First-Order Circuits

Transient response of RC & RL

Introduction

We have seen previously that inductors and capacitors have current or voltage dependencies on the rate of change of the voltage or current.

i.e.

$$i(t) = C\frac{dv(t)}{dt}$$

$$v(t) = L\frac{di(t)}{dt}$$

Need to examine the effect of voltage or current changes in circuits containing capacitors or inductors.

General Form of the Response Equations

(math revision)

Dealing with equations of the form

$$\frac{dx(t)}{dt} + ax(t) = f(t) \qquad (1)$$

Solution?

$$x(t) = x_p(t) \qquad \text{Particular integral}$$

is solution of (1). Also,

$$x(t) = x_c(t) \qquad \text{Complementary soln.}$$

is solution of

$$\frac{dx(t)}{dt} + ax(t) = 0$$

So let

$$\frac{dx_p}{dt} + ax_p(t) = A \qquad (2)$$

$$\frac{dx_c(t)}{dt} + ax_c(t) = 0 \qquad (3)$$

If A is a constant assume soln to (2) is a constant, K_1.

$\therefore$ Sub. K_1 for x_p into (2)

$$K_1 = \frac{A}{a}$$

Examining (3)

$$\frac{dx_c}{x_c} = -a\,dt$$

$$\therefore \quad \ln x_c(t) = -at + c$$

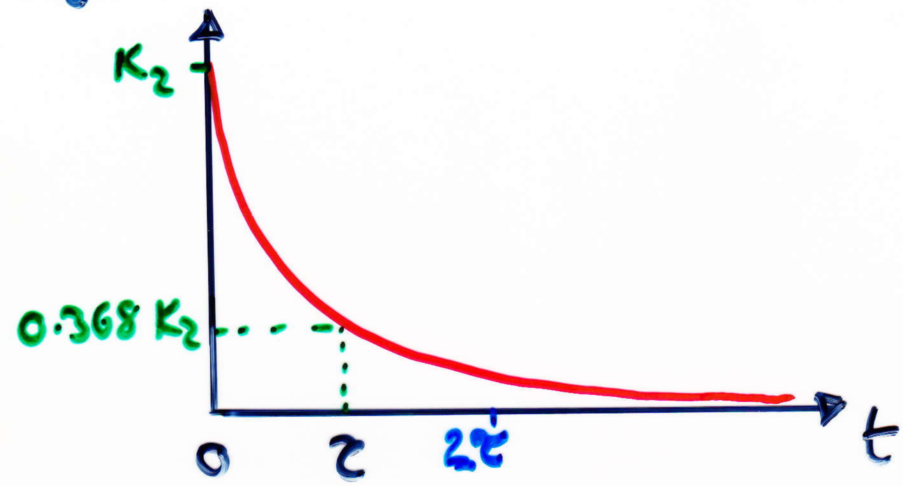
$$x_c(t) = K_2 e^{-at}$$

Remember, $x(t) = x_p + x_c(t)$

So, $$x(t) = \frac{A}{a} + K_2 e^{-at}$$

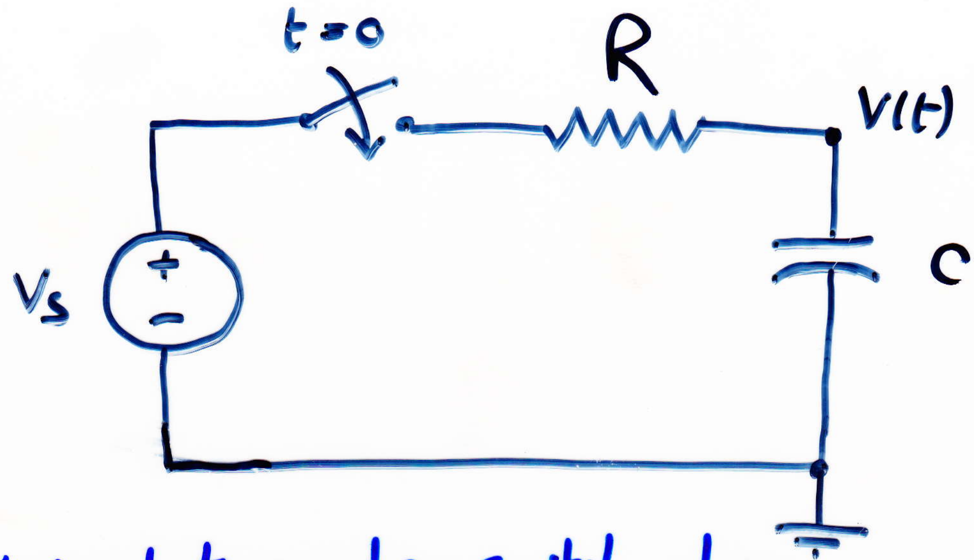
$$x(t) = K_1 + K_2 e^{-t/\tau}$$

$x_c = K_2 e^{-t/\tau}$



Curve drops 63.2% each time constant τ.

RC



Apply KCL at t=0 when switch closes

$$C\frac{dv(t)}{dt} + \frac{v(t) - V_s}{R} = 0$$

or

$$\frac{dv(t)}{dt} + \frac{v(t)}{RC} = \frac{V_s}{RC} \qquad (4)$$

Solution of this equation is of form

$$v(t) = K_1 + K_2 e^{-t/\tau}$$

Substituting this solution into (4)

$$-\frac{K_2}{\tau} e^{-t/\tau} + \frac{K_1}{RC} + \frac{K_2}{RC} e^{-t/\tau} = \frac{V_s}{RC}$$

Equating constants and exponential terms:

$$K_1 = V_s$$

$$\tau = RC$$

$$\therefore \quad v(t) = V_s + K_2 e^{-t/RC}$$

V_s is steady-state value.

K_2 ? Consider initial conditions - cap. is uncharged.

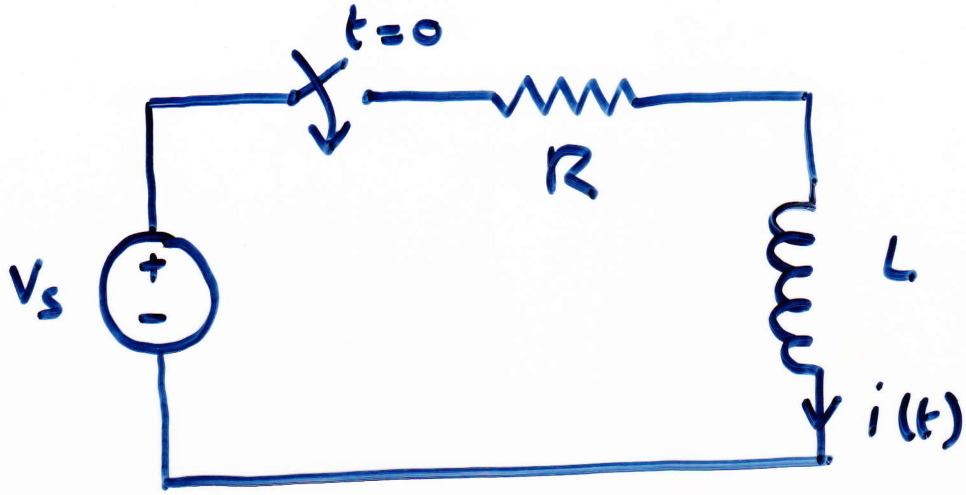
$$0 = V_s + K_2$$

$$K_2 = -V_s$$

$\therefore$ Complete solution is

$$v(t) = V_s - V_s e^{-t/RC}$$

RL



From KVL

$$L\frac{di(t)}{dt} + Ri(t) = V_s$$

$$i(t) = \frac{V_s}{R} + K_2 e^{-\left(\frac{R}{L}\right)t}$$

Since $\quad i(t) = K_1 + K_2 e^{-t/\tau}$ ← Apply..

$$-\frac{L}{\tau} K_2 e^{-t/\tau} + RK_1 + RK_2 e^{-t/\tau} = V_s$$

$$K_1 = \frac{V_s}{R}$$

$$\tau = \frac{L}{R}$$

$\frac{V_s}{R}$ is steady state and $\frac{L}{R}$ is circuit time constant.

So if no current at $t=0$

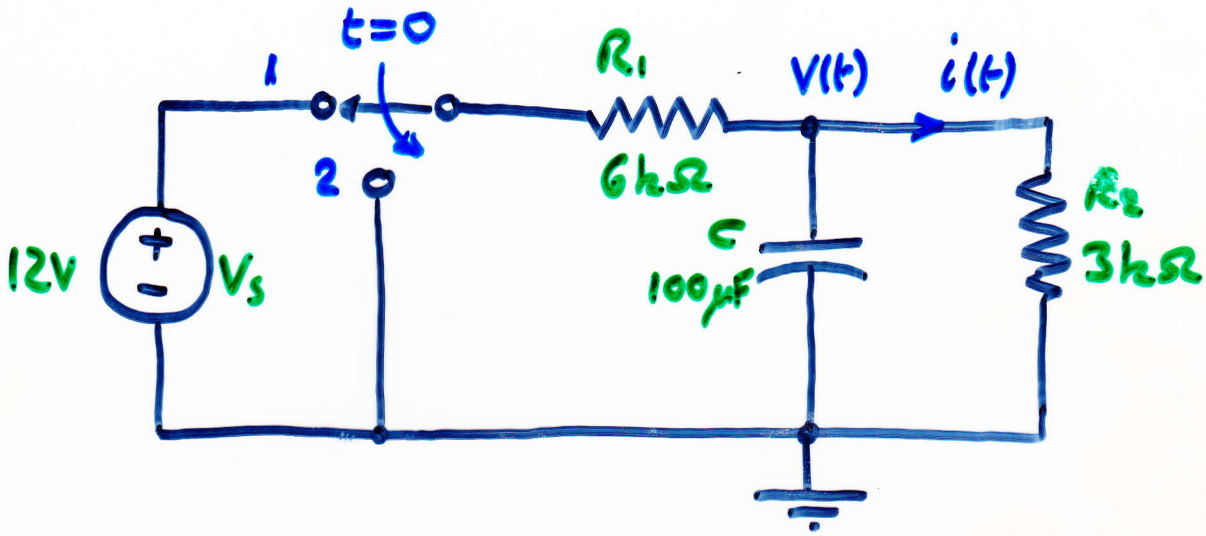
$$0 = \frac{V_s}{R} + K_2$$

$$K_2 = -\frac{V_s}{R}$$

$$\therefore \quad i(t) = \frac{V_s}{R} - \frac{V_s}{R} e^{-\frac{R}{L} t}$$

Examples: RC & RL Circuits.

① Irwin Example 7.1



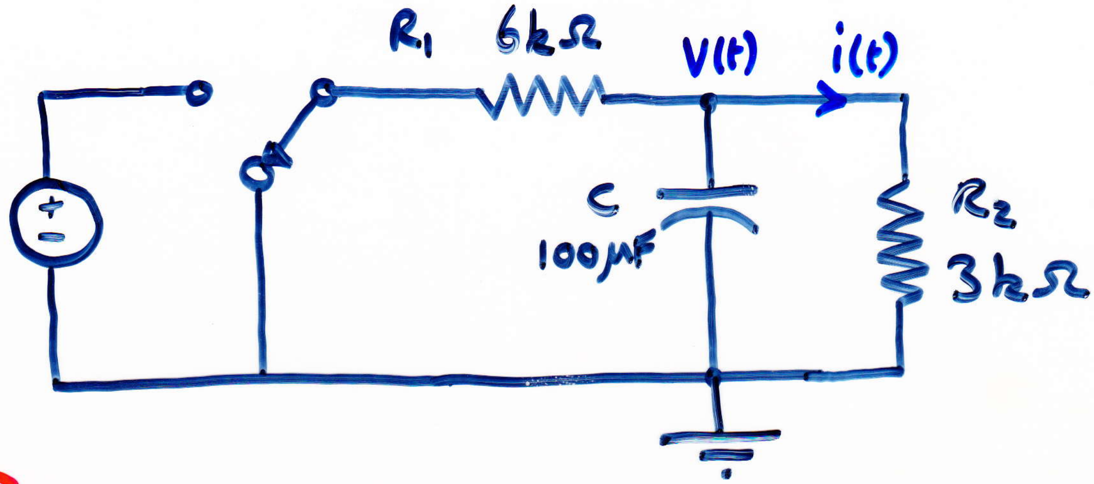
Determine $i(t)$ for $t > 0$.

Switch in position 1 for long time. At $t=0$ switched to position 2.

At $t=0$ C is fully charged.

No current passes through the capacitor (acts like open circuit to dc).

$$\therefore \; V_c(0) = 12\left(\frac{3k}{6k+3k}\right) = 4V$$



t > 0

From KCL

$$\frac{V(t)}{R_1} + C\frac{dv(t)}{dt} + \frac{V(t)}{R_2} = 0$$

$$\frac{V(t)}{CR_1} + \frac{dv(t)}{dt} + \frac{V(t)}{CR_2} = 0$$

Sub. in values

$$\frac{V(t)}{0.6} + \frac{dv(t)}{dt} + \frac{V(t)}{0.3} = 0 = \frac{dv}{dt} + \frac{10V(t)}{6} + \frac{20V(t)}{6}$$

$$\frac{dv}{dt} + 5v(t) = 0$$

Soln. to this eqtn is of the form $V(t) = K_2 e^{-t/\tau}$

So $\tau = \frac{1}{5} = 0.2s$

$$v(t) = K_2 e^{-t/0.2} \quad V$$

Now initially

$$v(0) = 4v$$

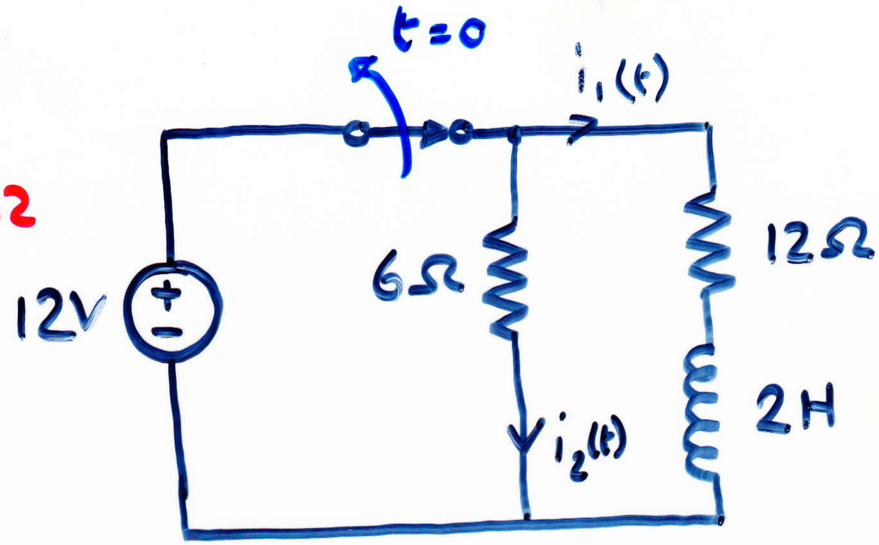
$$\therefore \quad v(t) = 4e^{-t/0.2} \quad V$$

$$i(t) = v(t)/R_2$$

$$i(t) = \frac{4}{3} e^{-t/0.2} \; mA$$

2

Irwin E6.2

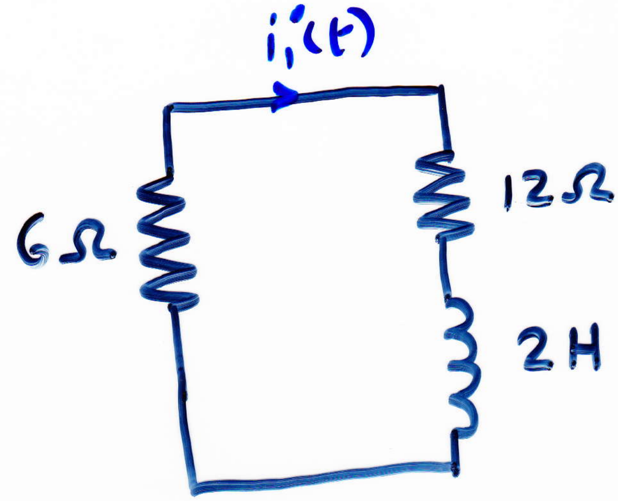


Switch opens at $t=0$. Find $i_1(t)$ for $t>0$

At $t=0$ system in steady state so inductor is like a short circuit.

$$\therefore\; i_1(t) = 12/12 = 1\text{ A}$$
$$i_2(t) = 12/6 = 2\text{ A}$$

When switch opens have:



From KVL

$$6\,i_1'(t) + 12\,i_1'(t) + 2\frac{di_1'(t)}{dt} = 0$$

$$\frac{di_1'(t)}{dt} + 9\,i_1' = 0$$

Soln. of form $i_1'(t) = K_2 e^{-t/\tau}$

$$\tau = \frac{1}{9}$$

Also at moment of switch opening $K_2 = 1\,A$

$$\therefore\ i_1'(t) = 1\,e^{-9t}\ A.$$