.

This is a limit equilibrium approach

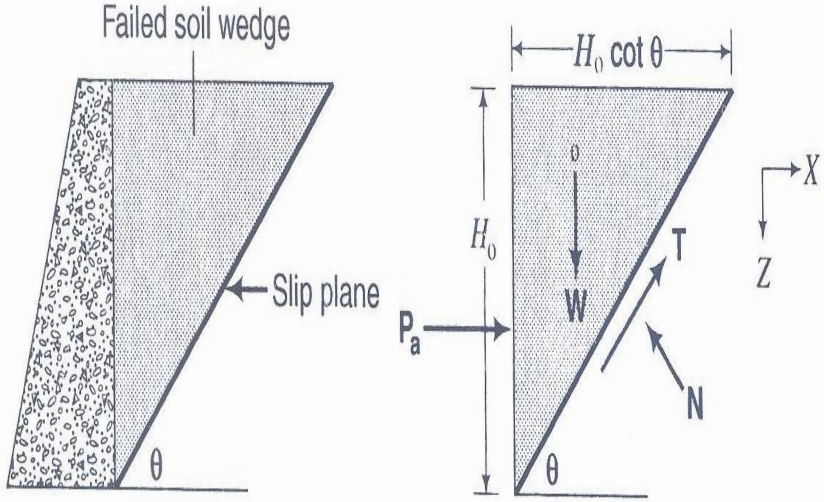
$$\Sigma F_x = P_a + T\cos\theta - N\sin\theta = 0$$

$$\Sigma F_z = W - T\sin\theta - N\cos\theta = 0$$

$$W = \frac{1}{2}\gamma H_o^{\ 2}\cot\theta$$

$$T = N\tan\phi'$$

$$P_a = \frac{1}{2}\gamma H_o^{\ 2}\cot\theta\tan(\theta - \phi')$$



(*a*) Retaining wall (*b*) Free-body diagram of failed soil wedge

FIGURE 10.8 Coulomb failure wedge.

$$\frac{\partial P_a}{\partial \theta} = \frac{1}{2}\gamma H_o^2[\cot\theta \sec^2(\theta - \phi') - \csc^2\theta \tan(\theta - \phi')] = 0$$

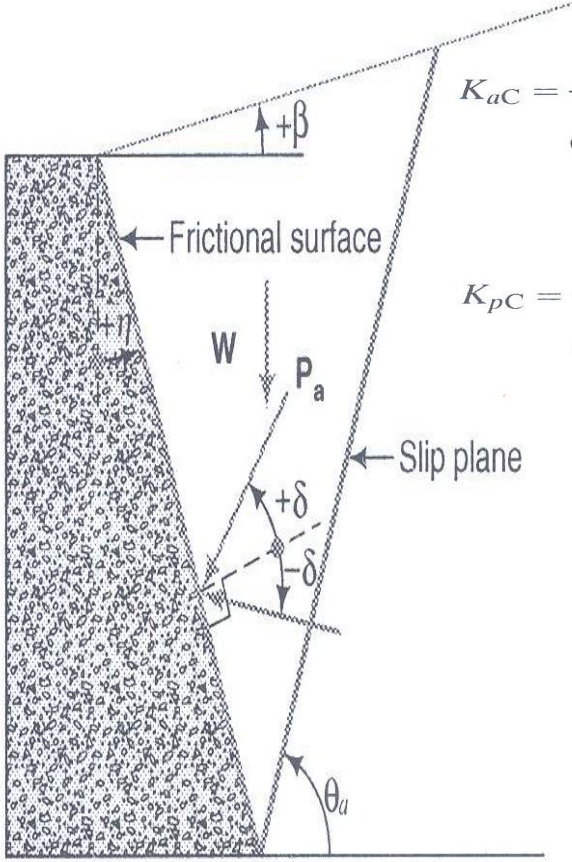
$$\theta = \theta_{cr} = 45^\circ + \frac{\phi'}{2} \tag{10.19}$$

$$P_a = \frac{1}{2}\gamma H_o^2 \tan^2\left(45^\circ - \frac{\phi'}{2}\right) = \frac{1}{2}K_a\gamma H_o^2 \tag{10.20}$$

Provides an upper bound solution (usually an estimate greater than the true value of P_a). However, this is equal to the lower bound solution provided by Rankine active state. This means it should be the true solution.

Coulomb's Earth Pressure Theory

(Sloping wall face, wall with friction, sloping backfill)



$$K_{aC} = \frac{\cos^2(\phi' - \eta)}{\cos^2\eta\cos(\eta+\delta)\left[1 + \left\{\dfrac{\sin(\phi'+\delta)\,\sin(\phi'-\beta)}{\cos(\eta+\delta)\,\cos(\eta-\beta)}\right\}^{1/2}\right]^2} \quad (10.21)$$

$$K_{pC} = \frac{\cos^2(\phi' + \eta)}{\cos^2\eta\,\cos(\eta-\delta)\left[1 - \left\{\dfrac{\sin(\phi'+\delta)\,\sin(\phi'+\beta)}{\cos(\eta-\delta)\,\cos(\eta-\beta)}\right\}^{1/2}\right]^2} \quad (10.22)$$

Inclination of slip surface to vertical

$$\tan\theta = \left[\frac{(\sin\phi'\cos\delta)^{1/2}}{\cos\phi'\{\sin(\phi'+\delta)\}^{1/2}}\right] \pm \tan\phi' \quad (10.23)$$

FIGURE 10.9 Retaining wall with sloping back, wall friction, and sloping soil surface for use with Coulomb's method for active condition.

Note:

$K_{ac} \neq 1/K_{pC}$, and C denotes Coulomb.

Correction charts available for logarithmic failure surface.

Coulomb's Earth Pressure Theory

(Sloping wall face, wall with friction, sloping backfill)

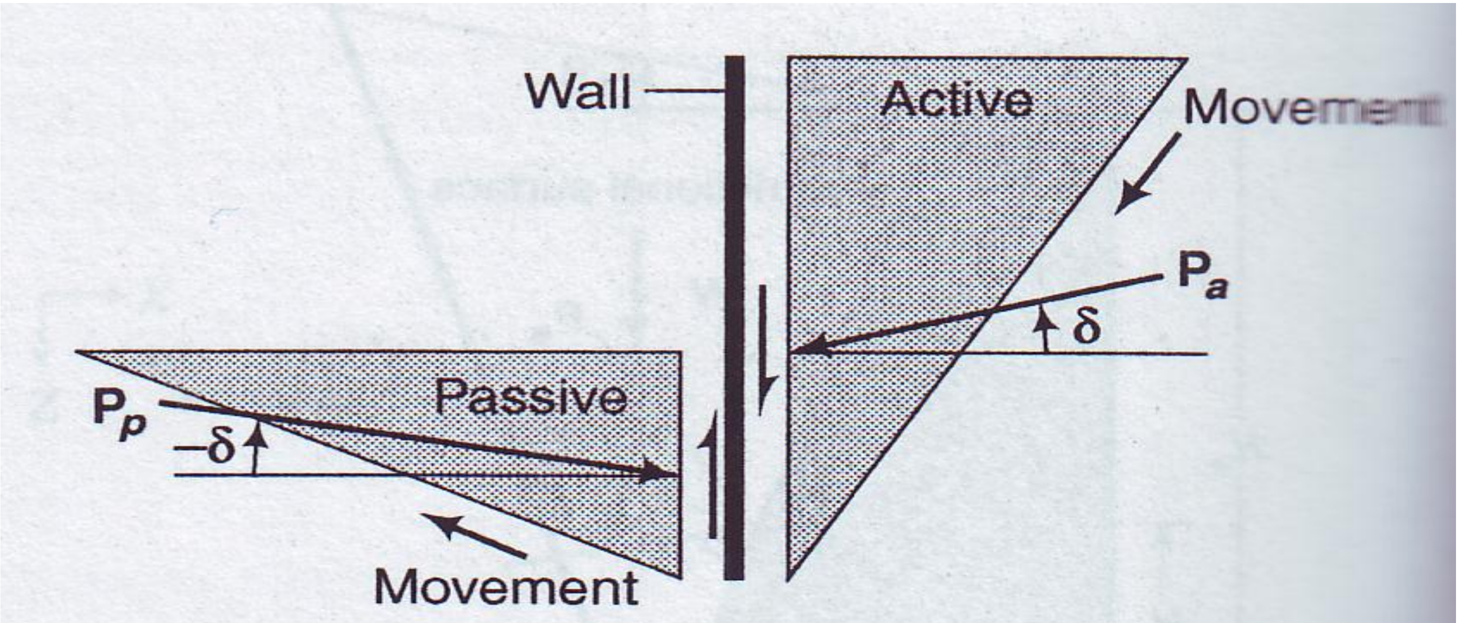


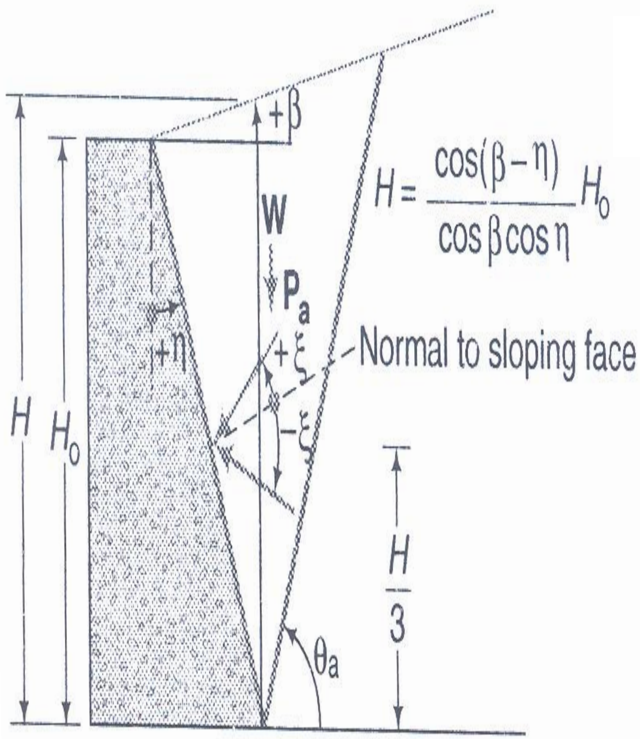
FIGURE 10.10 Directions of active and passive forces when wall friction is present.

Note:

- Active and passive forces are inclined at an angle δ to the normal to the sloping surface.
- Point of application is $H_0/3$ above from the base of wall.
- H0 = height of wall.

Rankine's Earth Pressure Theory

(Sloping wall face, wall with no friction, sloping backfill)



$$K_{aR} = \frac{\cos(\beta - \eta)\sqrt{1 + \sin^2\phi' - 2\sin\phi'\cos\varphi_a}}{\cos^2\eta\left(\cos\beta + \sqrt{\sin^2\phi' - \sin^2\beta}\right)} \quad (10.24)$$

$$K_{pR} = \frac{\cos(\beta - \eta)\sqrt{1 + \sin^2\phi' + 2\sin\phi'\cos\varphi_p}}{\cos^2\eta\left(\cos\beta - \sqrt{\sin^2\phi' - \sin^2\beta}\right)} \quad (10.25)$$

FIGURE 10.11 Retaining wall with sloping soil surface, frictionless soil-wall interface, and sloping back for use with Rankine's method.

Note:

See next page for definitions of some of the variables defined by the symbols.

Rankine's Earth Pressure Theory

(Sloping wall face, wall with no friction, sloping backfill)

$$\varphi_a = \sin^{-1}\left(\frac{\sin\beta}{\sin\phi'}\right) - \beta + 2\eta \quad (10.26)$$

$$\varphi_p = \sin^{-1}\left(\frac{\sin\beta}{\sin\phi'}\right) + \beta - 2\eta \quad (10.27)$$

$$\theta_a = \frac{\pi}{4} + \frac{\phi'}{2} + \frac{\beta}{2} - \frac{1}{2}\sin^{-1}\left(\frac{\sin\beta}{\sin\phi'}\right) \quad (10.28)$$

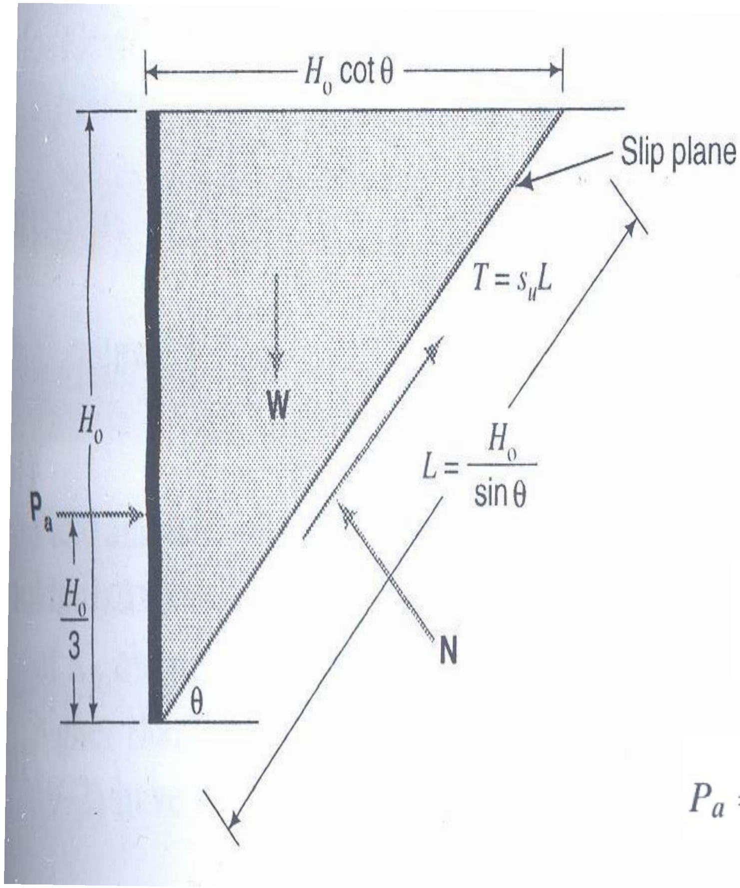
$$\theta_p = \frac{\pi}{4} - \frac{\phi'}{2} + \frac{\beta}{2} + \frac{1}{2}\sin^{-1}\left(\frac{\sin\beta}{\sin\phi'}\right) \quad (10.29)$$

$$\xi_a = \tan^{-1}\left(\frac{\sin\phi'\sin\varphi_a}{1 - \sin\phi'\cos\varphi_a}\right) \quad (10.30)$$

$$\xi_p = \tan^{-1}\left(\frac{\sin\phi'\sin\varphi_p}{1 + \sin\phi'\cos\varphi_p}\right) \quad (10.31)$$

Lateral Earth Pressure for Total Stress Analysis

(Vertical wall face, wall with no friction, horizontal backfill)



Active Case

$$P_a \cos\theta + T - W \sin\theta = 0 \tag{10.33}$$

$$P_a = \frac{1}{2}\gamma H_o^2 - \frac{s_u H_o}{\sin\theta \cos\theta} = \frac{1}{2}\gamma H_o^2 - \frac{2 s_u H_o}{\sin 2\theta}$$

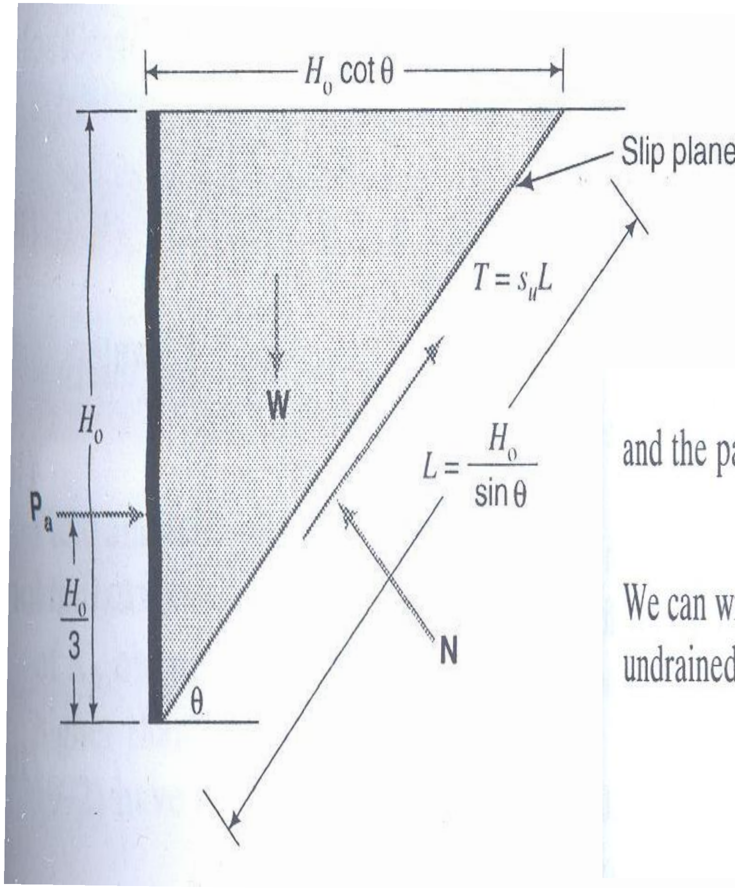
$$\frac{\partial P_a}{\partial\theta} = 4 s_u H_o \cot 2\theta \csc 2\theta = 0$$

$$P_a = \frac{1}{2}\gamma H_o^2 - 2 s_u H_o \tag{10.34}$$

$$(\sigma_x)_a = \gamma z - 2 s_u \tag{10.35}$$

Lateral Earth Pressure for Total Stress Analysis

(Vertical wall face, wall with no friction, horizontal backfill)



Passive Case

and the passive lateral pressure is

$$(\sigma_x)_p = \gamma z + 2s_u \quad (10.43)$$

We can write Eqs. (10.35) and (10.43) using apparent active and passive lateral earth pressures for the undrained condition as

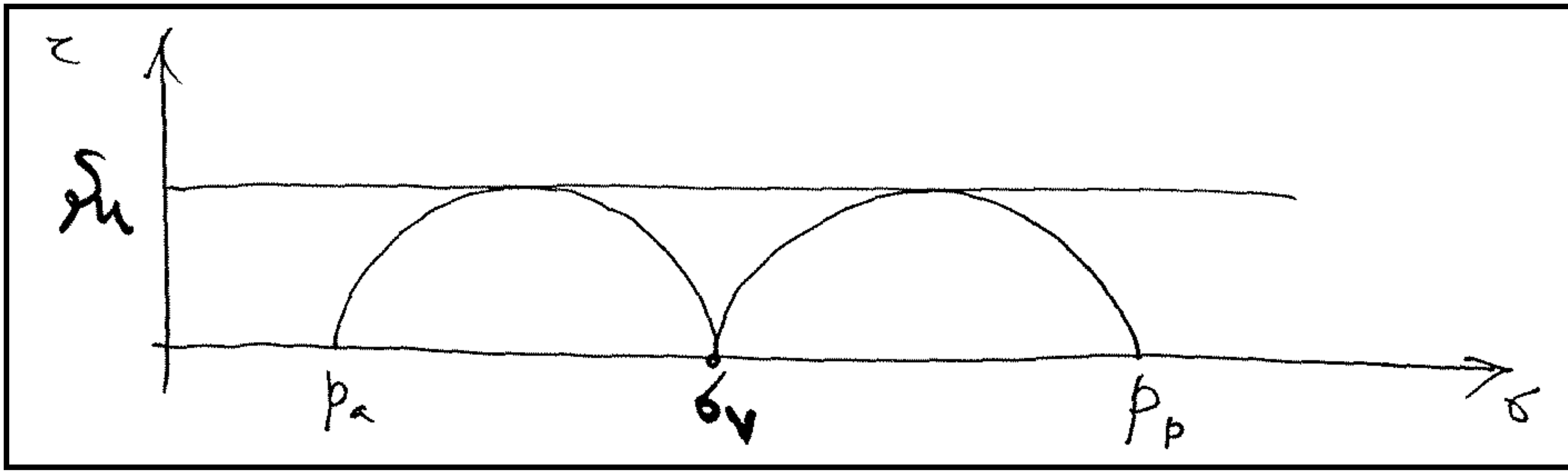
$$(\sigma_x)_a = \sigma_z - K_{au}s_u \quad (10.44)$$

$$(\sigma_x)_p = \sigma_z + K_{pu}s_u \quad (10.45)$$

$K_{au} = 2$; $K_{pu} = 2$

Total Stress Approach (Undrained Backfill)

⇒ extend the derivations for the c', φ' material directly c' = s_u : φ' = 0.



Assume uniform total unit weight = γ

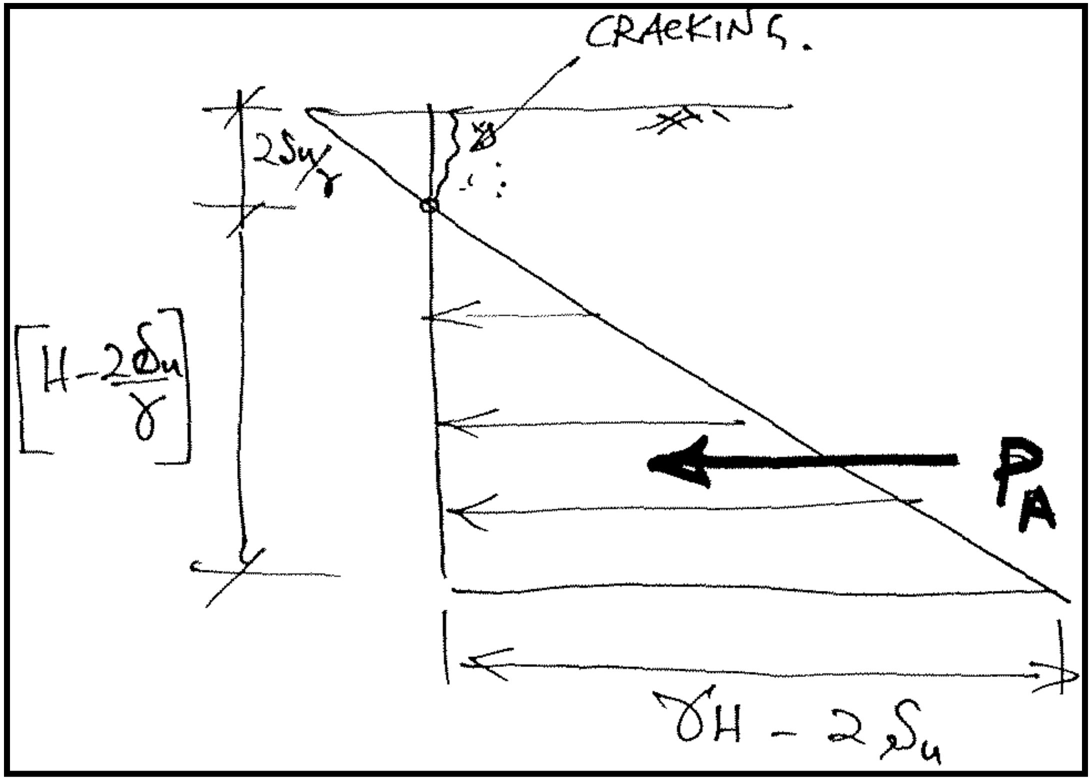
$$(\sigma_x)_a = \gamma z - 2s_u$$

$$(\sigma_x)_p = \gamma z + 2s_u$$

NOTE:

For φ′ = 0, $K_a = 1$ & $K_p = 1$

Total Stress Approach
Active Pressure (assume c = S_u)

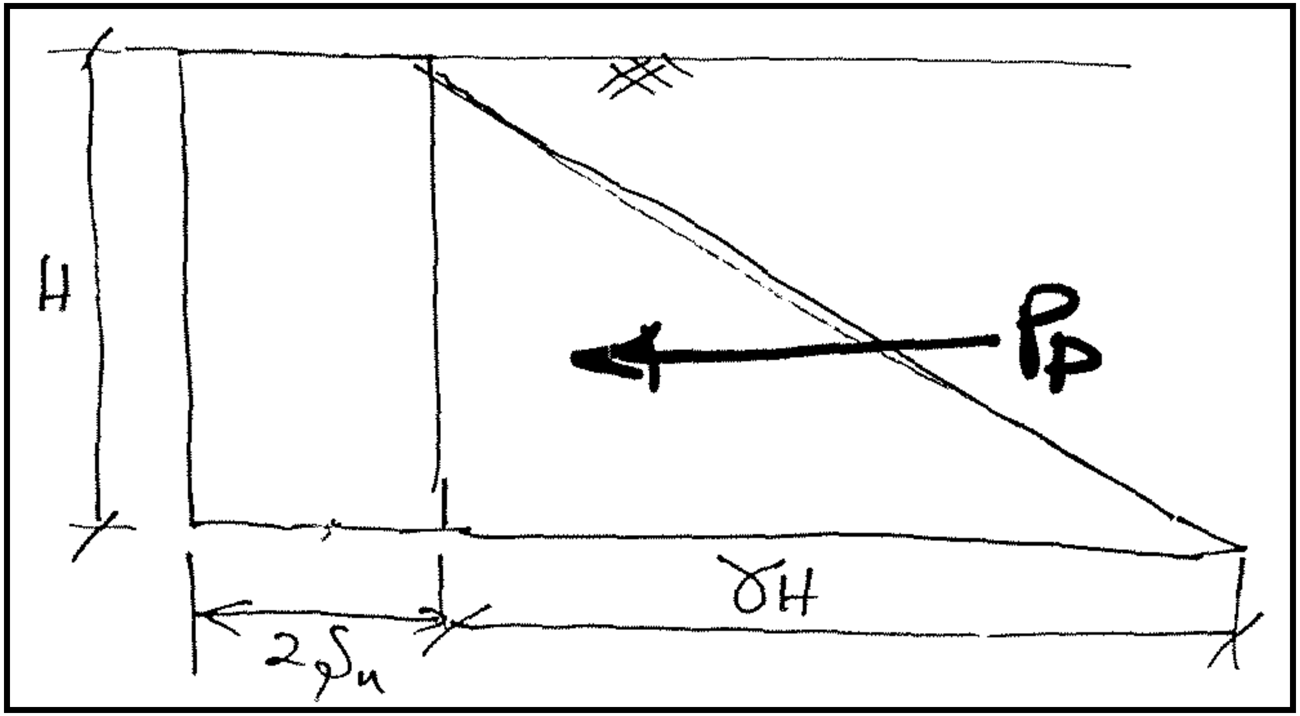


$(\sigma_x)_a = \gamma z - 2s_u$

$$Active: z_0 = \frac{2S_u}{\gamma}, z_0 = Crack\ depth$$

$$P_a = \frac{1}{2}\left[H - \frac{2s_u}{\gamma}\right][\gamma H - 2s_u]$$

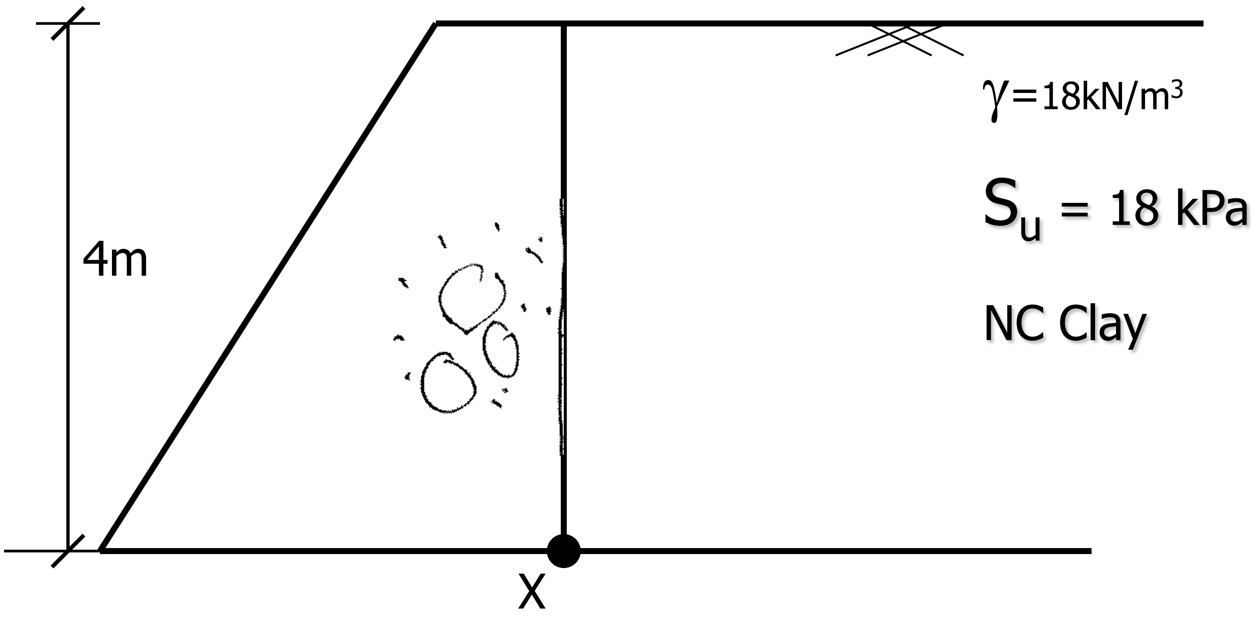
Total Stress Approach
Passive Pressure (assume $c = S_u$)



$(\sigma_x)_p = \gamma z + 2s_u$

$$P_p = \frac{1}{2} H[\gamma H] + [2s_u]H$$

Example: Active earth pressures Clayey backfill with groundwater



Find active pressure (p_a) distribution & total active force (P_a)

i.e., P_a = Resultant from pressure p_a distribution