Significant Figures (Sec 3-1)

Assume the last digit has uncertainty of ± 1

e.g., 40.1 ± 0.1 0.3948 ± 0.0001

(Sec 3-3)
Absolute error – margin of uncertainty assoc. with measurement
Relative error – uncertainty divided by the value

Quantity (g)	Sig fig	Absolute Uncertainty (±)	Relative uncertainty calc	Relative uncertainty	Percent Rel. uncertainty (%)
1.05	3	± 0.01	0.01/1.05	0.01	1
0.0105	3	± 0.0001	0.0001/0.0105	0.01	1
0.01	1	± 0.01	0.01/0.01	1	100
1050	(3 or) 4	± 1 or ± 10	1/1050	0.001	0.1
1.050×10^3	4	± 1	1/1050	0.001	0.1
1.05×10^3	3	±10	10/1050	0.01	1
4.44	3	± 0.01	0.01/4.44	0.0023	0.23

Rounding

If residue is > 5, round up.
Rounding 5.8503 to 2 sig figs =

If residue is < 5, round down.
Rounding 5.8499 to 2 sig figs =

If residue = 5, round so as to make the result even.
Round the following to 2 sig figs:
5.95 =
5.85 =
5.75 =
5.85000 =
5.85001 =

Sig Figs in Addition and Subtraction (Sec. 3-2)

Express the answer with the same number of decimal places as the number with the fewest decimal places.

e.g. pipet calibration performed properly:

34.141 7 g
- 24.198 3 g
g

e.g. pipet calib where top-loading balance used for initial weight

34.141 7 g
- 24.20 g
g

Practice: 5.4 + 3.2 + 6.8 =
14.6 + 0.0037 – 5.98 =
$7.62 \times 10^{-5} + 8.59 \times 10^{-4} + 5.334 \times 10^{-6} =$

Sig Figs in Multiplication/Division (Sec. 3-2)

Express the answer with the same number of significant figures as the number with the fewest sig figs.

e.g. calc of true volume:

True volume = (mass*buoy corr)/density
= (9.943 4 g)(1.0011)/(0.997 773 5 g/mL)
= 9.976 5500 53 mL

Practice: 40.1 x 0.3948 / 368.4 =
(21.6 * 0.317 / 4.1) x 16.37 =
(21.6 * 0.317 / 4.1) + 16.37 =

Sig Figs in logs and anti-logs (Sec. 3-2)

The number of digits in the mantissa of the logarithm is the number of decimals in the anti-log. e.g., 4.25×10^{-4} power (characteristic), mantissa

e.g. $\log(4.25 \times 10^{-4}) = -3.371\ 611 =$

pH = $-\log[H^+]$
pH = 2.01 $[H^+] =$
$[H^+] = 1.37 \times 10^{-5}$ pH =

Exercise:

5.4 + 3.2 + 6.8 =
14.6 + 0.0037 – 5.98 =
$7.62x10^{-5} + 8.59x10^{-4} + 5.334x10^{-6}$ =

40.1x0.3948/368.4 =
(21.6*0.317/4.1) x 16.37 =

Retain extra insignificant figures through calculations.
Only round off to correct number of sig figs at final answer.

Propagation of Random Error (Sec. 3-4)

Uncertainty for add & subtract:

$$error_{total} = \sqrt{e_1^2 + e_2^2 + e_3^2} \qquad \text{(eqn 3-5)}$$

1. 5.4 + 3.2 + 6.8 = 15.4
5.4±0.1 + 3.2±0.1 + 6.8±0.1 = 15.4

$$error_{total} = \sqrt{(0.1)^2 + (0.1)^2 + (0.1)^2} = 0.17$$

2. 14.6 + 0.0037 – 5.98 = 8.6
14.6±0.1 + 0.0037±0.0001 – 5.98±0.01 = 8.6 ±0.1

$$error_{total} = \sqrt{0.1^2 + 0.0001^2 + 0.01^2} = \sqrt{10^{-2} + 10^{-8} + 10^{-4}}$$

$$= \sqrt{1.01x10^{-2}} = 0.1005$$

3. $7.62x10^{-5} + 8.59x10^{-4} + 5.334x10^{-6} = 9.41x10^{-4}$
$(0.762±0.001)x10^{-4}+(8.59±0.01)x10^{-4}+(0.05334±0.00001)x10^{-4}= (9.41±0.1)\ x10^{-4}$

$$error_{total} = \sqrt{(10^{-7})^2 + (10^{-6})^2 + (10^{-9})^2} = \sqrt{1.01x10^{-12}} = 1.005x10^{-6}$$

Uncertainty for multiplication & division:

$$\%error_{total} = \sqrt{\%e_1^2 + \%e_2^2 + \%e_3^2} \qquad \text{(eqn 3-6)}$$

4. 40.1 x 0.3948 / 368.4 = 4.30x10^{-2} ±0.01x10^{-2}
±0.1 ±0.0001 ±0.1
±0.25% 0.025% ±0.027%

$$\%error_{total} = \sqrt{(0.25\%)^2 + (0.025\%)^2 + (0.027\%)^2} = 0.253\%$$

Absolute error = (relative error) x (value)
Absolute error = (0.253%/100%) x (4.30x10^{-2}) = 0.0109 x10^{-2}

5. (21.6*0.317/4.1) x 16.37 = 27

$$error_{total} = \sqrt{(0.46\%)^2 + (0.31\%)^2 + (2.44\%)^2} = 2.50\%$$

Absolute error = 0.6_8 Correct answer = 27.3 ± 0.7

6. (21.6*0.317/4.1) + 16.37 = 18.0
Propagate error through brackets first (use relative error as mult&div) and then convert to absolute error to propagate through addition

$$\%e_{brackets} = \sqrt{\left(\tfrac{0.1}{21.6} \bullet 100\%\right)^2 + \left(\tfrac{0.001}{0.317} \bullet 100\%\right)^2 \left(\tfrac{0.1}{4.1} \bullet 100\%\right)^2}$$

$$= \sqrt{0.4_6{}^2 + 0.3_2{}^2 + 2._{44}{}^2} = 2.5_0\%$$

Bracket term = 1.6_{70} ± 2.5_0%
= 1.6_{70} ± (1.6_{70}x2.5_0%/100%) = 1.6_{70} ± 0.04_2
Answer = (1.6_{70} ± 0.04_2) + 16.37 ± 0.01 = 18.0_4 ± ??

$$e_{addition} = \sqrt{(0.04_2)^2 + (0.01)^2} = 0.043$$

Lab Rule: If the error of an operation is ≤1/3 of required accuracy, don't worry about it.

Assume one source of error causes an error equal to 1
Then, a second operation causes an error of 1/3
Let's propagate errors to see the effect of the second operation

$$e_{total} = \sqrt{(1)^2 + (0.333)^2} = 1.05$$

Due to the squaring of the errors, the effect of the smaller error is minimized, such that it contributes very little to the ultimate overall error. Hence we don't need to worry about errors that are less than 1/3 of others or of our target accuracy

Propagation of Systematic Errors (Sec. 3-5)

This is the big concern about systematic errors. Because they always occur in a specific direction, their effect on our result will build up quickly.

Statistics (Chapter 4)
Reporting Results

Repeated weighings: 1.012, 1.007, 1.022, 1.020, 1.017 g

Mean – average of a set of results

$$\bar{x} = \frac{\sum_i x_i}{n} \quad \text{(Eqn. 4-1)} = \frac{1.012+1.007+1.022+1.020+1.017}{5}$$

= **1.016**

Range – difference between highest and lowest value in set
= 1.022 – 1.007 = **0.015**

Median – the middle value of an ordered list of data values
Ordered weighings: 1.007, 1.012, **1.017**, 1.020, 1.022 g

Repeated weighings: 1.012, 1.007, 1.022, 1.020, 1.170 g

Mean – average of a set of results

$$\bar{x} = \frac{\sum_i x_i}{n} \quad \text{(Eqn. 4-1)} = \frac{1.012+1.007+1.022+1.020+1.170}{5}$$

= **1.046**

Range – difference between highest and lowest value in set
= 1.170 – 1.007 = **0.163**

Median – the middle value of an ordered list of data values
Ordered weighings: 1.007, 1.012, **1.020**, 1.022, 1.170 g

note: Median less influenced than mean by bad data point (outlier)

Grubbs-test for Bad Data (Sec. 4.6)

Replicate titrations: 12.53, 12.56, 12.47, 12.67, 12.48 mL
Is 12.67 a bad point (an outlier)?

First determine the mean and standard deviation of the entire data set (5 results):

mean = 12.54_2 stdev = 0.08_0

$$G_{calc} = \frac{\text{ques value} - \bar{x}}{s}$$ (Eqn. 4-13)

$$G_{calc} = \frac{12.67 - 12.542}{0.080} = 1.6$$

Compare G_{calc} with G_{table} (Table 4-5).
If $G_{calc} < G_{table}$, the questionable point should be retained.

No. of data	95% confidence	99% confidence
3	1.1531	1.1546
4	1.4625	1.4925
5	1.6714	1.7489
6	1.8221	1.9442
7	1.9381	2.0973
8	2.0317	2.2208
9	2.1096	2.3231
10	2.1761	2.4097
11	2.2339	2.4843
12	2.285	2.5494
13	2.3305	2.607
14	2.3717	2.6585