Quadratic Poles or Zeros.

Form: $1 + 2\zeta(j\omega\tau) + (j\omega\tau)^2$

Now, dependent on ω AND ζ – damping ratio

If $\zeta > 1$ or $\zeta = 1$ roots are real and unequal or real & equal.

If $\zeta < 1$ roots are complex conjugates.

For $\omega\tau \ll 1$ quadratic factor is 0 dB.

For $\omega\tau \gg 1$

$$20\log_{10}\left|1-(\omega\tau)^2 + j\zeta(\omega\tau)\right| \simeq 20\log_{10}\left|(\omega\tau)^2\right|$$

$$= 40\log_{10}|\omega\tau|$$

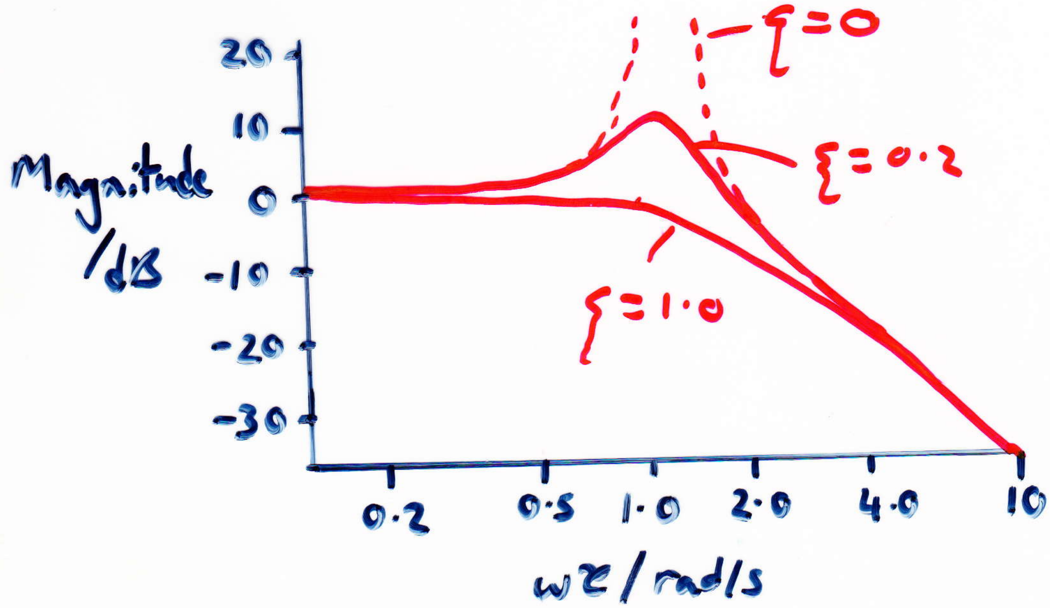
When plotting against $\omega\tau$ have a slope of:

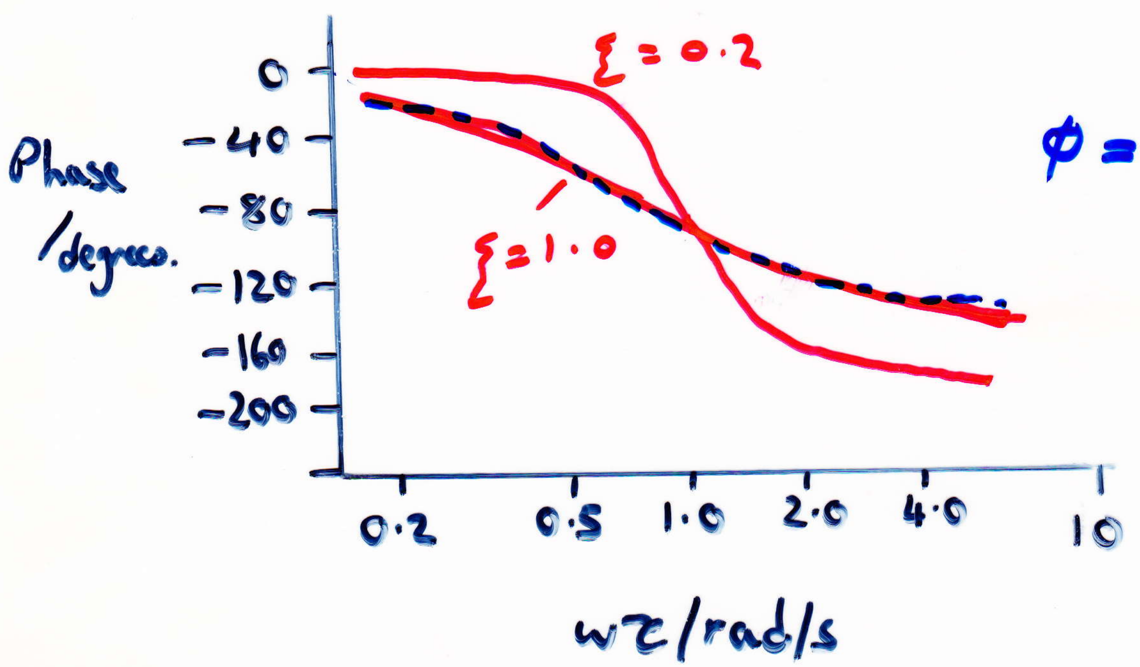
+40 dB/decade ZERO

−40 dB/decade POLE

Between $\omega\tau \ll 1$ & $\omega\tau \gg 1$ behaviour depends on ζ.

POLE





$$\phi = -\tan^{-1} \frac{2\zeta\omega\tau}{1-(\omega\tau)^2}$$

For **zeros** curves are inverted.

Example

$$G_r(j\omega) = \frac{10(0.1j\omega + 1)}{(j\omega + 1)(0.02j\omega + 1)}$$

$K_o = 10$, $20\,\text{Log}_{10}(10) = 20$

Considering: $0.1j\omega + 1$.

0dB for $0.1\omega \ll 1$

+20 dB/decade for $0.1\omega \gg 1$

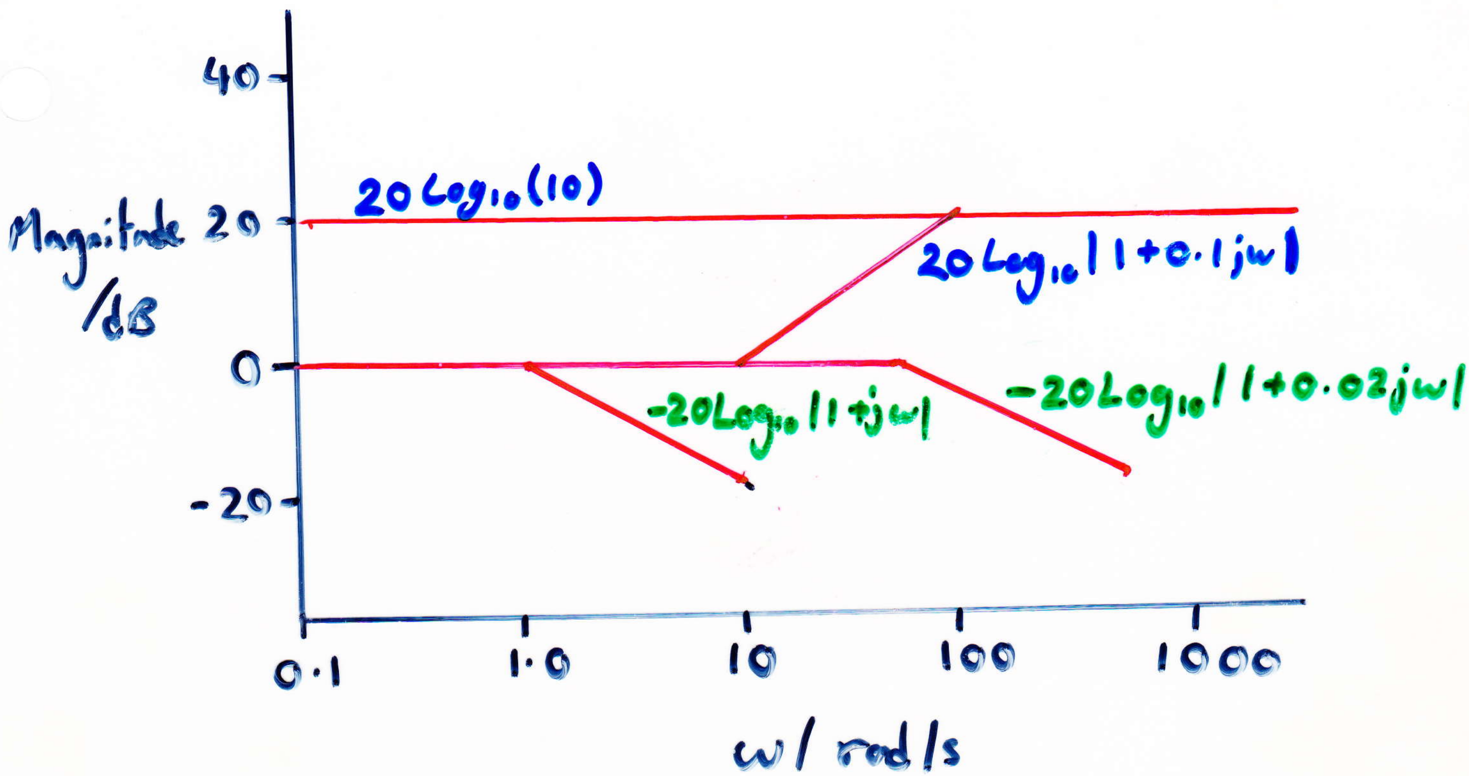
Zero Break point at $\omega = 10$ $(0.1\omega = 1)$

Considering: $j\omega + 1$

Pole Break freq. at $\omega = 1$

Considering: $0.02j\omega + 1$

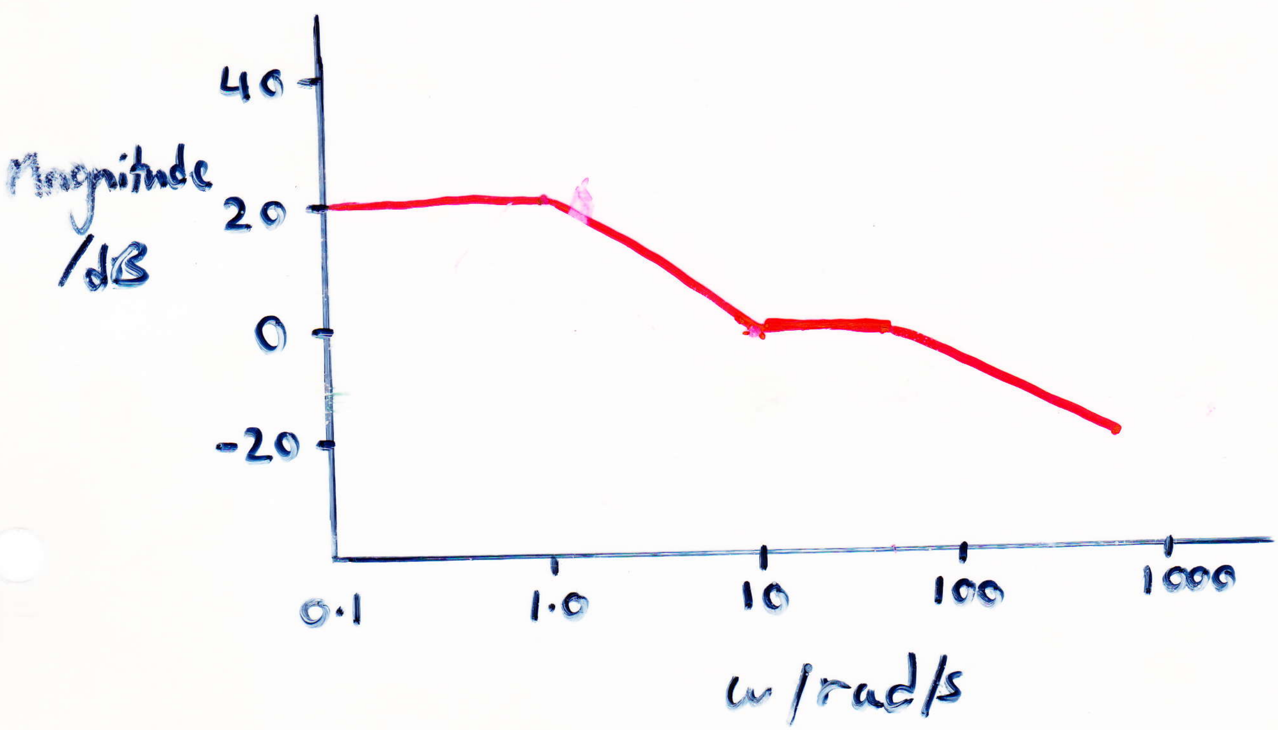
Pole Break freq. at $\omega = 50$.



<u>Break points</u>

Zeros: $\omega = 10$

Poles: $\omega = 1$ & $\omega = 50$



Phase :

Zeros

K_0 :- no phase contribution.

$(1+0.1j\omega)$:- $\phi = +\tan^{-1} 0.1\omega$

ϕ	$0°$	$+45°$	$+90°$
0.1ω	$\ll 1$	$\omega = 10$	$\gg 1$

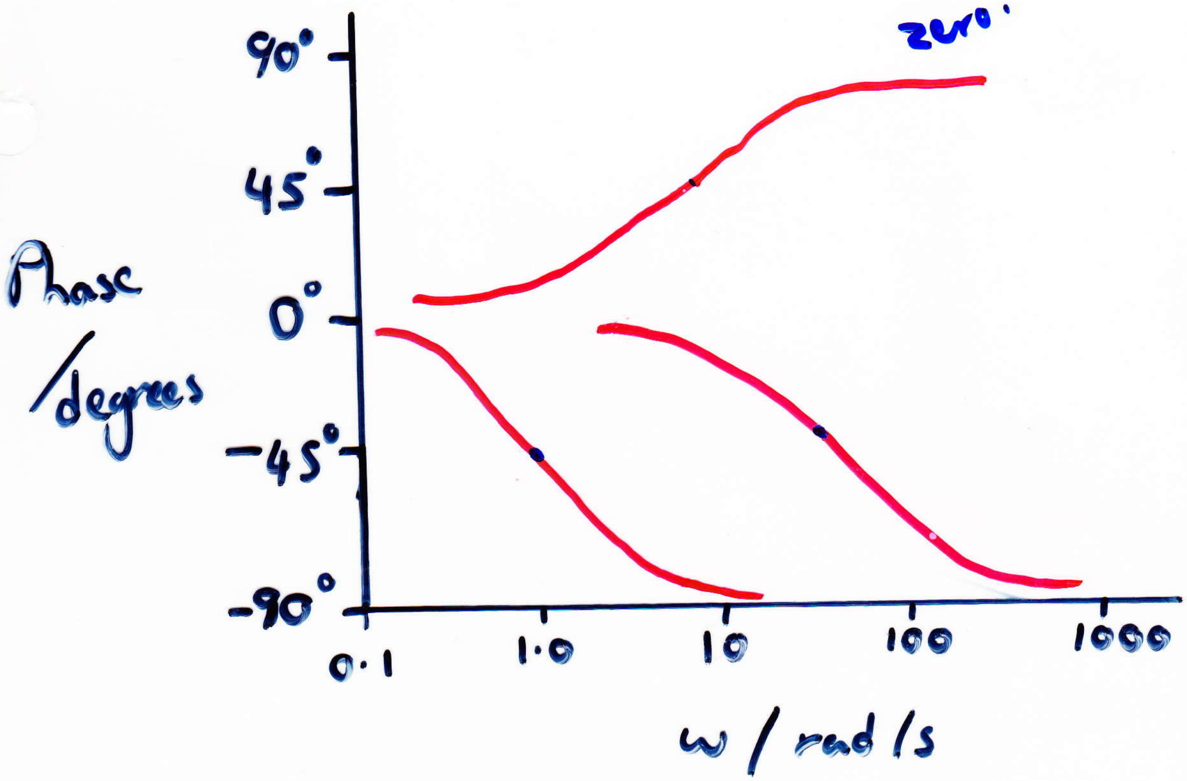
Poles

$(1+j\omega)$ $\phi = -\tan^{-1} \omega$

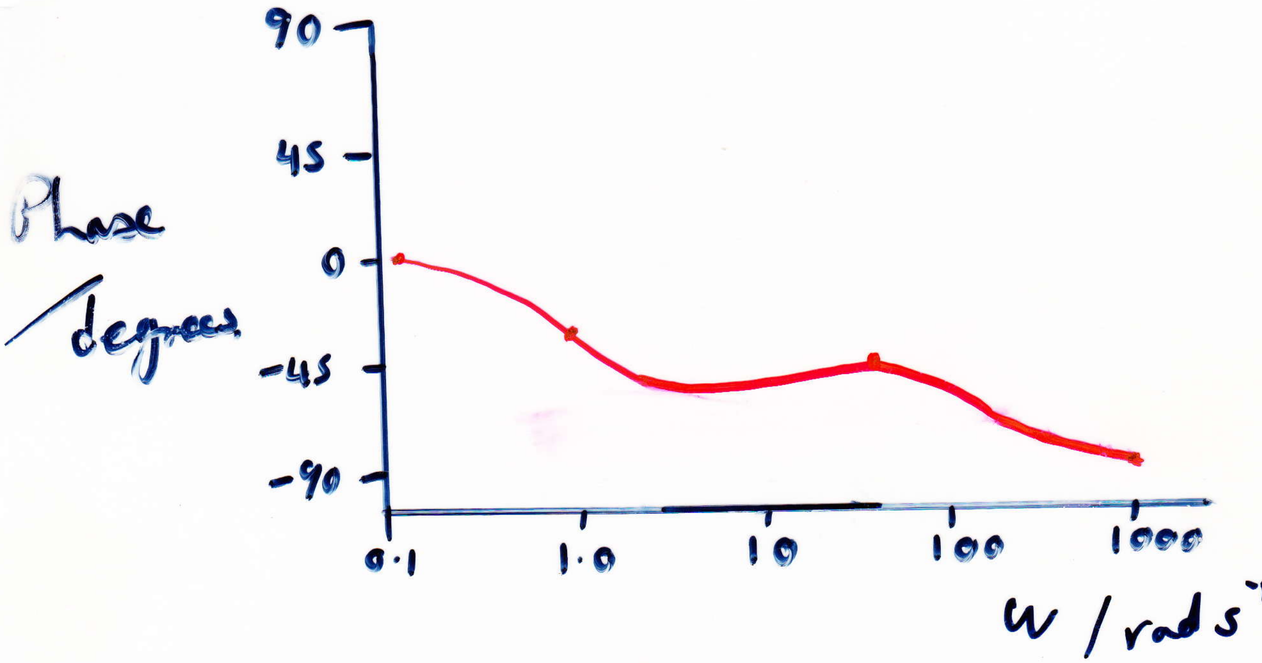
$(1+0.02j\omega)$ $\phi = -\tan^{-1} 0.02\omega$

ϕ	$0°$	$-45°$	$-90°$
ω	$\ll 1$	$\omega = 1$	$\gg 1$
0.02ω	$\ll 1$	$\omega = 50$	$\gg 1$

24.4



zero
90°
45°
0°
-45°
-90°
Phase /degrees
0.1
1.0
10
100
1000
w / rad/s



90
45
0
-45
-90
Phase /degrees
0.1
1.0
10
100
1000
w / rad s⁻¹

Example (12.4).

$$G_v(j\omega) = \frac{25\,(j\omega+1)}{(j\omega)^2(0.1j\omega+1)}$$

$K_0 = 25$,

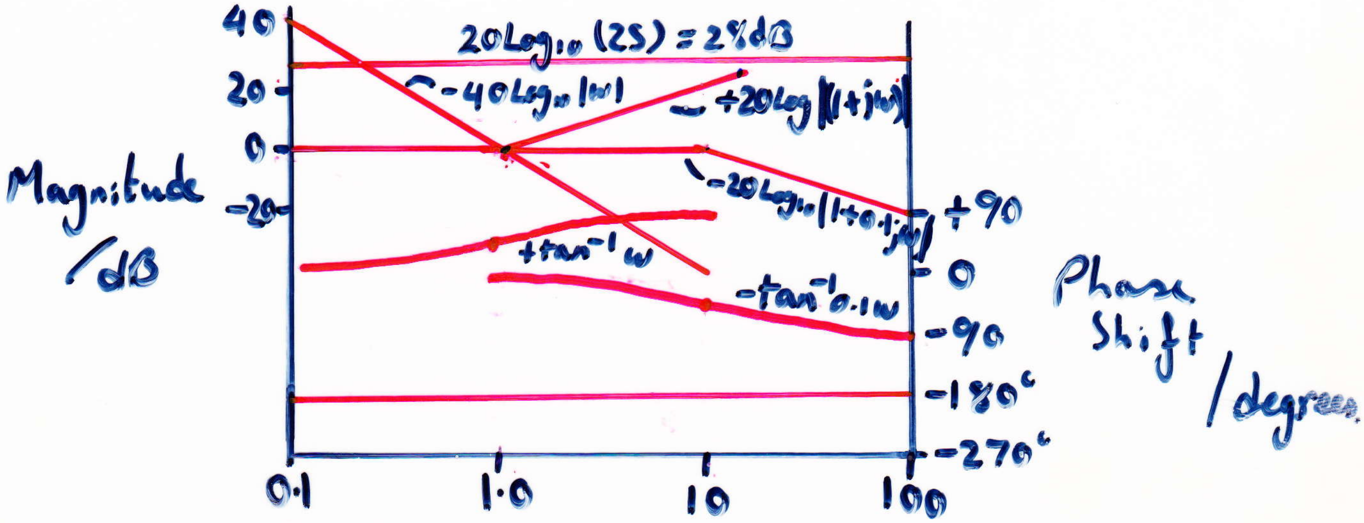
$(j\omega+1)$ Break point $\omega = 1$

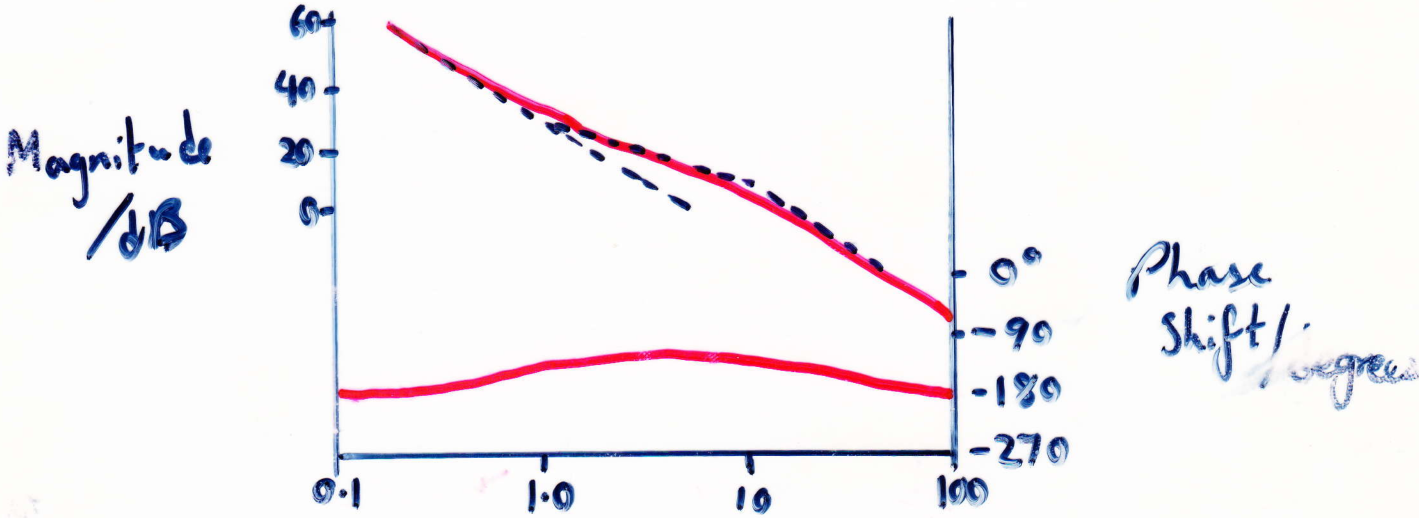
Poles $(0.1j\omega+1)$ Break point $\omega = 10$

$(j\omega)^2$ Magnitude zero @ $\omega = 1$

Irwin 11.4.

27.6


40
20
0
-20
Magnitude /dB
20Log10 (25) = 28dB
-40Log10 |w|
+20Log |1+jw|
-20Log10 |1+0.1jw|
+tan^-1 w
-tan^-1 0.1w
+90
0
-90
-180°
-270°
Phase Shift /degrees
0.1
1.0
10
100


60
40
20
0
Magnitude /dB
0°
-90
-180
-270
Phase Shift/degrees
0.1
1.0
10
100