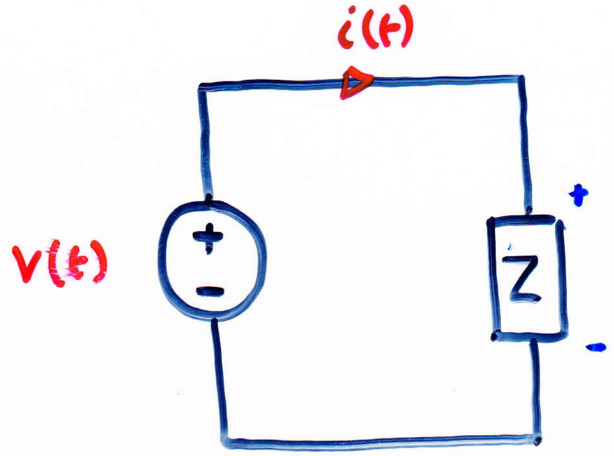
Instantaneous & Average Power



Steady-state:

$$v(t) = V_m \cos(\omega t + \theta_v)$$

$$i(t) = I_m \cos(\omega t + \theta_i)$$

$\therefore$ Instantaneous power

$$p(t) = v(t)\, i(t)$$

$$= V_m I_m \cos(\omega t + \theta_v) \cos(\omega t + \theta_i)$$

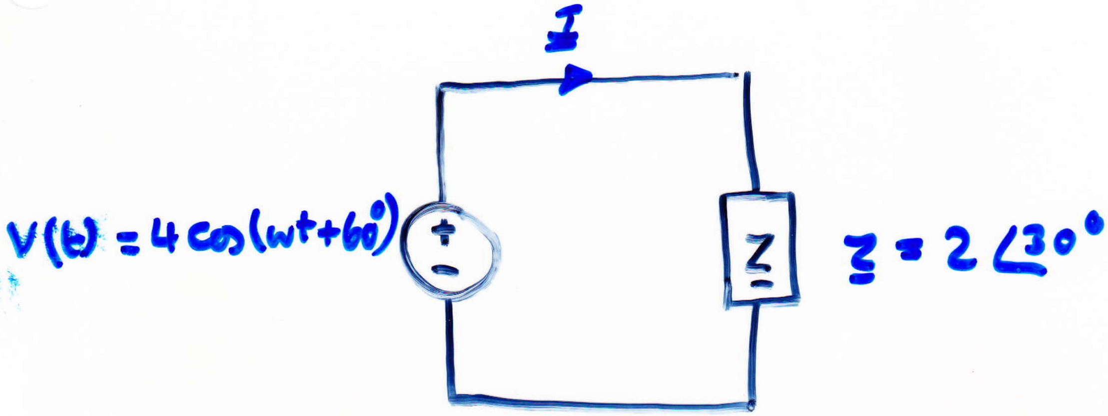
Now

$$\cos\phi_1 \cos\phi_2 = \tfrac{1}{2}\left[\cos(\phi_1 - \phi_2) + \cos(\phi_1 + \phi_2)\right]$$

$$\therefore\ p(t) = \frac{V_m I_m}{2}\left[\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)\right]$$

Note: Time independent   Twice freq. of input.

Example (Irwin 9.1)



Determine

i) current,

ii) instantaneous power.

i) $$\underline{I} = \frac{4\angle 60^\circ}{2\angle 30^\circ}$$

$$= 2\angle 30^\circ \text{ A.}$$

So $i(t) = 2\cos(\omega t + 30^\circ)$

ii) $$p(t) = 4[\cos(30^\circ) + \cos(2\omega t + 90^\circ)]$$

$$= 3.46 + 4\cos(2\omega t + 90^\circ) \text{ W}$$

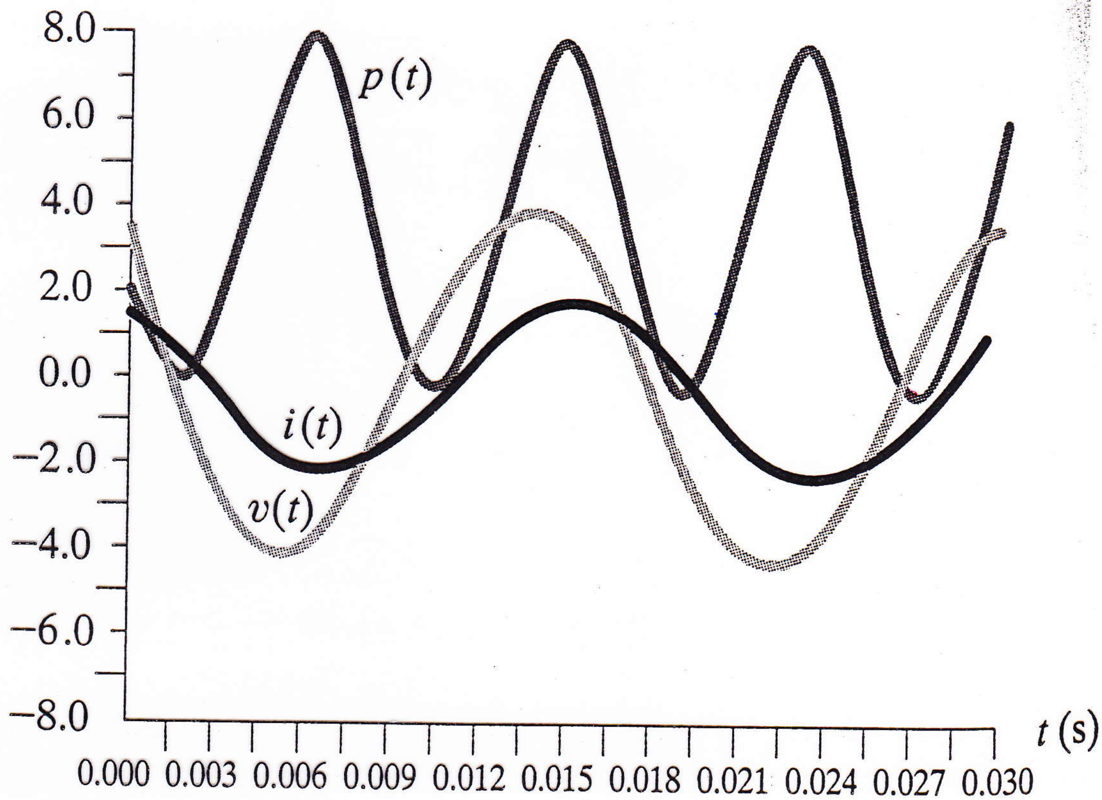


Figure 9.2 *Plots of $v(t)$, $i(t)$, and $p(t)$ for the circuit in Example 9.1 using $f = 60$ Hz.*

Average Power.

Power is varying sinusoidally so ~~average~~ integrate over a period and divide by that period.

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} p(t)\,dt$$

$$= \frac{1}{T}\int_{t_0}^{t_0+T} V_m I_m \cos(\omega t+\theta_v)(\cos(\omega t+\theta_i))dt$$

where $T = \frac{2\pi}{\omega}$

Now

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} \frac{V_m I_m}{2}\left[\underbrace{\cos(\theta_v-\theta_i)}_{\text{Independent of } t\text{, so can be taken outside of integral.}} + \underbrace{\cos(2\omega t+\theta_v+\theta_i)}_{\text{zero over a period or integer number of periods.}}\right]dt$$

$$\therefore \quad P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Considering purely <u>resistive</u> circuits

$$P = \frac{1}{2} V_m I_m$$

For purely <u>reactive</u> circuit

$$P = \frac{1}{2} V_m I_m \cos(90^\circ)$$

$$= 0.$$

<u>Example.</u>

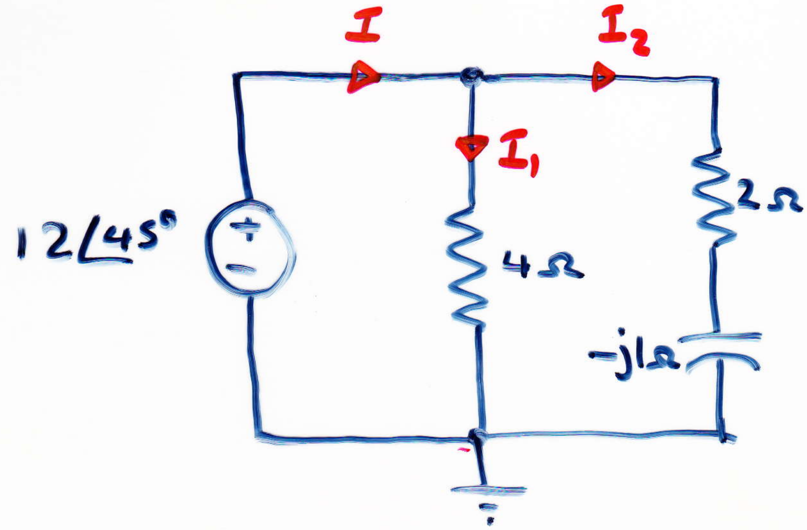
i) $\underline{V} = 2\angle 30^\circ \text{ V}$ and $\underline{I} = 2\angle 30^\circ \text{ A}$

ii) $\underline{V} = 2\angle 30^\circ \text{ V}$ and $\underline{I} = 2\angle 120^\circ \text{ A}$

iii) $\underline{V} = 2\angle 30^\circ \text{ V}$ and $\underline{I} = 2\angle 60^\circ \text{ A}$

i) 2 W ii) 0 W iii) 1.732 W

Example. (Irwin Example 9.3)



Determine total average power absorbed and total average power supplied.

$$\underline{I}_1 = \frac{12\angle 45^\circ}{4} = 3\angle 45^\circ \text{ A}$$

$$\underline{I}_2 = \frac{12\angle 45}{2-j1} = \frac{12\angle 45^\circ}{2.24\angle -26.57}$$

$$= 5.36\angle 71.57 \text{ A}$$

So $\underline{I} = \underline{I}_1 + \underline{I}_2$

$$= 3\angle 45^\circ + 5.36\angle 71.57$$

$$= 8.15\angle 62.10^\circ \text{ A}$$

Average power in 4Ω resistor

$$P_4 = \tfrac{1}{2} V_m I_m = \frac{12 \times 3}{2} = 18W$$

Average power of 2Ω resistor.

$$P_2 = \tfrac{1}{2} I_m^2 R = \frac{(5.36)^2 \times 2}{2} = 28.7W$$

$\therefore$ Total absorbed is

$$P_A = 18 + 28.7 = \underline{46.7W}$$

Total supplied

$$P_S = \tfrac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

$$= \tfrac{1}{2} \times 12 \times 8.15 \times \cos(45^\circ - 62.10^\circ)$$

$$= \underline{46.7W}$$

Effective or RMS values.

So far have seen that the average power absorbed is dependent on the type of source

dc supply $\rightarrow$ $I^2 R$

sinusoidal $\rightarrow$ $\frac{1}{2} I_m^2 R$

What if another type of waveform is used?

Recall we looked at RMS values of waveforms

$\downarrow$

root mean squared.

This is what is used.

Define

$$P = I_{eff}^2 R \quad (1)$$

I_{eff} is effective constant current of current waveform.

Considering the average power due to the waveform

$$P = \frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\, R\, dt \quad (2)$$

T. period.

Equating (1) & (2)

$$I_{eff} = \sqrt{\frac{1}{T}\int_{t_0}^{t_0+T} i^2(t)\, dt} \qquad \text{RMS!}$$

$\therefore \quad P = I_{RMS}^2 R$

Similarly $\quad P = \dfrac{V_{RMS}^2}{R}$

Example.

Determine the rms value of

$$i(t) = I_m \cos(\omega t - \theta)$$

Period, $T = \dfrac{2\pi}{\omega}$

Now

$$I_{rms} = \left[\frac{1}{T}\int_0^T I_m^2 \cos^2(\omega t - \theta)\right]^{\frac{1}{2}}$$

Using $\cos^2\phi = \frac{1}{2} + \frac{1}{2}\cos 2\phi$

$$\therefore\ I_{rms} = I_m \left\{\frac{\omega}{2\pi}\int_0^{2\pi/\omega}\left[\frac{1}{2} + \frac{1}{2}\cos(2\omega t - 2\theta)\right]dt\right\}^{1/2}$$

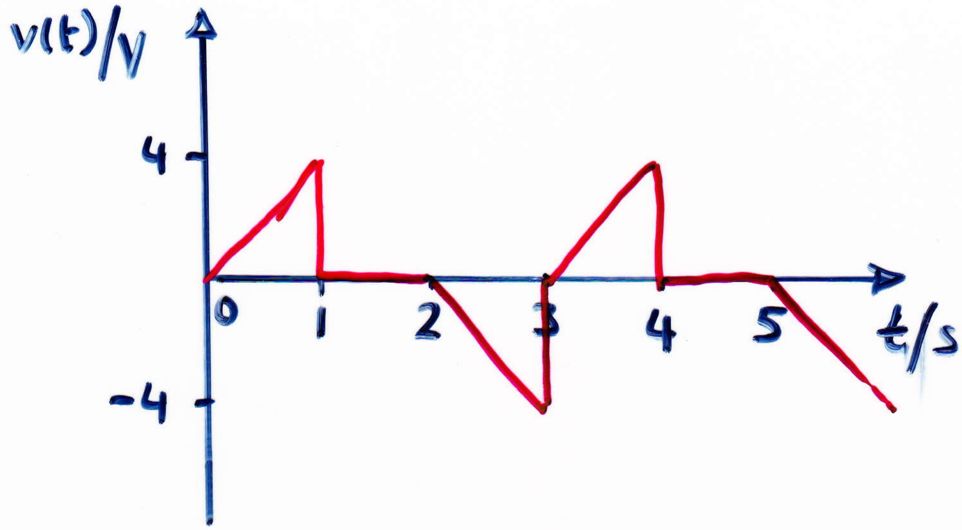
$$I_{rms} = I_m \left(\frac{\omega}{2\pi}\int_0^{2\pi/\omega} \frac{1}{2}\, dt\right)^{\frac{1}{2}}$$

$$= I_m \left\{\left[\frac{\omega}{2\pi}\left(\frac{t}{2}\right)\right]_0^{2\pi/\omega}\right\}^{1/2} = \frac{I_m}{\sqrt{2}}$$

Example.

~ 2.8

~ 2.0



$$v(t) = \begin{cases} 4t \text{ V} & 0 < t \leq 1\text{s} \\ 0 \text{ V} & 1 < t \leq 2\text{s} \\ -4t + 8\text{V} & 2 < t \leq 3\text{s} \end{cases}$$

$$V_{rms} = \left\{ \frac{1}{3} \left[\int_0^1 (4t)^2 dt + \int_1^2 (0)^2 dt + \int_2^3 (8-4t)^2 dt \right] \right\}^{1/2}$$

$$= \left\{ \frac{1}{3} \left(\left[\frac{16t^3}{3} \right]_0^1 + \left[64t - \frac{64t^2}{2} + \frac{16t^3}{3} \right]_2^3 \right) \right\}^{1/2}$$

$$= 1.89 \text{V}$$