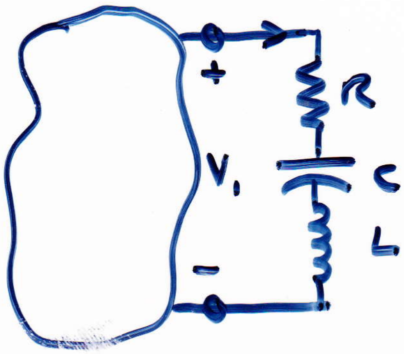
Resonant Circuits.

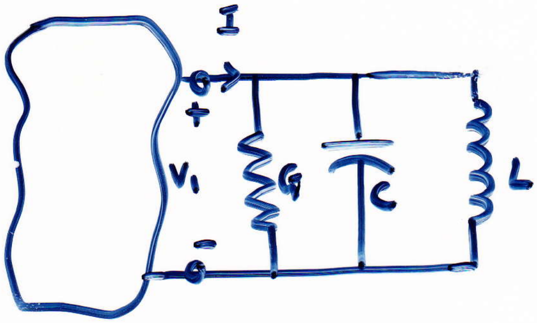
Consider RLC circuits

SERIES



• $\underline{Z}(j\omega) = R + j\omega L + \frac{1}{j\omega C}$ (1)

PARALLEL



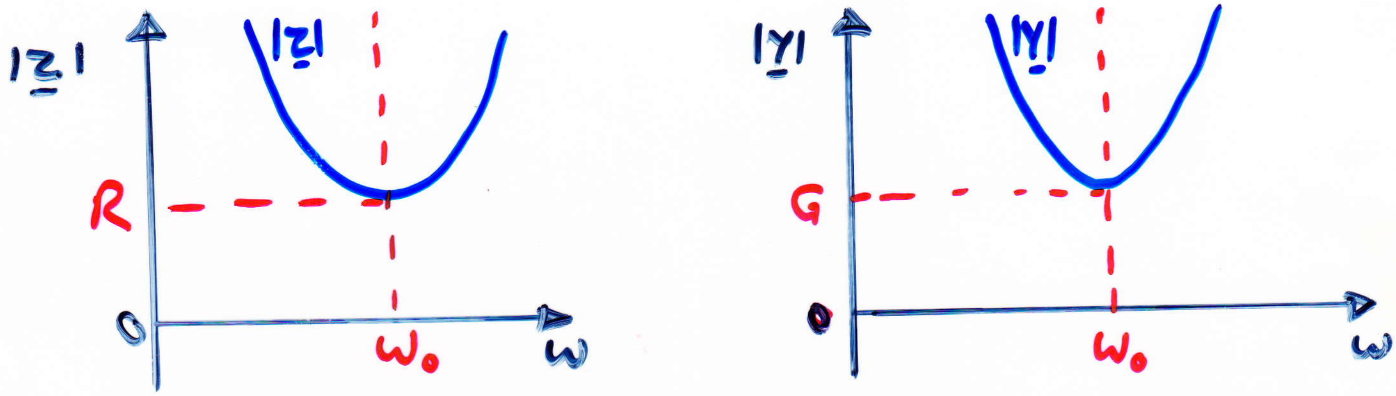
• $\underline{Y}(j\omega) = G + j\omega C + \frac{1}{j\omega L}$ (2)

Equations (1) and (2) have same basic form

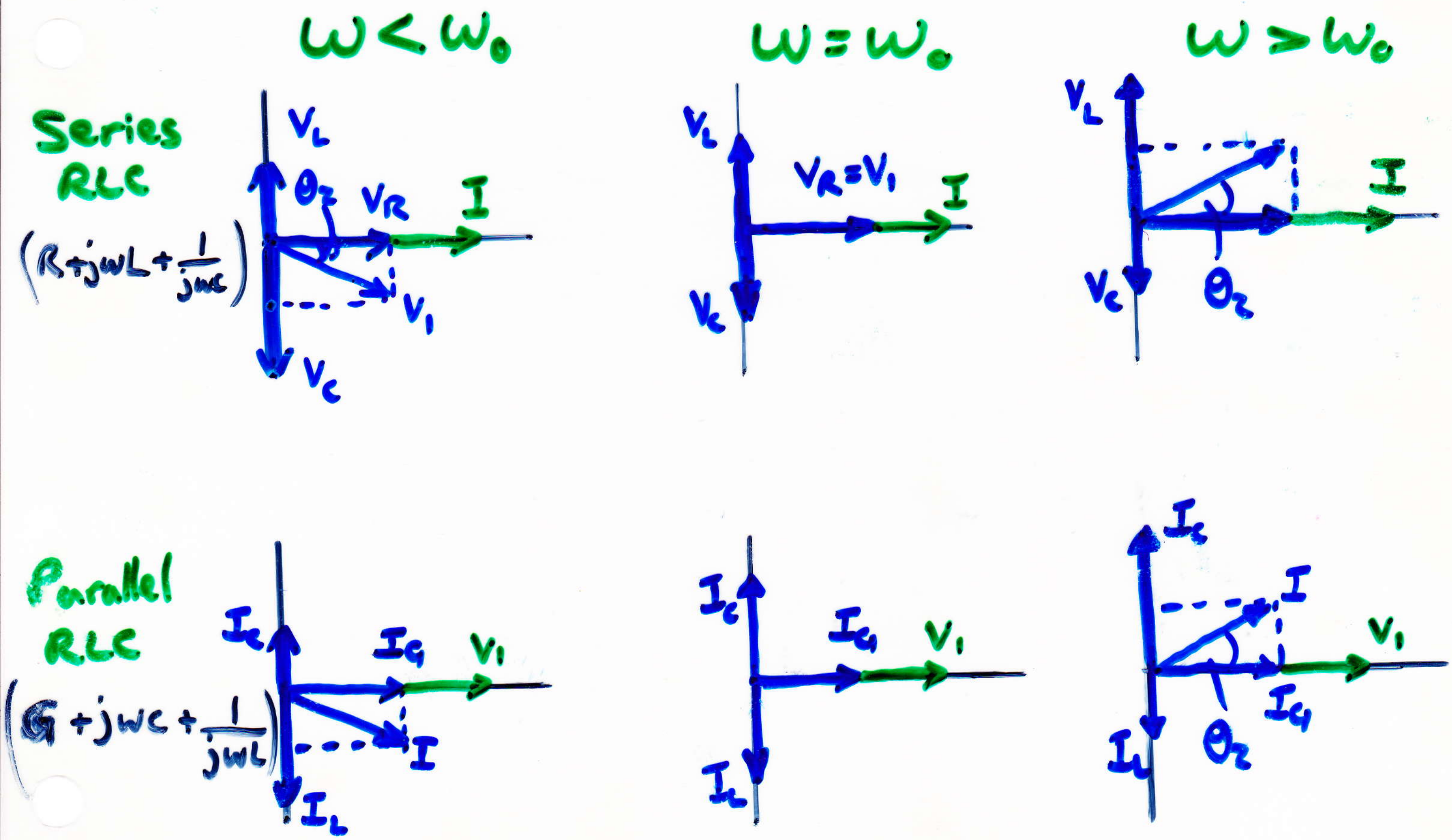
If $\omega L = \frac{1}{\omega C}$ then imaginary terms equal zero.

$\therefore \omega_0 = \frac{1}{\sqrt{LC}}$ RESONANT FREQUENCY

At this freq. $\underline{Z}(j\omega_0) = R$ for SERIES circuits
$\underline{Y}(j\omega_0) = G$ " PARALLEL "



At resonance voltage and current in phase, therefore phase angle is zero.



Q

Consider series RLC. Can define a quantity known as the quality factor, Q:

$$Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 C R} = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (3)$$

[Question: what would be the similar expression for parallel circuit?]

Considering (3), if R is small the Q is high, if R is large Q is low.

To look at Q graphical let us do a little more mathematical manipulation.

Consider the admittance of the series circuit.

$$\underline{Y}(j\omega) = \frac{1}{R(1 + j(1/R)(\omega L - 1/\omega C))}$$

$$= \frac{1}{R(1 + j\frac{\omega L}{R} - j/\omega C R)}$$

$$= \frac{1}{R\left[1 + jQ\left(\frac{\omega L}{RQ} - 1/\omega CRQ\right)\right]}$$

Since $Q = \omega_0 L/R = 1/\omega_0 CR$

Then $$\underline{Y}(j\omega) = \frac{1}{R\left(1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)\right)}$$

Since $\underline{I} = \underline{Y}\,\underline{V}_1$ & $\underline{V}_R = \underline{I}\,R$

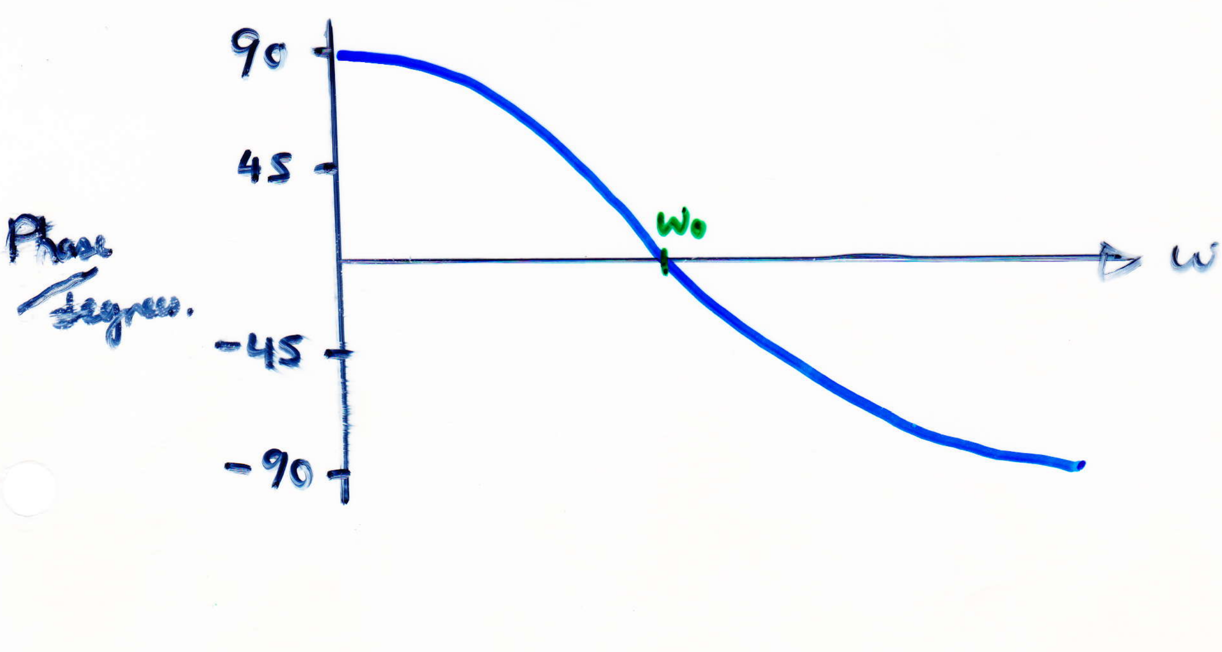
then $$\frac{V_R}{V_1} = \underline{G}v(j\omega) = \frac{1}{1 + jQ\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)}$$

Transfer function.

Looking at magnitude & phase

$$M(\omega) = \frac{1}{\left[1 + Q^2(\omega/\omega_0 - \omega_0/\omega)^2\right]^{1/2}}$$

$$\phi(\omega) = -\tan^{-1} Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right)$$



Let magnitude $= \frac{1}{\sqrt{2}} = \left| \frac{1}{1 + jQ(\omega/\omega_0 - \omega_0/\omega)} \right|$

$$\therefore \quad Q\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right) = \pm 1$$

Solving this can see have four freq.

$$\omega = \pm \frac{\omega_0}{2Q} \pm \omega_0 \sqrt{\left(\frac{1}{2Q}\right)^2 + 1}$$

Taking +ve freq.

$$\therefore \quad \omega_{LO} = \omega_0 \left[-\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right] \qquad (4)$$

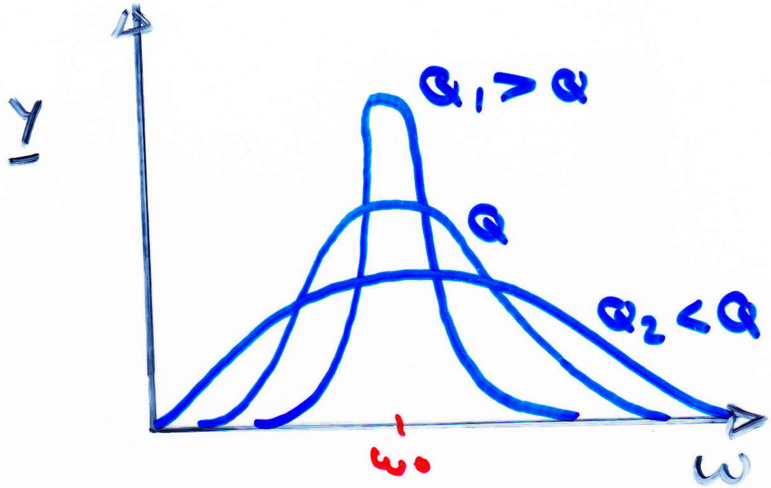
$$\omega_{HI} = \omega_0 \left[\frac{1}{2Q} + \sqrt{\left(\frac{1}{2Q}\right)^2 + 1} \right] \qquad (5)$$

Subtracting (4) from (5)

Bandwidth, $BW = \omega_{HI} - \omega_{LO} = \frac{\omega_0}{Q}$

Multiplying (4) & (5)

$$\omega_0^2 = \omega_{LO}\,\omega_{HI}$$



Q also defined as

$$Q = 2\pi \frac{W_S}{W_D}$$ GENERAL DEFN

W_D energy dissipated per cycle

W_S max. energy stored at resonance

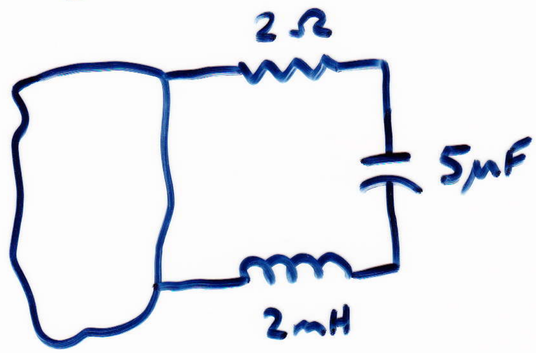
7th / 9th

Example. Irwin, Ex 11.9 / 12.9

Series circuit with $R = 2\Omega$, $L = 2mH$ and $C = 5\mu F$.

Determine the resonant freq., the quality factor and bandwidth.

What is the change in Q and BW if R is changed from 2Ω to 0.2Ω?



$$\omega_0 = \frac{1}{\sqrt{LC}} = \frac{1}{(2\times10^{-3}\times5\times10^{-6})^{1/2}}$$

$$= 10^4 \text{ rad/s}$$

$$\therefore\ f_0 = \frac{10^4}{2\pi} = \underline{1592 \text{ Hz.}}$$

$$Q = \frac{\omega_0 L}{R} = \frac{10^4\times2\times10^{-3}}{2} = \underline{10}$$

$$BW = \frac{\omega_0}{Q} = \frac{10^4}{10} = 10^3 \text{ rad/s.}$$

If R becomes 0.2Ω then:

$$Q = \frac{10^4 \times \cancel{2} \times 10^{-3}}{\cancel{0.2}\ 0.1}$$

$$= 100$$

$$BW = \frac{\omega_0}{Q} = \frac{10^4}{10^2}$$

$$= 10^2 \text{ rad/s.}$$