# *SOIL MECHANICS II*
# *CIVL 311*
## COURSE NOTES
## 2012

# *Module 2*
# *Stresses in Soils*

Instructor: Dr. D. Wijewickreme, P. Eng.
Department of Civil Engineering
University of British Columbia
Vancouver, B.C.
Canada

# Module 2
# Stresses in Soils
## *Overall Learning Objectives*

- Stresses in soils
  - Types of stresses
  - General approach to determination of stresses
- Methods to determine stresses in soils
- Stress at a point (Mohr Circle) - Review only

# Stresses in Soils

In general, stresses within a soil mass are induced due to:

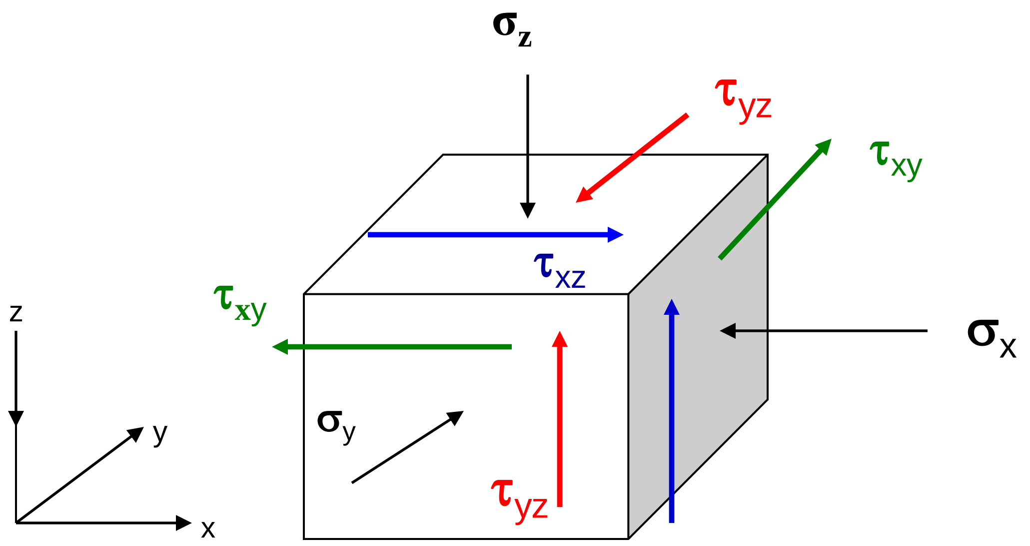
(i) gravity;

(ii) change in groundwater conditions; and

(iii) application of external loads.

- Due to self weight – Below horizontal ground
  - $\sigma_v = \Sigma \gamma_i d_i$
  - $u$ = pore pressure
  - $\sigma'_v = \sigma_v - u$

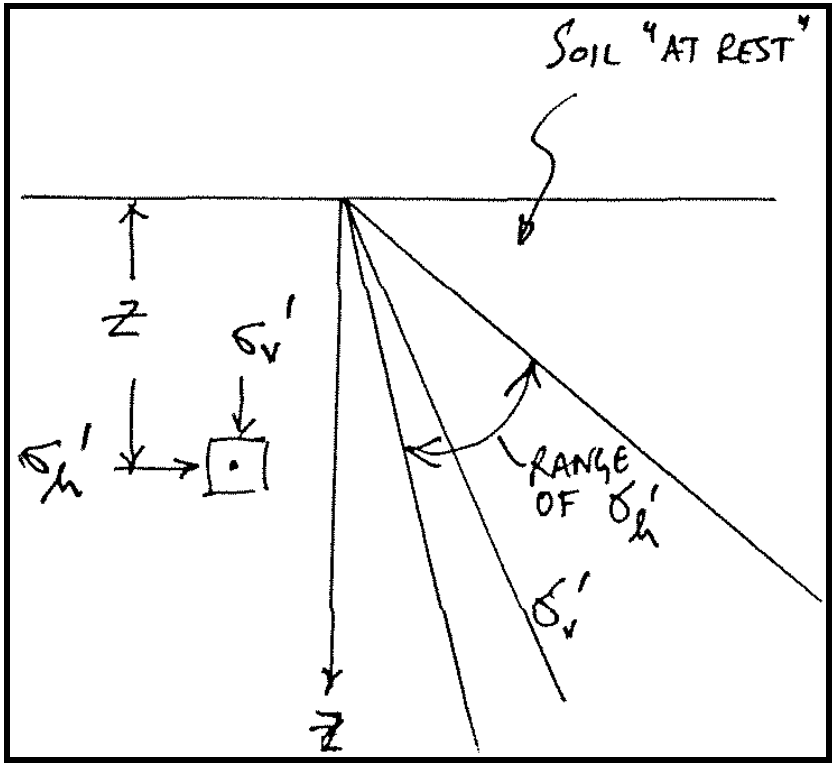
- Stresses $\sigma_z$ , $\sigma_x$ , $\sigma_y$ , $\tau_{xy}$ , $\tau_{yz}$ , $\tau_{zx}$ (or their increments $\Delta\sigma_z$, $\Delta\sigma_x$ , $\Delta\sigma_y$ , $\Delta\tau_{xy}$ , $\Delta\tau_{yz}$ , $\Delta\tau_{zx}$ due to a given loading situation) generally varies from point to point. Stress at a point:




$\Delta\sigma_z$ most important for settlements

Sign Convention
Compression = +ve
Extension = - ve

# Geostatic Stresses




**Stresses in soil due to self wt. below horizontal ground**

$\sigma'_z = \sigma'_v$ ; $\sigma'_h = K_0\sigma'_v$

where $K_0$ = Coefficient lateral earth pressure "at rest":

NC Soils:

$K_o^{nc} = 1 - \sin\phi_{cs}'$

Jaky's formula (1944)

OC Soils:

$K_0 = (1 - \sin\phi_{cs}')OCR^{(1 - \sin\phi'_{cs})}$

Mayne & Kulhawy (1982)

# Stresses in Soils Due to External Loading

- Point load
- Linear load
- Strip load
- Uniformly loaded circular area
- Uniformly loaded rectangular area
- Uniformly loaded arbitrary area

# Stresses in Soils

- Stresses are generally computed assuming soil:
  - as a continuum (not as a particulate material)
  - linear elastic (Hooke's law)
  - homogeneous
  - isotropic
  - semi-infinite soil mass (half space)

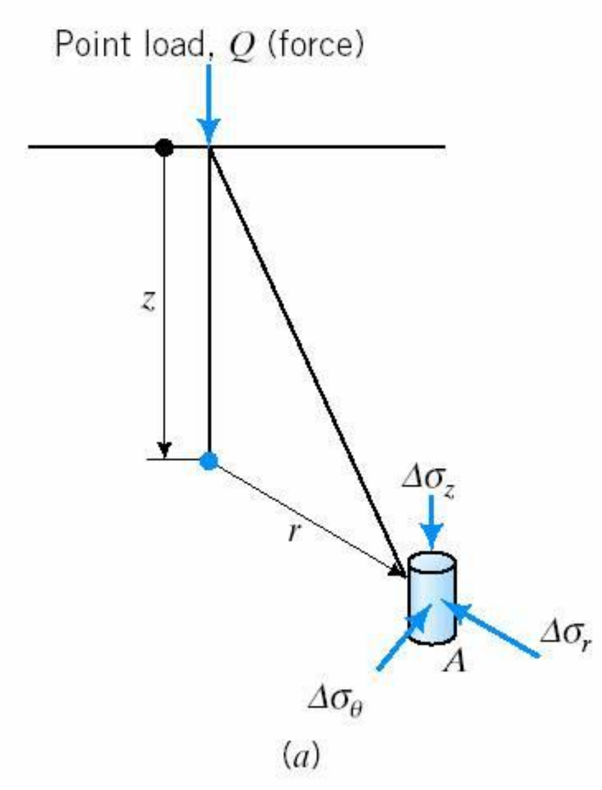
# Stresses Under Uniformly-loaded Footings

Several approaches:

(i) analytical (closed–form);
(ii) numerical analyses*; and
(iii) Charts.

*using readily available computer codes.
e.g. Geostudio – Sigma/W
http://www.geo-slope.com/products/geostudio/

# Concentrated Vertical Loads




Theoretical (closed-form) solutions
Stresses Beneath a Point Load,

$$\Delta\sigma_z = \frac{Q}{z^2} * I_q$$

$$I_q = \frac{3}{2\pi}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

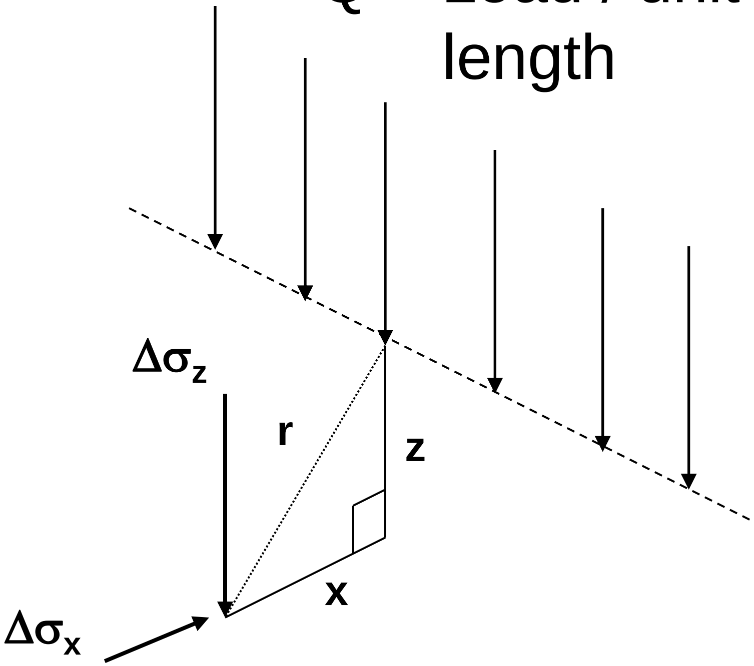
for $\Delta\sigma_\theta$ , $\Delta\sigma_r$ , $\Delta\tau_{rz}$ see Budhu

**Fig-5_22**

# Line Load of Infinite Length

$$\Delta\sigma_z = \frac{2Q}{\pi}\left( \frac{z^3}{\left(x^2 + z^2\right)^2} \right)$$

Q = Load / unit length




For other conditions

See Budhu's book

# Strip Loading (finite width and infinite length)

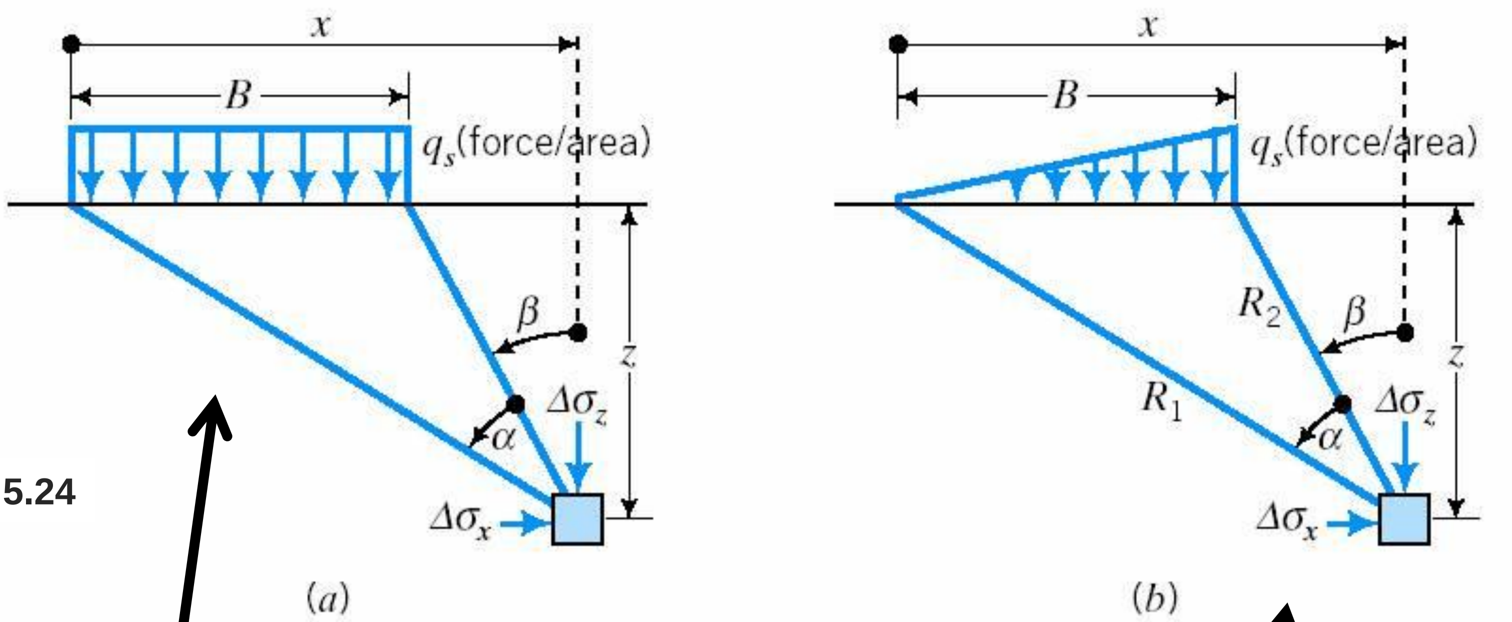



Fig 5.24

See Equation 5.66 of Budhu Text for the solution for this case (a)

$$\Delta\sigma_z = \frac{q_s}{\pi}\left(\frac{x}{B}\alpha - \frac{1}{2}\sin 2\beta\right) \qquad (5.70)$$

# Contours of vertical stress below uniformly-loaded square footing

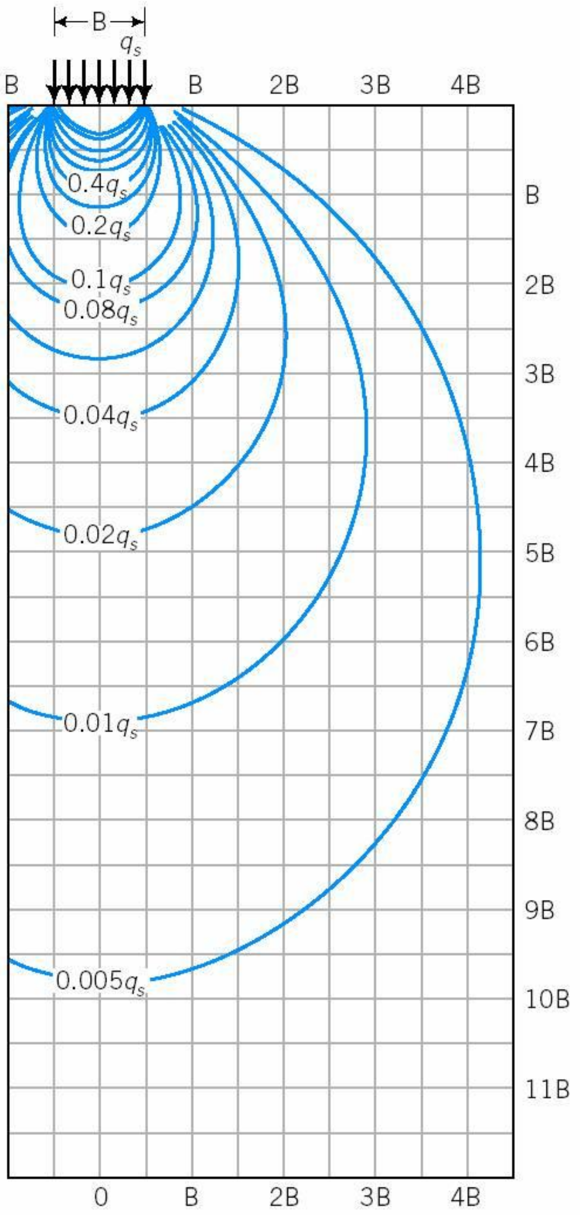
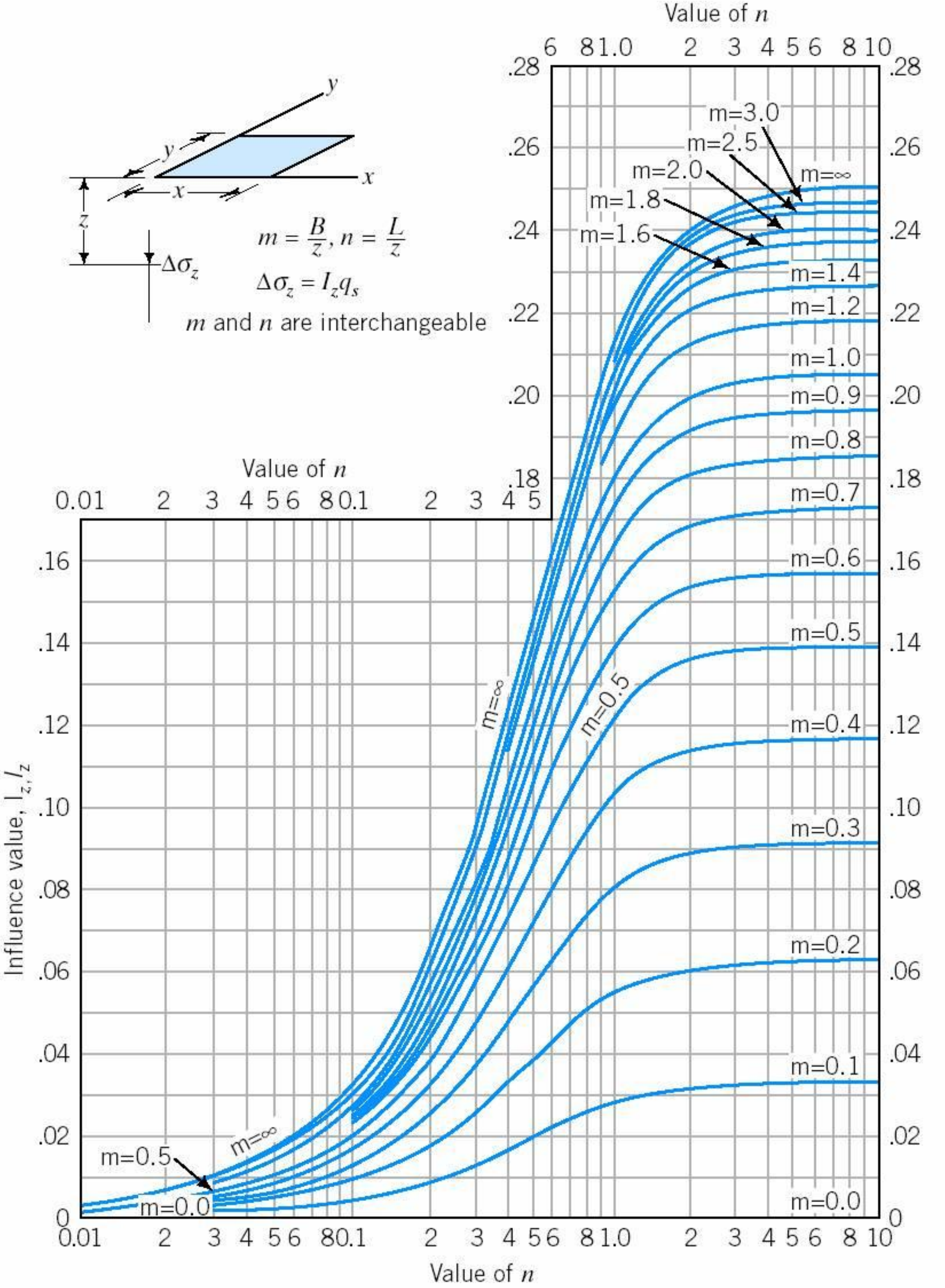



**Fig 5.25**

# Vertical Stress Below a Corner of Uniformly-Loaded Rectangular Area

**Fig 5.26**

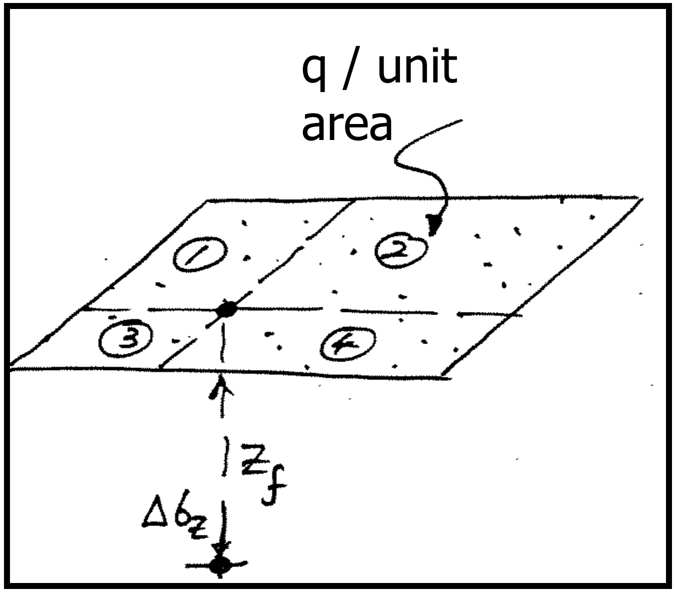
$$\Delta\sigma_z = I_z q_z$$



# How to compute σz when the point of interest is not below the corner:

Any general point inside footprint ==> common corner to 4 rectangles using superposition:

$$\sigma_z = q_z * \left[ (I_z)_1 + (I_z)_2 + (I_z)_3 + (I_z)_4 \right]$$

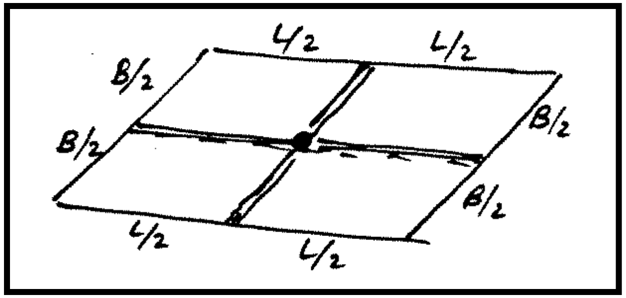



NOTE: Point beneath the centroid
[conveniently use 4 x ($I_z$ for one quadrant)

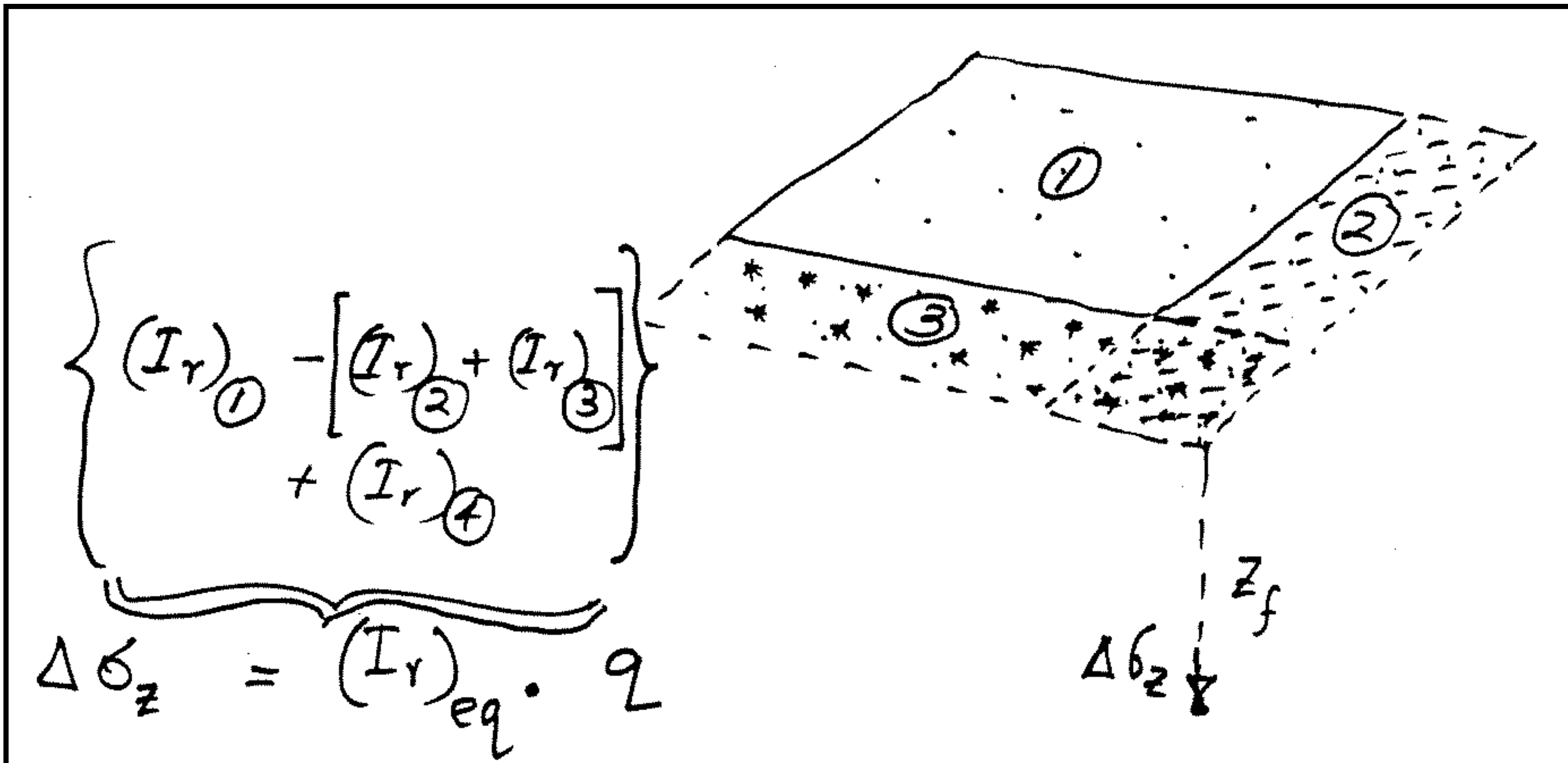
If $m \rightarrow \infty,\ n \rightarrow \infty$

large loaded area

⇨ $I_z = 0.25$



How to compute $\sigma_z$ when the point of interest is outside the footprint of foundation:



# Another approximate solution:

# Assume load spread at -> 2V:1H

Rectangular footing

$$\Delta\sigma_z = q_z\,[(B*L)/\,(B+z)(L+z)]$$

Square footing

$$\Delta\sigma_z = q_z\,[(B*B)/\,(B+z)(L+z)]$$

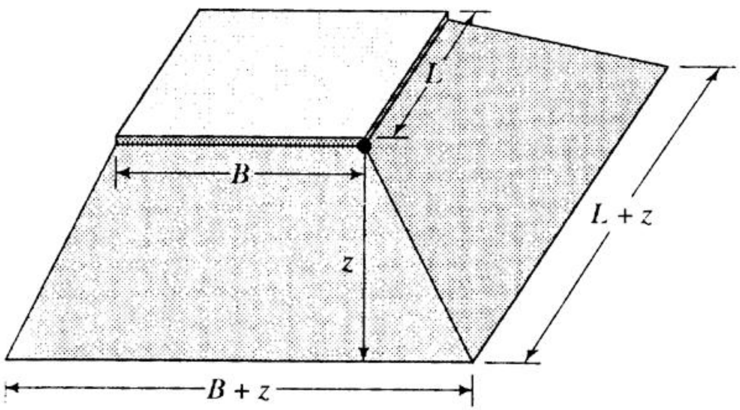


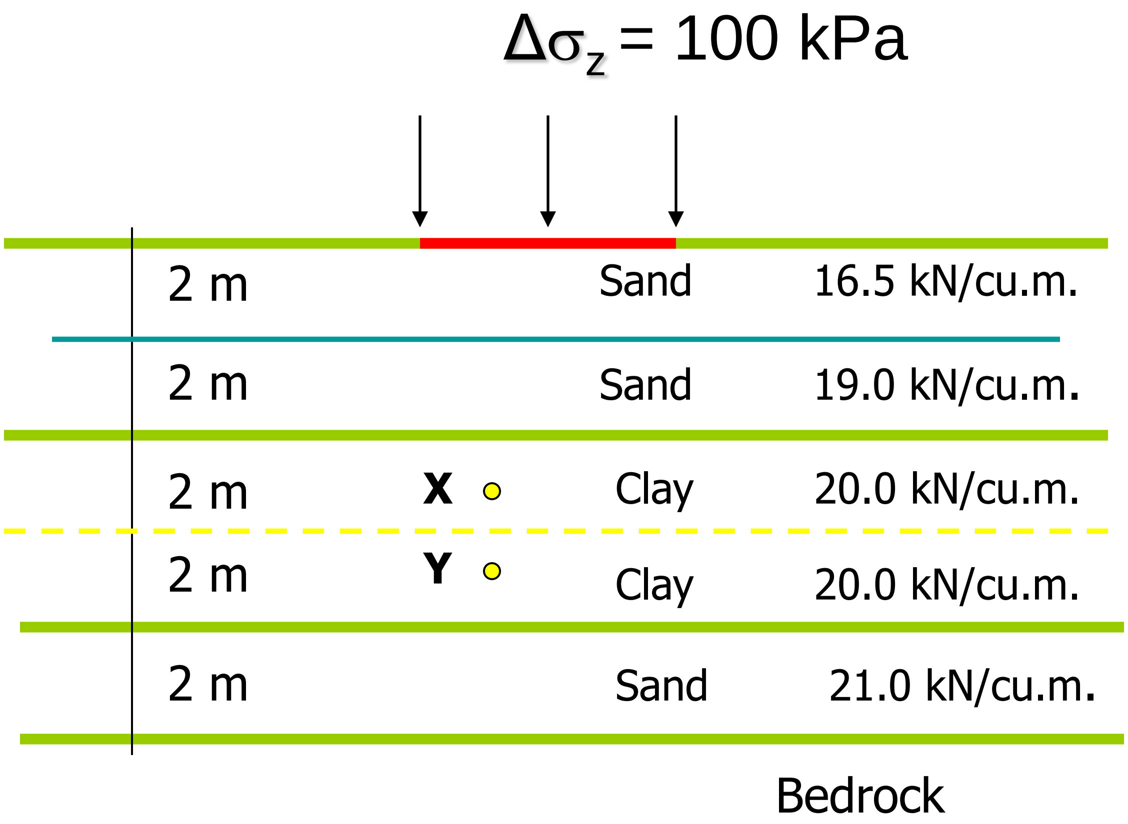

FIGURE 5.27 Dispersion of load for approximate increase in vertical stress under a rectangle.

# Example Problem

**4m dia. circular tank** (not wide-area load):
Need to compute –

Total settlement ($S_c$)
Time for settlement @ U=90%

From a lab test of a sample taken at the mid point of clay layer, after geometric construction of field curve, it is given:
Soil is NC
$C_c = 0.55$;
$e = 1.0 – 0.3 \log (\sigma'_z /100)$
$c_v = 1.0 \times 10^{-4}$ cm$^2$/sec



## Ref: Uniformly loaded circular area

## Budhu Textbook Section 5.11.5

Vertical Stress along the center beneath a circular area loaded with $q_z$ / unit area →

$$\Delta\sigma_z = q_z \left[ 1 - \left\{ \frac{1}{1 + \left[ \frac{r_0}{z} \right]^2} \right\}^{3/2} \right]$$

$$If\ r_0 >>> z \quad \Delta\sigma_z \approx q$$

$\sigma'_{z0}$ at two depth locations X and Y $\Rightarrow$

@X $\sigma_{z0}$ =

u =

$\sigma'_{zo}$ =

@Y $\sigma_{z0}$ =

u =

$\sigma'_{z0}$=

Given N.C. conditions, from constructed field compression curve $\Rightarrow$

$$e = 1.0 – 0.3 \log (\sigma'_z /100)$$

$\Delta\sigma_z$ calculations at the two depth locations X and Y knowing bearing pressure at the surface = 100 kPa

@X $(r_0/z)$=

$I_c$ =

$\Delta\sigma_z$ =

@Y $(r_0/z)$ =

$I_c$ =

$\Delta\sigma_z$ =

Final consolidation settlement $(\delta_c)_{ult}$ (compute and add the settlements for the two layers)

@ X

$$\Delta e = C_c \log \frac{\sigma'_{zo} + \Delta\sigma_z}{\sigma'_{z0}}$$

$$=$$

$$(\rho_{pc}) = \frac{\Delta e}{1+e_0} H_0$$

$$=$$

@ Y

$$\Delta e = C_c \log \frac{\sigma'_{z0} + \Delta\sigma_z}{\sigma'_{z0}}$$

$$=$$

$$(\rho_{pc}) = \frac{\Delta e}{1+e_0} H_0$$

$$=$$

# Client wants time for U=90% settlement

$$T_\upsilon = \frac{C_\upsilon t_{90}}{d^2} = 0.848$$

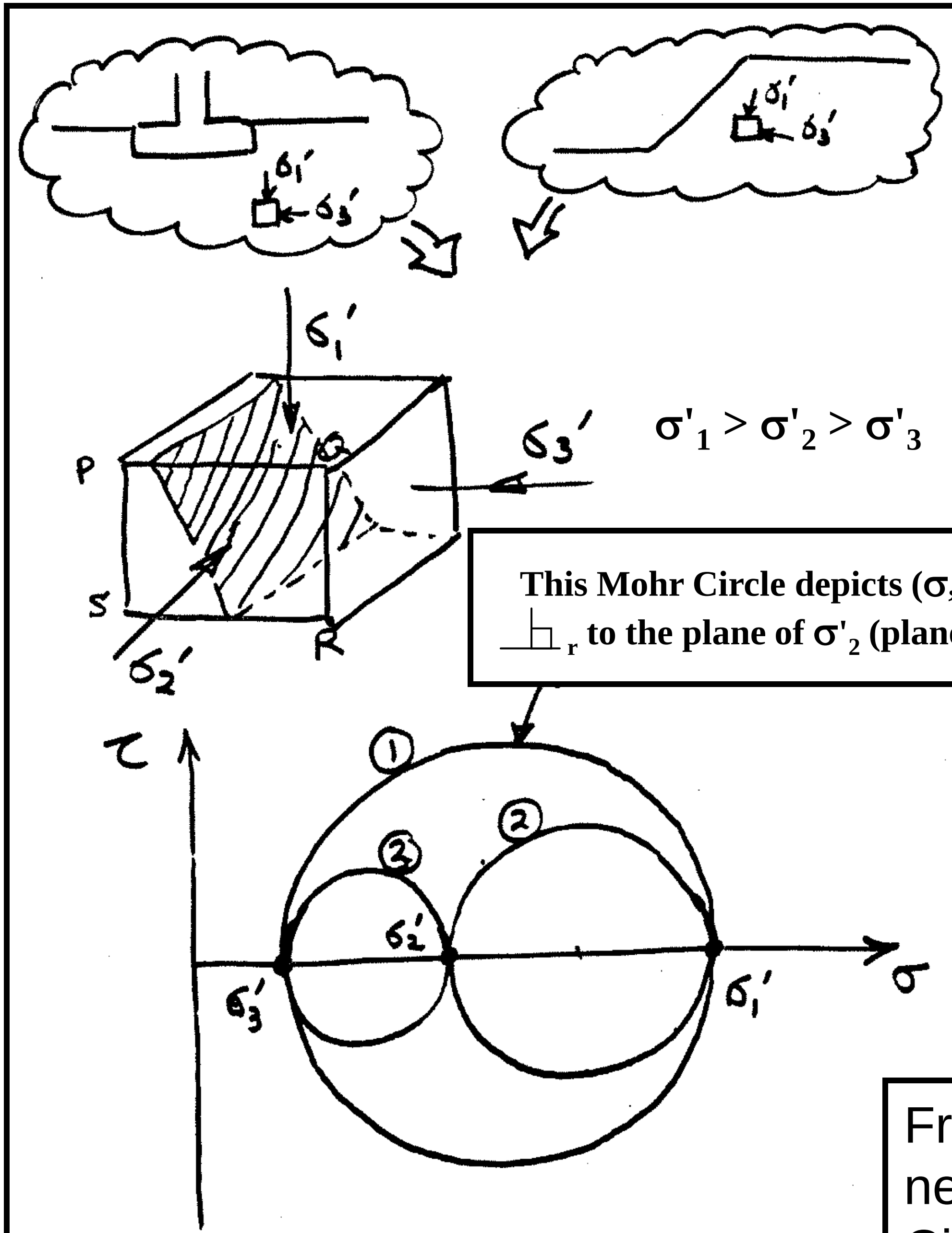
$T_v$ = $(c_v * t) / (H_{dr})^2$

$c_v$ = $1.0 \times 10^{-4}$ $cm^2/s$

$H_{dr}$ =

$t_{90}$ =

# Stress at a point and Mohr circle



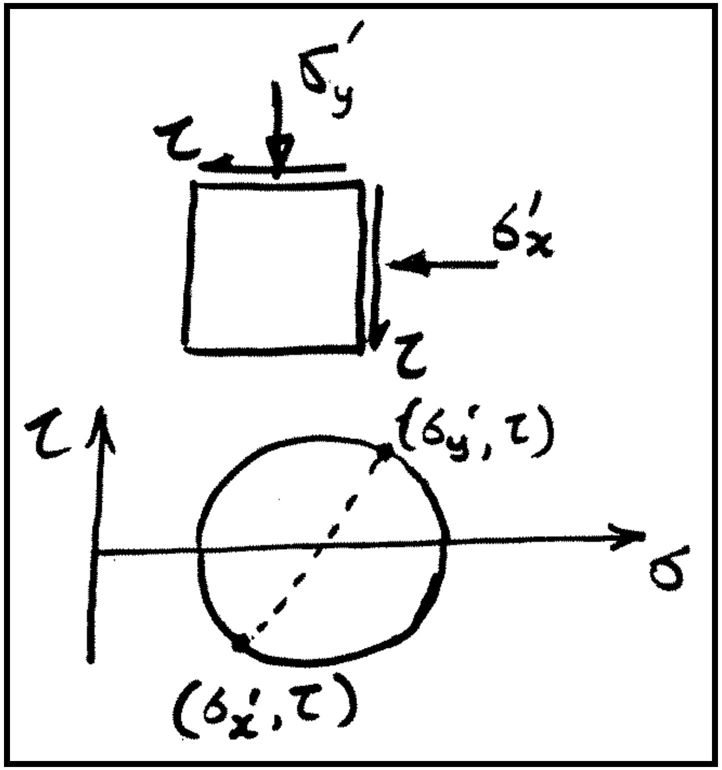

$\sigma'_1 > \sigma'_2 > \sigma'_3$

**This Mohr Circle depicts ($\sigma$, $\tau$) values on all planes ⊥r to the plane of $\sigma'_2$ (plane PQRS)**

From shear stress point of view → need to examine circle ① only. Circles ② and ③ less critical.

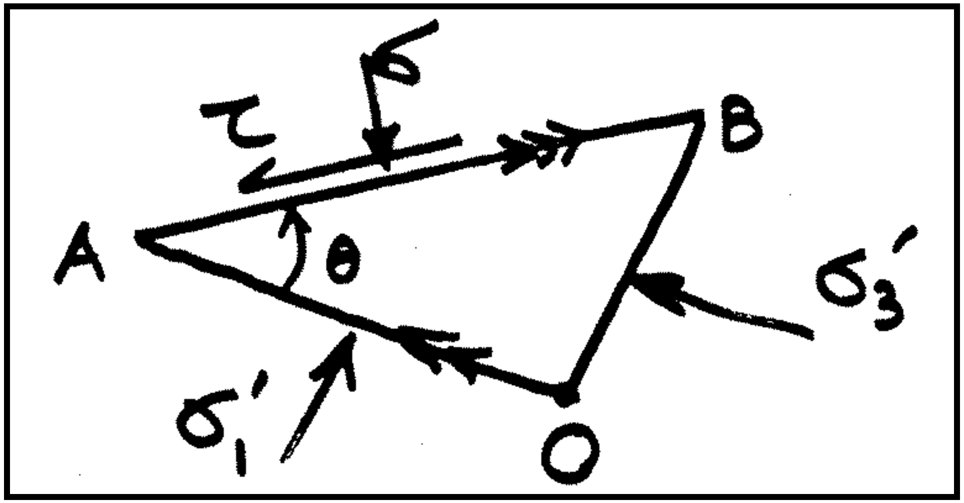
**Sign Convention:**

- Compressive stresses + ve
- Counterclockwise shear + ve




**Let us review**:

Given $\sigma'_1$, $\sigma'_3$ at a point O
Find ($\sigma$, $\tau$) on an arbitrary plane (AB) through O.



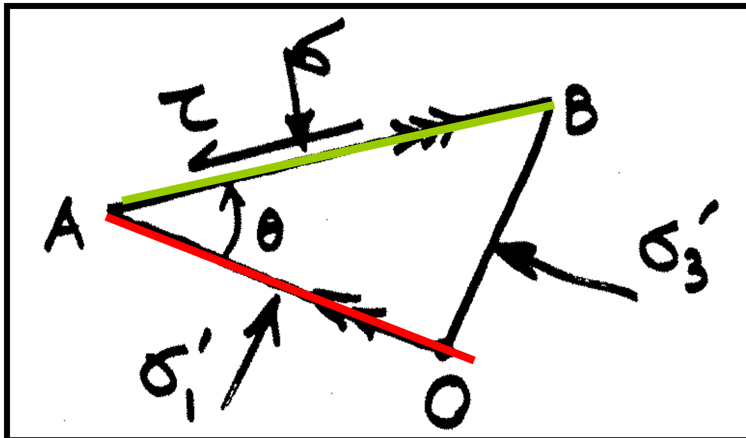

$$\sigma = \frac{\sigma'_1 + \sigma'_3}{2} + \frac{\sigma'_1 - \sigma'_3}{2}\cos 2\theta$$

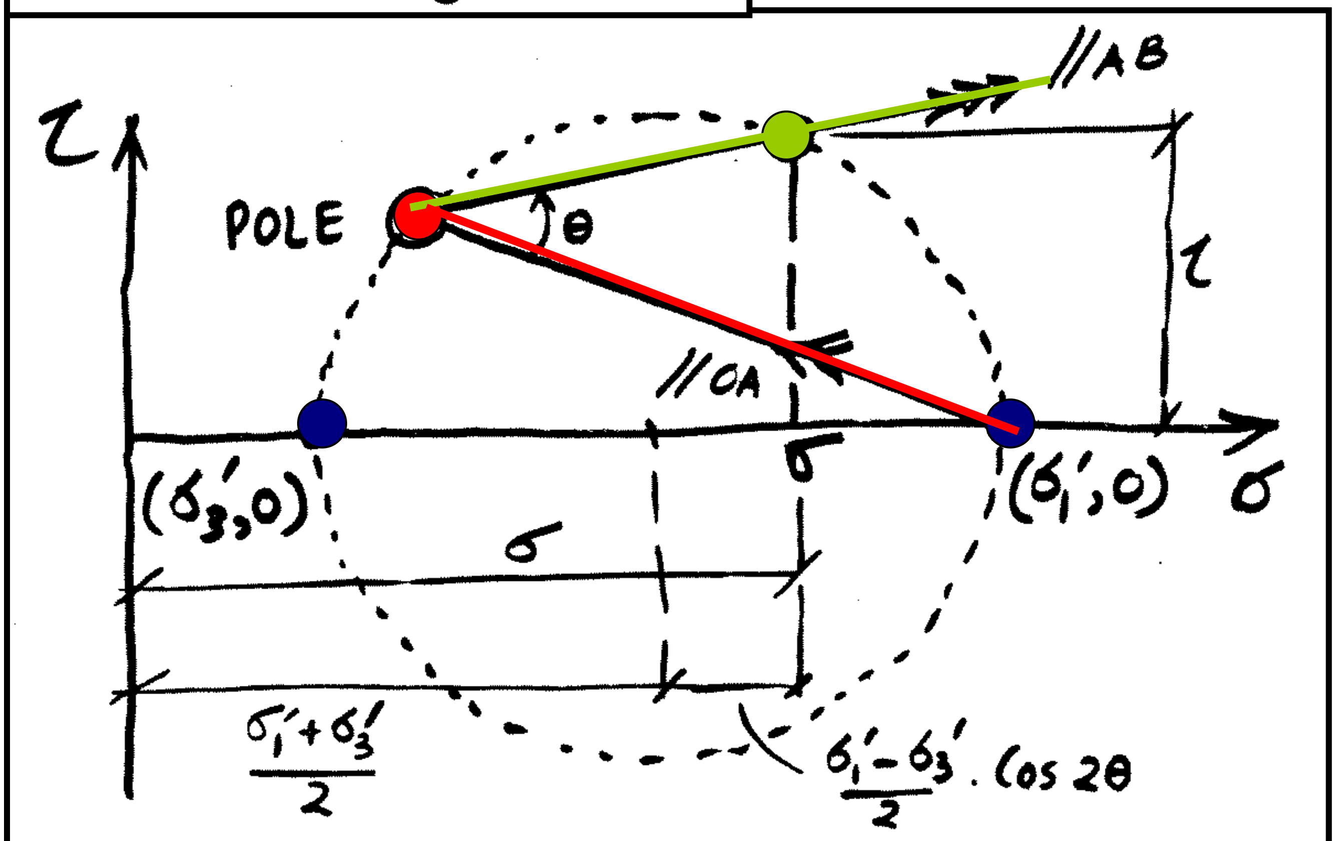
$$\tau = \frac{\sigma'_1 - \sigma'_3}{2}\sin 2\theta$$

# Pole Method

$$\tau = \frac{\sigma_1' - \sigma_3'}{2} \sin 2\theta$$

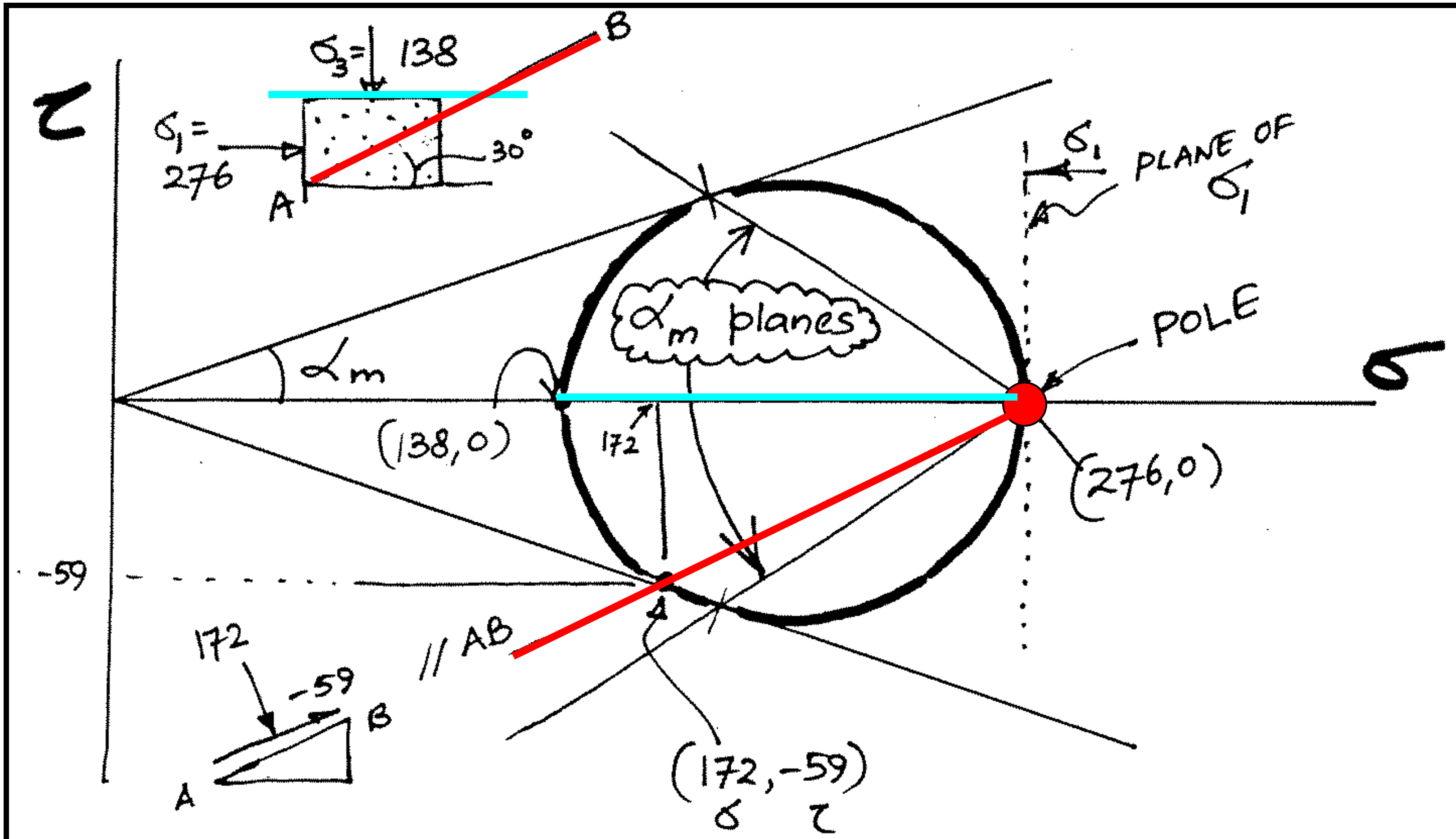
$$\sigma = \frac{\sigma_1' + \sigma_3'}{2} + \frac{\sigma_1' - \sigma_3'}{2} \cos 2\theta$$



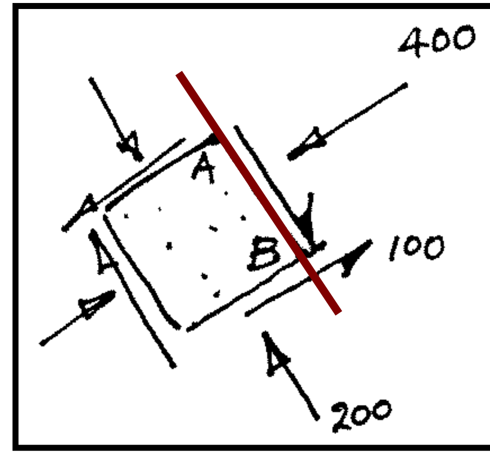



1. Given σ'$_1$ and σ'$_3$ as below, find:

- Stresses on a plane at 30º to horizontal
- planes having maximum obliquity of stresses

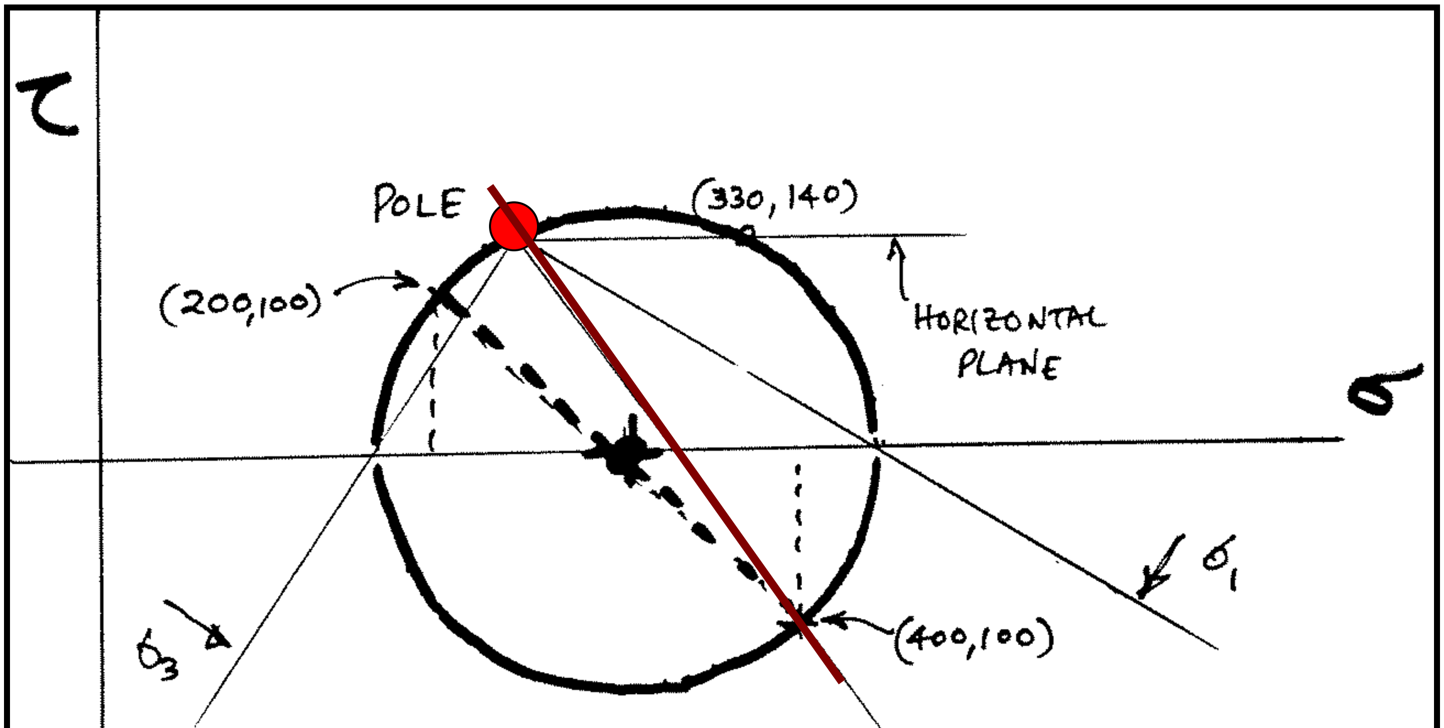


2. Given the stress condition




Find:

- $\sigma'_1$, $\sigma'_3$ and their directions
- Stresses on horizontal plane




On Horizontal Plane $\sigma = 330$; $\tau = 140$

# Module 2
# Stresses in Soils
### *Overall Learning Objectives*

- Stresses in soils
  - Types of stresses
  - General approach to determination of stresses
- Methods to determine stresses in soils
- Stress at a point (Mohr Circle) - Review only