3

In Example 7.3

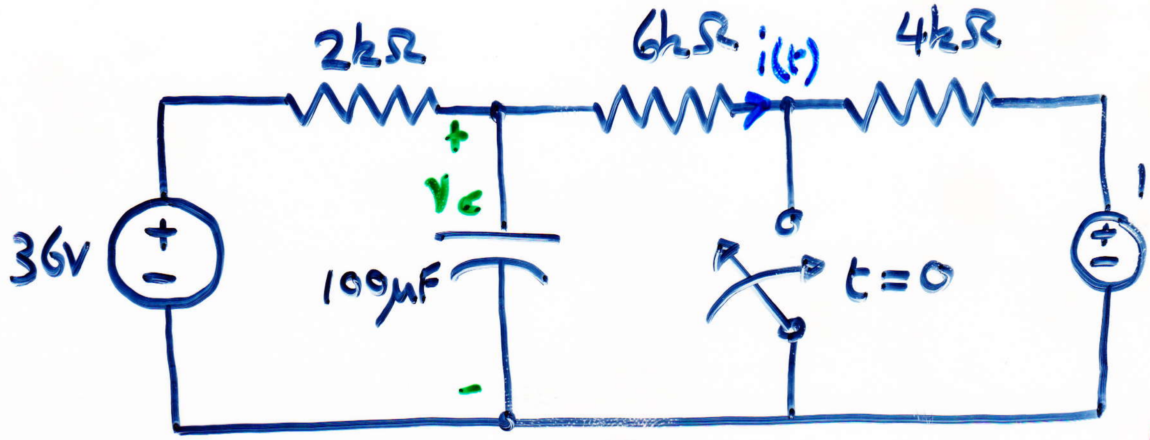


Fig 1

Circuit in steady state form before $t=0$.
Find $i(t)$ for $t>0$

First find steady state voltage across capacitor

$$36 - V_c = 2k\, i(t)$$

But $i(t) = (36-12)/12k = 2\,mA$

$$\therefore V_c = 36 - 2\cancel{k} \times 2 \times \cancel{10}^{-3}$$

$$= \underline{32\,V} \quad (\text{at } t = 0^-)$$

When switch shuts, at $t = 0^+$, capacitor acts as a 32V supply momentarily.

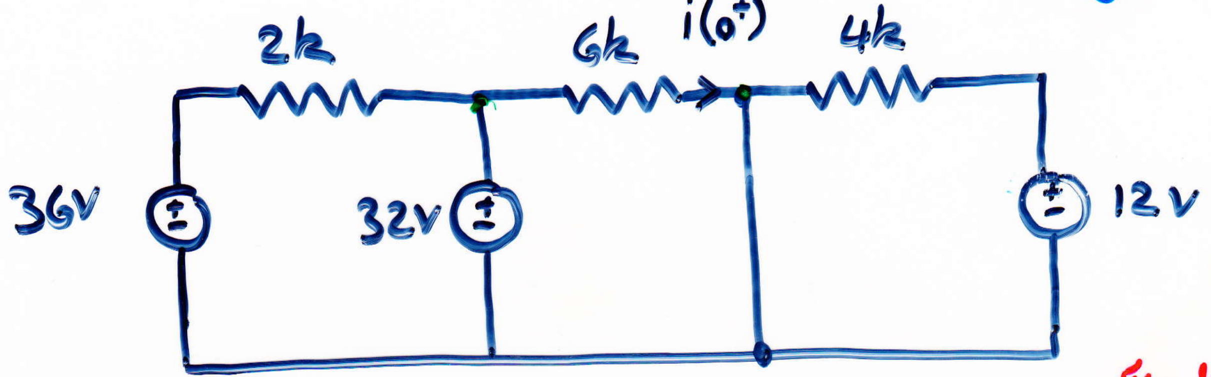


Fig 1a

$$\therefore\ i(0^+) = \frac{32\text{V}}{6\text{k}} = \frac{16}{3}\text{ mA}$$

At $t = \infty$

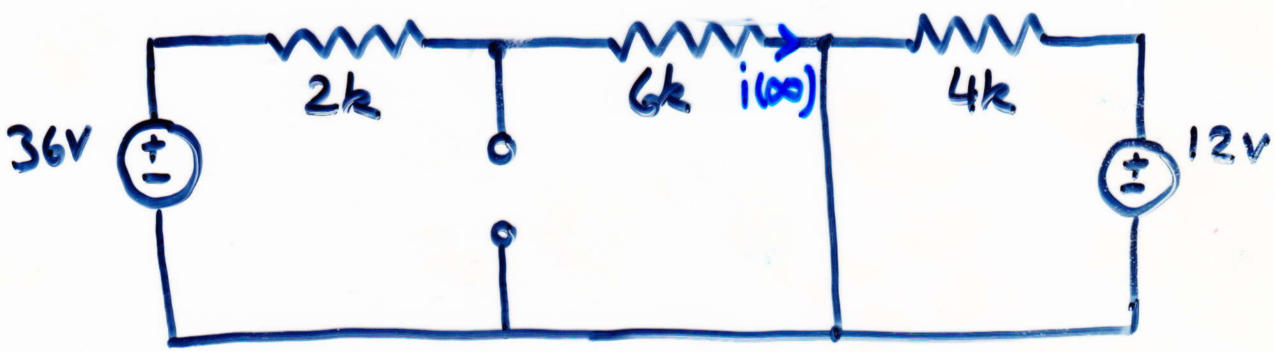
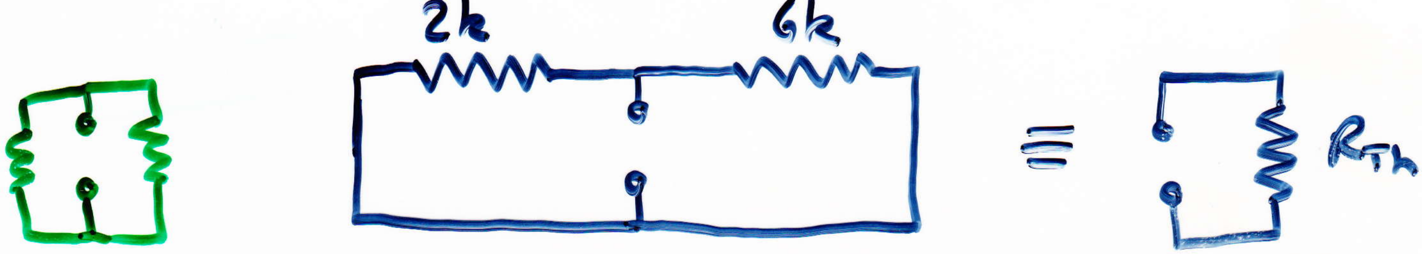


Fig 1b

$$\therefore\ i(\infty) = \frac{36\text{V}}{8\text{k}} = \frac{9}{2}\text{ mA.}$$

Now consider the Thévenin equivalent resistance for open-circuit terminals of the capacitor



$$R_{Th} = \frac{2k \; 6k}{2k + 6k} = \frac{12}{8} k$$

$$R_{Th} = \frac{3}{2} k$$

Now, we know the effective resistance the capacitor sees, so the time constant is

$$\tau = R_{Th} C$$

$$= \frac{3}{2} \times 10^{3} \times 100 \times 10^{-6}$$

$$= 0.15 \, s$$

Making use of the solution of the form

$$i(t) = K_1 + K_2 e^{-t/\tau}$$

$$K_1 = i(\infty) = \frac{9}{2}\ mA$$

$$K_2 = i(0^+) - K_1$$

$$= \frac{16}{3} - \frac{9}{2} = \frac{32-27}{6}$$

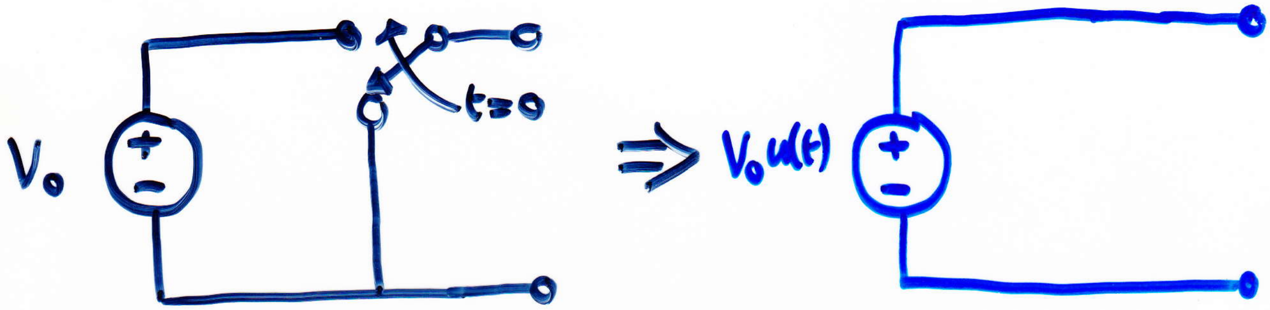
$$= \frac{5}{6}\ mA$$

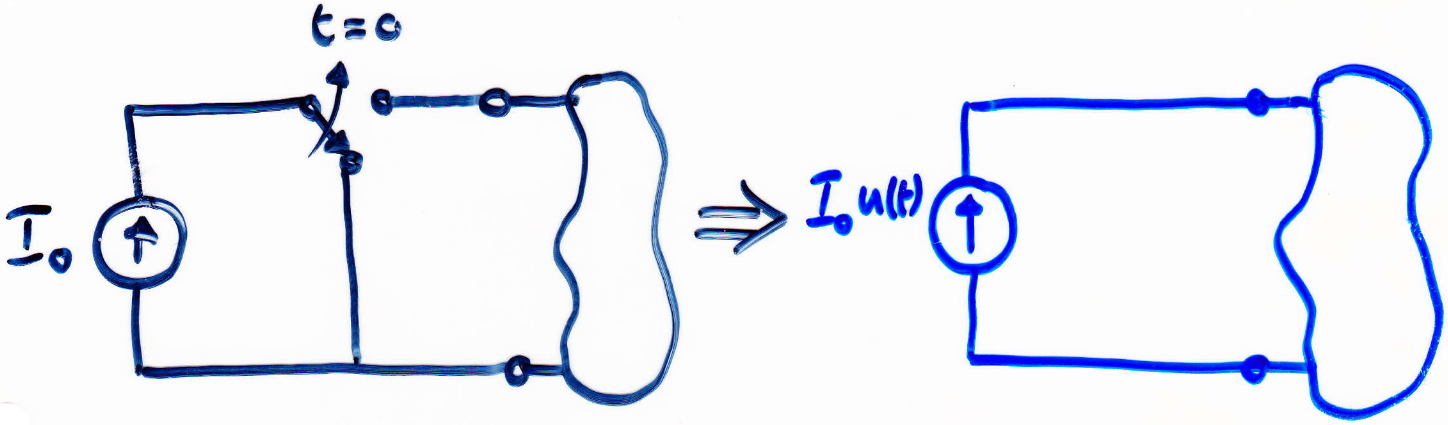
And we determine $\tau = 0.15\ s$

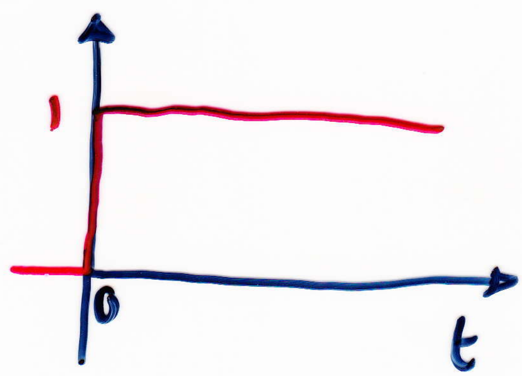
$$\therefore\ i(t) = \frac{36}{8} + \frac{5}{6} e^{-t/0.15}\ mA$$

Pulse Response of RC & RL Circuits

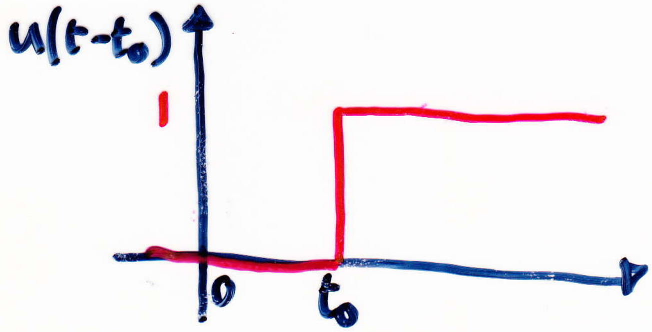
So far have considered the abrupt application of a voltage or current. Can this discontinuous derivative be described mathematically?





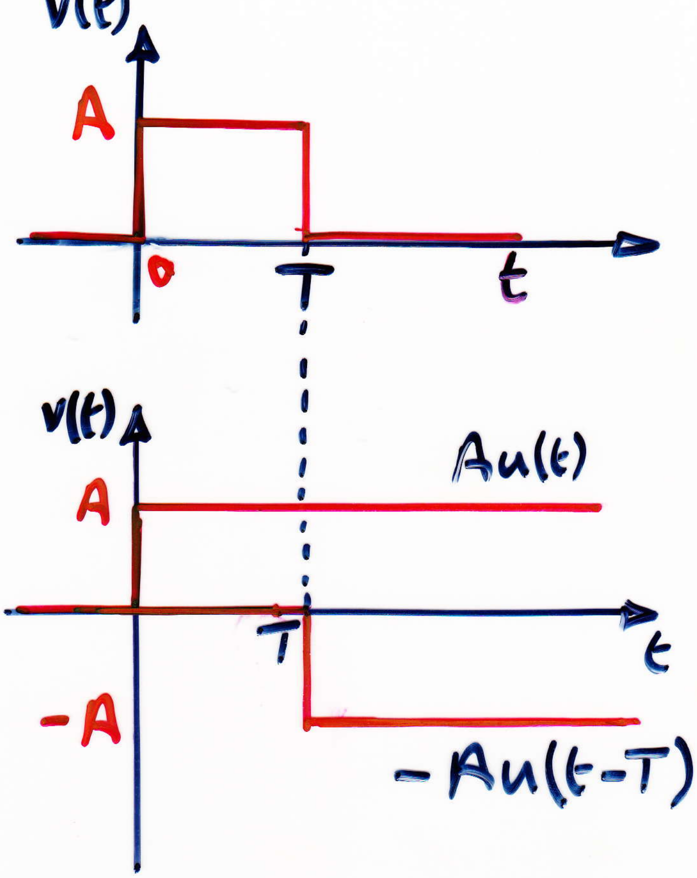


$$u(t) = \begin{cases} 0 & t<0 \\ 1 & t>0 \end{cases}$$



$$u(t-t_0) = \begin{cases} 0 & t<t_0 \\ 1 & t>t_0 \end{cases}$$

Step Function

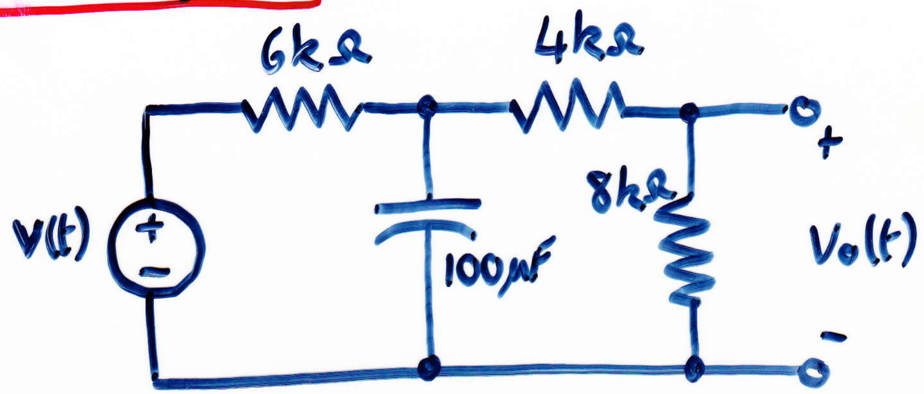


$$v(t) = A\left[u(t) - u(t-T)\right]$$

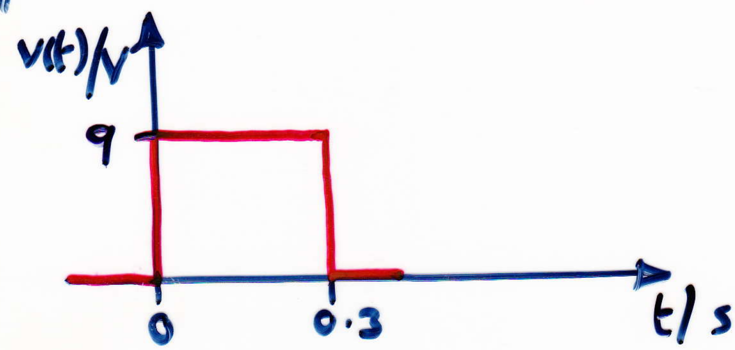
If the pulse starts at $t = t_0$ and with width T the equation is

$$v(t) = A\left\{u(t-t_0) - u\left[t-(t_0+T)\right]\right\}$$

Example.



Input



Determine the expression for $v_o(t)$

Initially voltage across capacitor is zero

$$v_o(0^+) = 0$$

If after $t=0$ and $v(t) = 9v$, consider situation that $v(t)$ is unchanged

In this situation

$$V_0(\infty) = \frac{9}{6k + 4k + 8k}(8k) = 4V$$

From the point of view of the capacitor the Thévenin equivalent resistance is

$$R_{Th} = \frac{6k \times 12k}{6k + 12k}$$

$$= 4k\Omega$$

So circuit time constant is

$$\tau = R_{Th} C$$
$$= 4 \times 10^3 \times 100 \times 10^{-6}$$
$$= 0.4s$$

So response of $V_0(t)$ for $0 < t < 0.3s$

$$V_0 = 4 - 4e^{-t/0.4} \; V \qquad 0 < t < 0.3s$$

Can see v_c relates to $v_o(t)$ through

$$v_o(t) = \frac{2}{3} v_c(t)$$

So $v_c(t) = \frac{3}{2}\left(4 - 4e^{-t/0.4}\right)$ V

Since the capacitor voltage is continuous

$$v_c(0.3^-) = v_c(0.3^+)$$

$$\therefore v_o(0.3^+) = \frac{2}{3} v_c(0.3^-)$$

$$= 4\left(1 - e^{-0.3/0.4}\right)$$

$$= 2.11\text{V}$$

When $t > 0.3$s source is zero.

So when $t \to \infty$ $v_o = 0$.

$$v_o(t) = 2.11\, e^{-(t-0.3)/0.4} \text{ V}$$

$t > 0.3$s

So solution

$$v_o(t) = 4\left(1-e^{-t/0.4}\right)u(t) - 4\left(1-e^{-(t-0.3)/0.4}\right)u(t-0.3)\ \text{V}$$

or

$$v_o(t) = \begin{cases} 0 & t<0 \\ 4(1-e^{-t/0.4})\ \text{V} & 0<t<0.3\text{s} \\ 2.11e^{-(t-0.3)/0.4}\ \text{V} & t>0.3\text{s} \end{cases}$$

or

$$v_o(t) = 4(1-e^{-t/0.4})\left[u(t)-u(t-0.3)\right]$$
$$+ 2.11e^{-(t-0.3)/0.4}\,u(t-0.3)\ \text{V}$$