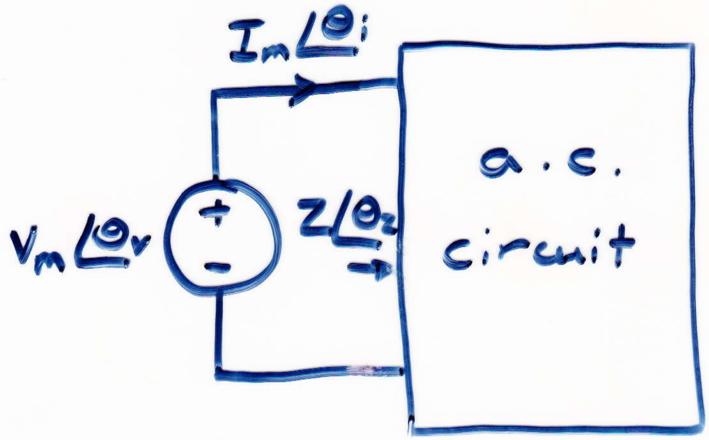
Impedance & Admittance

Definition

Two-terminal input IMPEDANCE,

$$\underline{Z} = \frac{\underline{V}}{\underline{I}}$$

$$\underline{Z} = \frac{V_m \angle \theta_v}{I_m \angle \theta_i} = \frac{V_m}{I_m} \angle \theta_v - \theta_i = Z \angle \theta_z$$

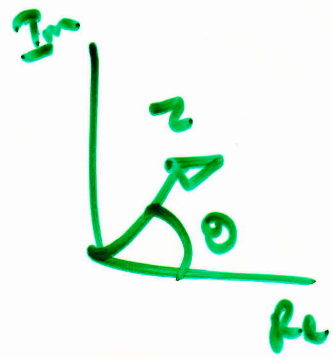


$$Z(\omega) = \underbrace{R(\omega)}_{\text{Resistive}} + j\underbrace{X(\omega)}_{\text{Reactive}}$$

$$Z\angle\theta_Z = R + jX$$

$$Z = \sqrt{R^2 + X^2}$$

$$\theta_Z = \tan^{-1}\frac{X}{R}$$



$$R = Z\cos\theta_Z$$

$$X = Z\sin\theta_Z$$

Consider individual components

R $\qquad Z = R$

L $\qquad Z = j\omega L = jX_L = \omega L \angle 90^\circ$

$$X_L = \omega L$$

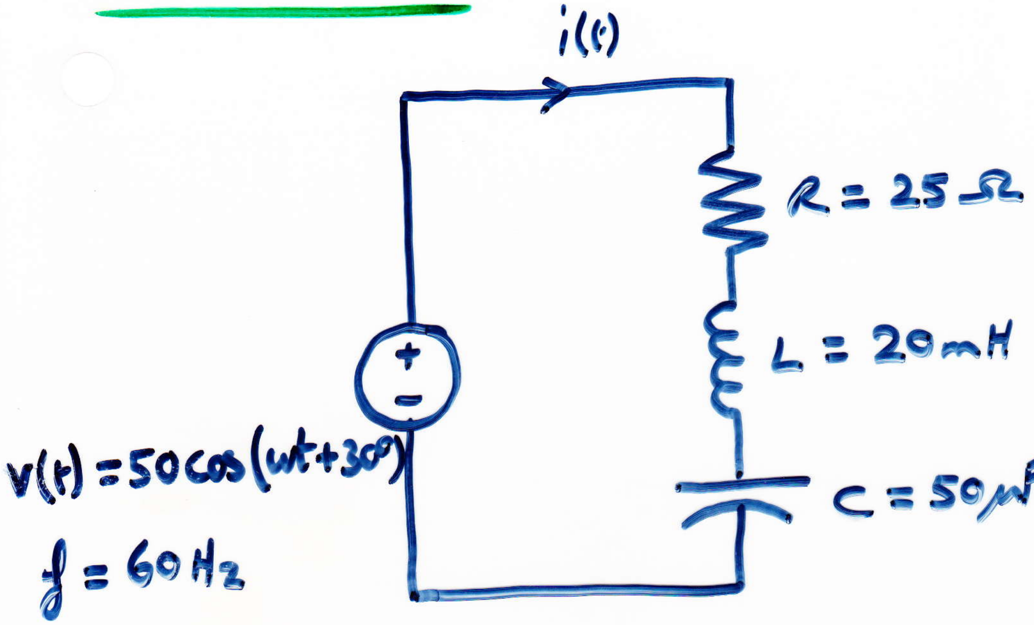
C $\qquad Z = \frac{1}{j\omega C} = \underline{jX_C} = -\frac{1}{\omega C}\angle 90^\circ$

$$\left[\frac{1}{j\omega C}\left(\times\frac{-j}{-j}\right)\right]$$

$$X_C = -\frac{1}{\omega C}$$

Note KCL & KVL are valid, as are (resistor) combination rules.

Example



Determine

i) current $i(t)$

ii) equivalent impedance if f is changed to 400 Hz.

$$\underline{V} = 50\angle 30^\circ$$

$$Z_R = 25\,\Omega$$

$$Z_L = j\omega L = j(2\pi \times 60)(20 \times 10^{-3}) = j7.54\,\Omega$$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{(2\pi \times 60)(50 \times 10^{-6})} = -j53.05\,\Omega$$

$$\underline{Z} = \underline{Z}_R + \underline{Z}_L + \underline{Z}_C = 25 - j45.51\,\Omega$$

$$= 51.93 \angle{-61.22^\circ}$$

$$\underline{I} = \frac{\underline{V}}{\underline{Z}} = \frac{50 \angle 30^\circ}{51.93 \angle{-61.22}}$$

$$= 0.96 \angle 91.22^\circ \quad A$$

In time domain $i(t) = 0.96 \cos(377t + 91.22^\circ)$

If $f = 400\,Hz$?

$$Z_R = 25\,\Omega$$

$$Z_L = j\omega L = j\,50.27\,\Omega$$

$$Z_C = \frac{-j}{\omega C} = -j7.96\,\Omega$$

$$z = 25 + j42.31 = 49.14 \angle 59.42^\circ\ \Omega$$

Admittance

Definition

$$\underline{Y} = \frac{1}{\underline{Z}} = \frac{\underline{I}}{\underline{V}}$$

$$\underline{Y} = Y_m \angle \theta_y$$

$$\underline{Y} = \underbrace{G}_{\text{conductance}} + j\underbrace{B}_{\text{susceptance}} = \frac{1}{R + jX}$$

Can show

$$G = \frac{R}{R^2 + X^2}, \quad B = \frac{-X}{R^2 + X^2}$$

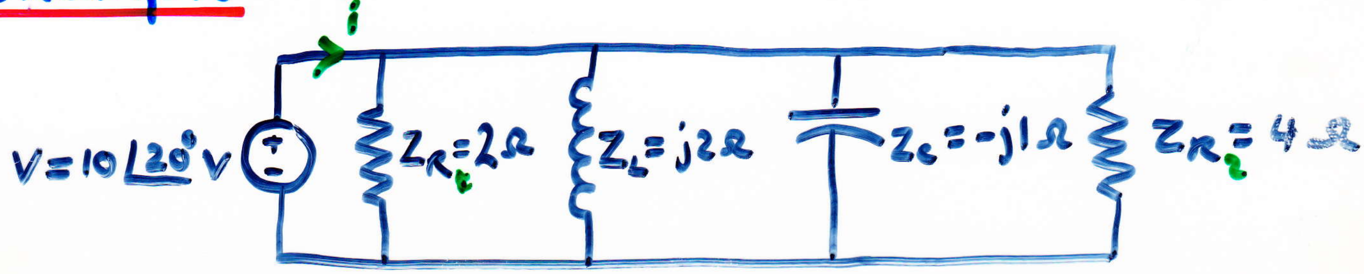
$$R = \frac{G}{G^2 + B^2}, \quad X = \frac{-B}{G^2 + B^2}$$

Combining admittance

SERIES $\quad \frac{1}{\underline{Y}_s} = \frac{1}{\underline{Y}_1} + \frac{1}{\underline{Y}_2} + \frac{1}{\underline{Y}_3} + \dots + \frac{1}{\underline{Y}_n}$

PARALLEL $\quad Y_p = \underline{Y}_1 + \underline{Y}_2 + \underline{Y}_3 + \dots \underline{Y}_n$

Example (E8.9)



$$Y_{R_1} = \frac{1}{2} = 0.5\,S \qquad Y_{R_2} = \frac{1}{4}S$$

$$Y_L = \frac{1}{j2} = -0.5j\,S$$

$$Y_C = \frac{1}{-j} = j\,S$$

$$\therefore\ Y_P = 0.5 + 0.25 - 0.5j + j = 0.75 + 0.5j$$

$$\underline{I} = \underline{Y_P}\,\underline{V}$$

$$= \left(\frac{3}{4} + \frac{1}{2}j\right)\underline{V}$$

$$Y_P = \sqrt{\left(\frac{3}{4}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{9}{16} + \frac{1}{4}} = \sqrt{\frac{13}{16}} = 0.901$$

$$\theta_Y = \tan^{-1}\left(\frac{1/2}{3/4}\right) = \tan^{-1}(2/3) = 33.7°$$

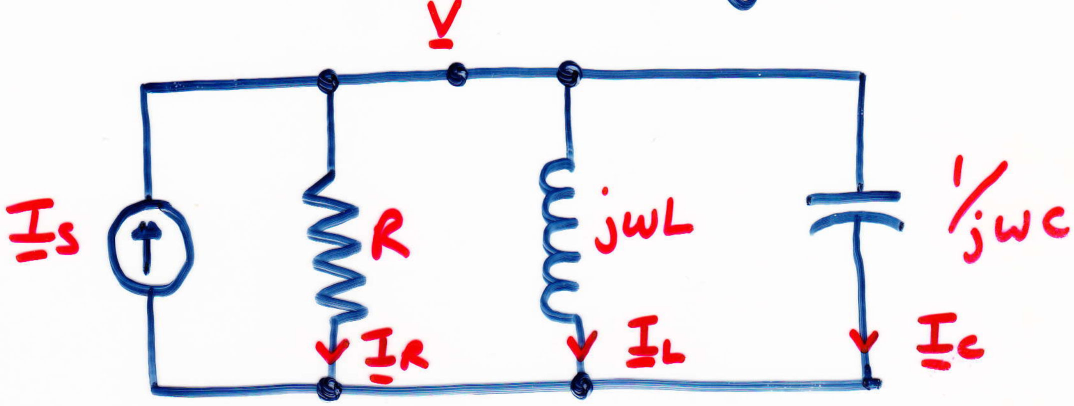
$$\therefore\ \underline{I} = 0.901\angle 33.7° \times 10\angle 20°$$

$$\underline{I} = 9.01\angle 53.7°\ A$$

Phasor Diagrams & Application of Kirchhoff's Laws

Phasor Diagrams

Consider the RLC arrangement.



Let $\underline{V}$ be a reference phasor with an arbitrarily assigned phase of 0°,

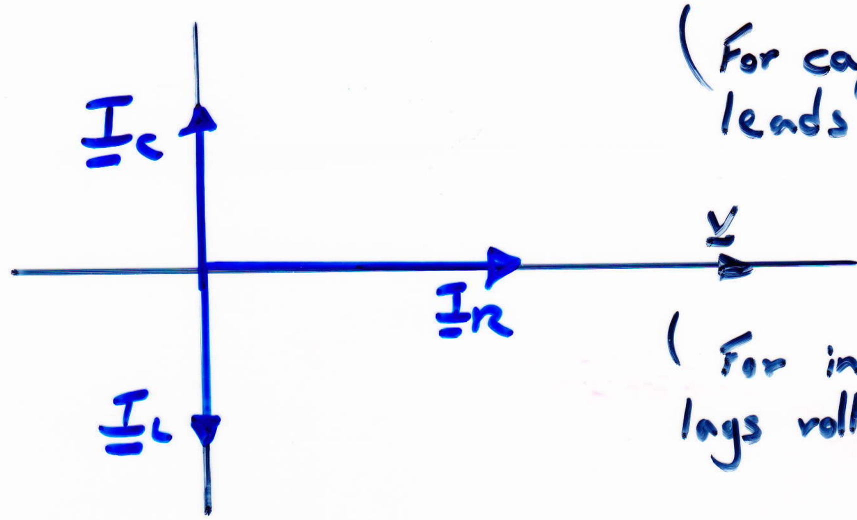
so $\underline{V} = V_m \angle 0°$

At upper node have from KCL

$$\underline{I}_s = \underline{I}_R + \underline{I}_L + \underline{I}_C = \frac{\underline{V}}{R} + \frac{\underline{V}}{j\omega L} + \frac{\underline{V}}{1/j\omega C}$$

$$\therefore \; \underline{I}_s = \frac{V_m \angle 0^\circ}{R} + \frac{V_m \angle -90^\circ}{\omega L} + V_m \omega C \angle 90^\circ$$

Representing these individual contributions graphically we have:

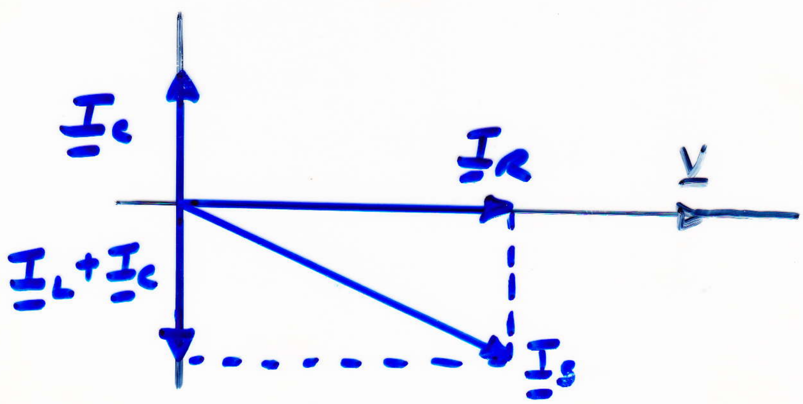


(For capacitor current leads voltage by 90°)

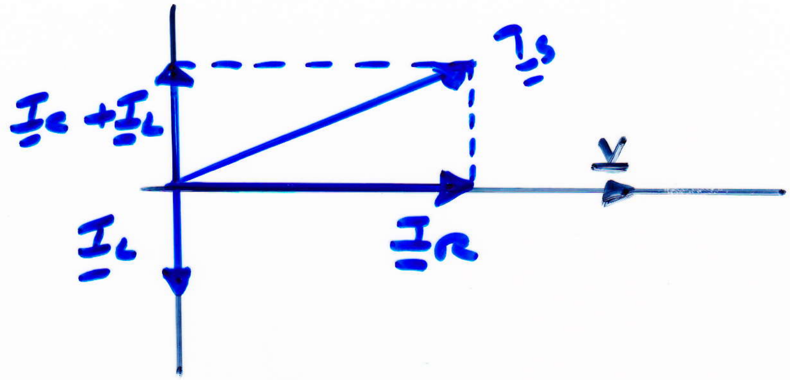
(For inductor current lags voltage by 90°)

Considering relative sizes if:

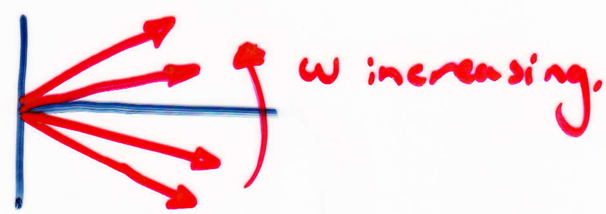
ω is small, then $\underline{I}_L$ is greater than $\underline{I}_c$



ω is large, then $\underline{I}_C$ is greater than $\underline{I}_L$



As frequency increases phasor moves from 4th quadrant to 1st quadrant.



Now recall

$$\underline{I} = \left[\frac{1}{R} + j\left(\omega C - \frac{1}{\omega L}\right)\right]\underline{V}$$

Can see when $\omega C = \frac{1}{\omega L}$, i.e. $\underline{I}_C = \underline{I}_L$ then $\underline{I}_S$ is in phase with $\underline{V}$

$$\omega L = \frac{1}{\omega C}$$

$$\therefore \boxed{\omega = 1/\sqrt{LC}}$$