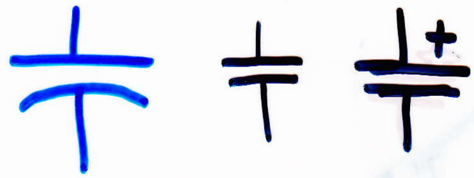
Capacitors & Inductors.

Introduction

Two new circuit elements

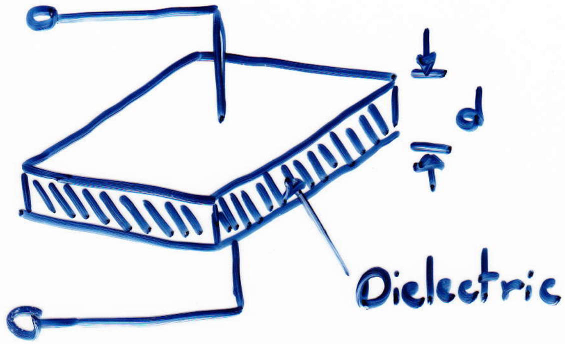
Capacitor 

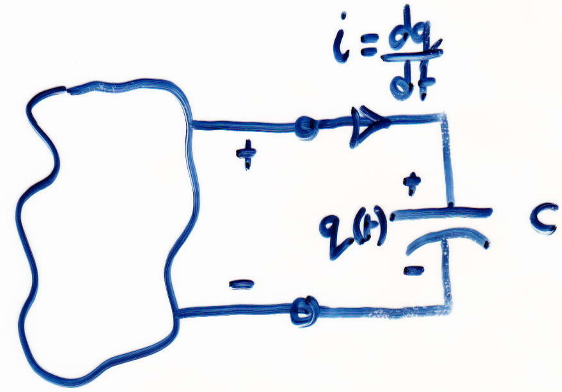
Inductor 

Devices can store energy.

Element	Stores energy...	Energy in form of...
Capacitor	when voltage across the element.	electric field
Inductor	when current passes through it.	magnetic field.

Capacitors.





From physics know ...

$$C = \frac{\varepsilon A}{d}$$ for parallel plate capacitor

$\varepsilon = \varepsilon_0 \varepsilon_r$, permittivity of the dielectric

For vacuum $\varepsilon_0 = 8.85 \times 10^{-12}\ Fm^{-1}$ ($\approx \varepsilon_{air}$)

Relative permittivity ε_r

e.g.

Material	ε_r
air	~ 1
mica	5.7 − 6.7
polystyrene	2.55

Also

$$q = Cv$$

q is charge on capacitor.

What about current-voltage characteristics?

$$i = \frac{dq}{dt} = \frac{dCv}{dt}$$

$$i = C\frac{dv}{dt}$$

$$dv = \frac{1}{C}\, i\, dt$$

$$v(t) = \frac{1}{C}\int_{-\infty}^{t} i(x)\,dx$$

rewriting

$$v(t) = \frac{1}{C}\int_{-\infty}^{t_0} i(x)\,dx + \frac{1}{C}\int_{t_0}^{t} i(x)\,dx$$

$$= v(t_0) + \frac{1}{C}\int_{t_0}^{t} i(x)\,dx$$

Energy stored ?

Considering power

$$p(t) = v(t)i(t) = C v(t) \frac{dv(t)}{dt}$$

Energy then is

$$W_c(t) = C \int_{-\infty}^{t} v(x) \frac{dv(x)}{dx} \, dx$$

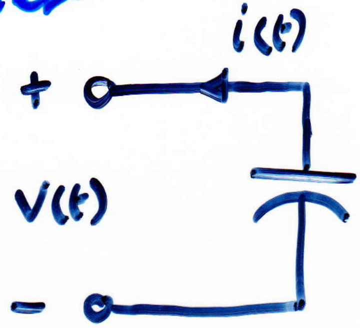
$$= C \int_{v(-\infty)}^{v(t)} v(x) \, dv(x) = \left[\frac{1}{2} C v^2(x) \right]_{v(-\infty)}^{v(t)}$$

$$\underline{W_c(t) = \frac{1}{2} C v^2}$$

Since $v(-\infty) = 0$

$$\underline{W_c(t) = \frac{1}{2} \frac{q^2(t)}{C}}$$

Consider



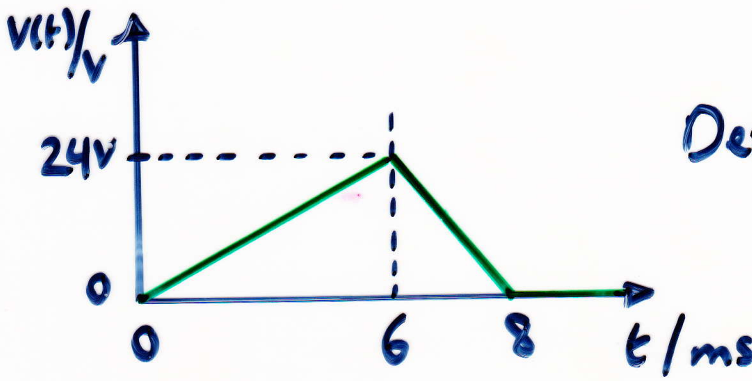
$i(t)$?

$$i(t) = -C\frac{dv(t)}{dt}$$

Example

Irwin Example 5.2

$5\mu F$ capacitor has following voltage waveform



Determine current waveform.

$$v(t) = \frac{24}{6\times10^{-3}}t \qquad 0 \leq t < 6ms$$

and

$$= \frac{-24}{2\times10^{-3}}t + 96 \qquad 6 \leq t < 8ms$$

$$= 0 \qquad t \geq 8ms$$

Now $i(t) = C\dfrac{dv(t)}{dt}$

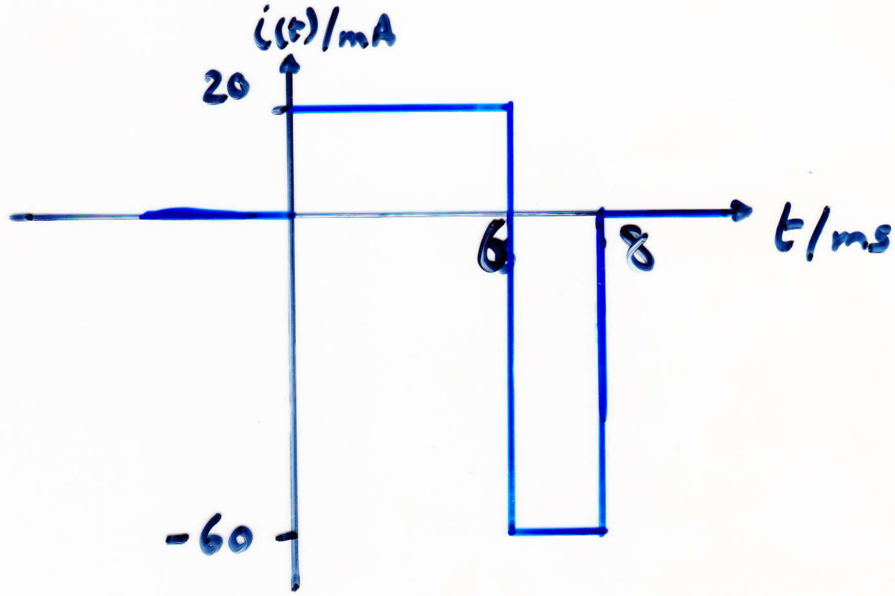
$= 5\times10^{-6} \times 4\times10^{3} \qquad 0 \le t < 6ms$

$= 20mA \qquad 0 \le t < 6ms$

$i(t) = 5\times10^{-6}\times(-12\times10^{3}) \qquad 6 \le t < 8ms$

$= -60mA \qquad 6 \le t < 8ms$

$i(t) = 0 \qquad t \geq 8ms$

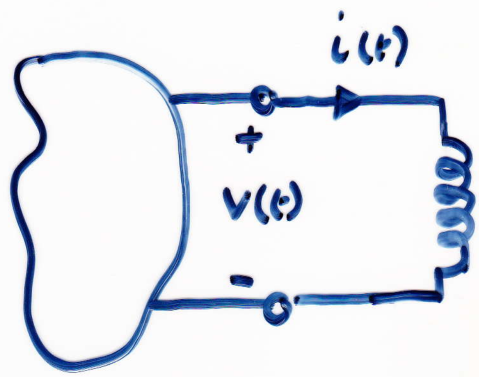


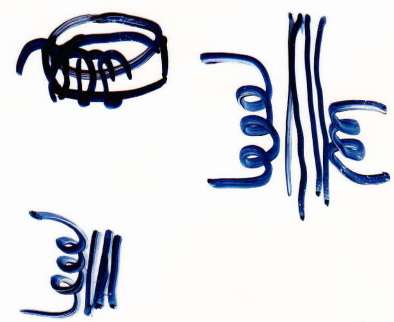
Inductors

Usually conducting wire in form of a coil.

Material on which they are wound does affect the inductance.

Material can be air, non-magnetic, iron or ferrite.





$$v(t) = L \frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L} \int_{-\infty}^{t} v(x)\, dx$$

$$i(t) = i(t_0) + \frac{1}{L} \int_{t_0}^{t} v(x)\, dx$$

Power?

$$p(t) = v(t)\, i(t)$$

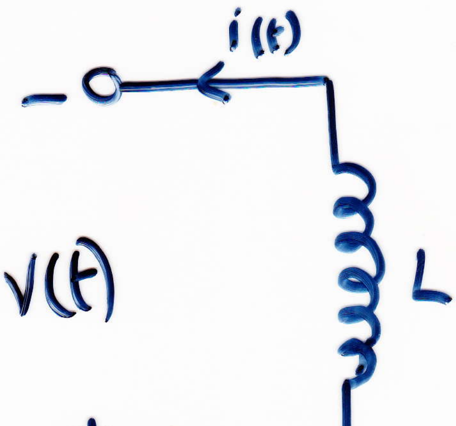
$$= L \frac{di(t)}{dt}\, i(t)$$

Energy?

$$w_L(t) = \int_{-\infty}^{t} L \left(\frac{di(x)}{dx} \right) i(x)\, dx$$

$$\therefore\ w_L(t) = \frac{1}{2} L i^2(t)$$

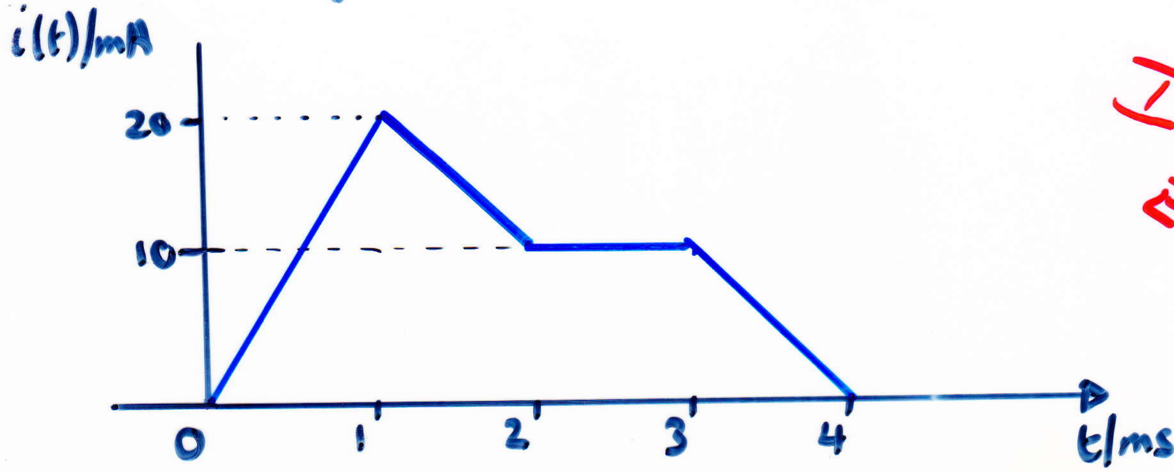
Consider



$$v(t) = L \frac{di(t)}{dt}$$

Example 5mH inductor has following current waveform. Determine inductor voltage waveform.

Irwin E6.4



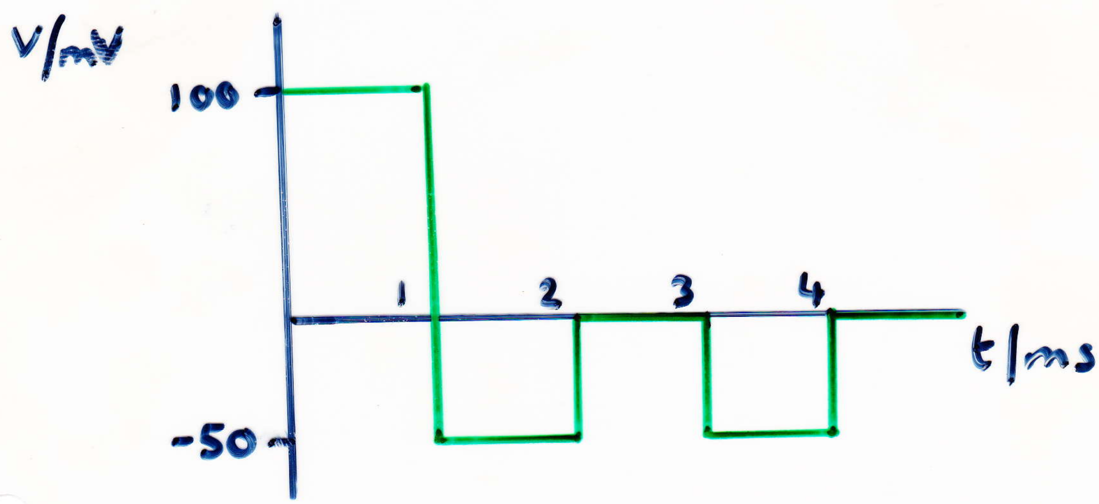
$$v(t) = L\frac{di(t)}{dt}$$

$i(t) = 20t \qquad 0 \le t < 1\text{ms}$

$i(t) = -10t + 30 \qquad 1 \le t < 2\text{ms}$

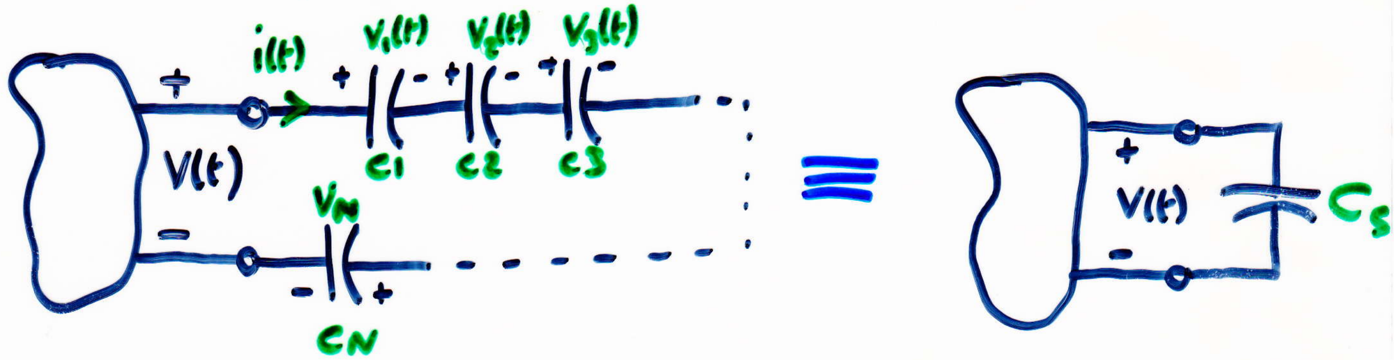
$i(t) = 10 \qquad 2 \le t < 3\text{ms}$

$i(t) = -10t + 40 \qquad 3 \le t < 4\text{ ms}$



Capacitor & Inductor Combinations

Series Capacitors



$$v(t) = v_1(t) + v_2(t) + v_3(t) + \ldots + v_N(t)$$

$$v_i(t) = \frac{1}{C_i}\int_{t_0}^{t} i(t)\,dt + v_i(t_0)$$

$$v(t) = \left(\sum_{i=1}^{N} \frac{1}{C_i}\right)\int_{t_0}^{t} i(t)\,dt + \sum_{i=1}^{N} v_i(t_0)$$

$$= \frac{1}{C_s}\int_{t_0}^{t} i(t)\,dt + v(t_0)$$

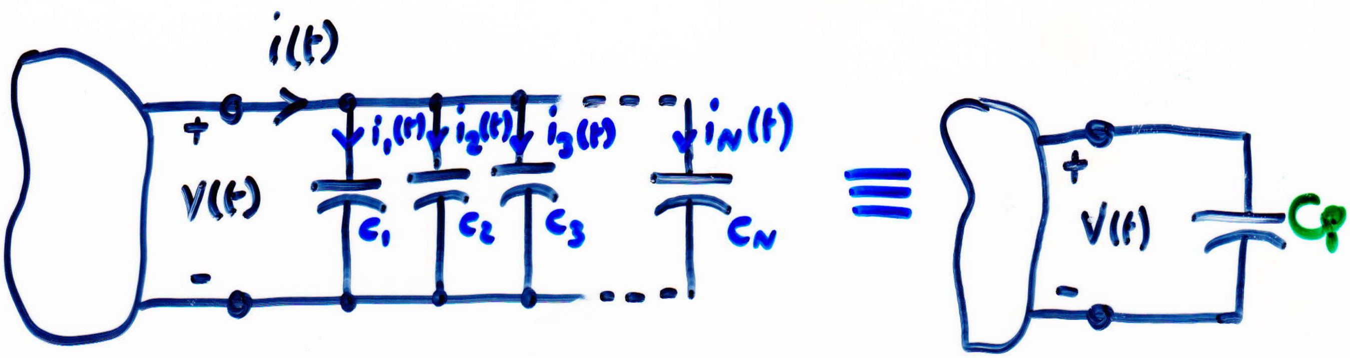
where $$v(t_0) = \sum_{i=1}^{N} v_i(t_0)$$

and

$$\frac{1}{C_s} = \sum_{i=1}^{N} \frac{1}{C_i} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_N}$$

Note: current same through each capacitor, so each capacitor gains same charge in the same time period. Voltage across each capacitor depends on this charge & capacitance of the element.

Parallel Capacitors



$$i(t) = i_1(t) + i_2(t) + i_3(t) + \ldots + i_N(t)$$

$$= C_1 \frac{dv(t)}{dt} + C_2 \frac{dv(t)}{dt} + C_3 \frac{dv(t)}{dt} + \ldots + C_N \frac{dv(t)}{dt}$$

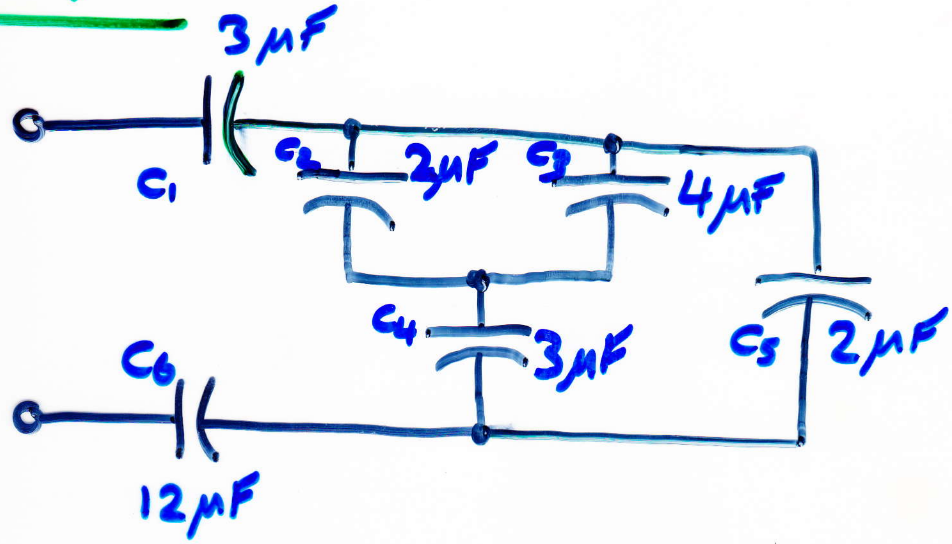
$$= \left(\sum_{i=1}^{N} C_i \right) \frac{dv(t)}{dt}$$

$$= C_p \frac{dv(t)}{dt}$$

where

$$\underline{C_p = C_1 + C_2 + C_3 + \ldots + C_N}$$

Example



Combine C_2 & C_3 , $C_{23} = 2\mu F + 4\mu F = 6\mu F$

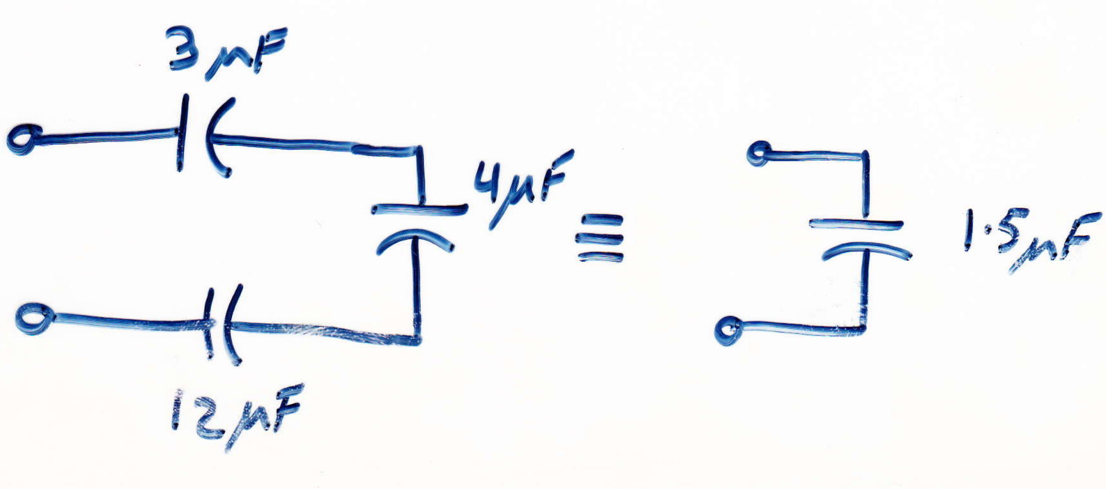
" $C_{23} + C_4$,

$$\frac{1}{C_{234}} = \frac{1}{C_{23}} + \frac{1}{C_4} = \frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

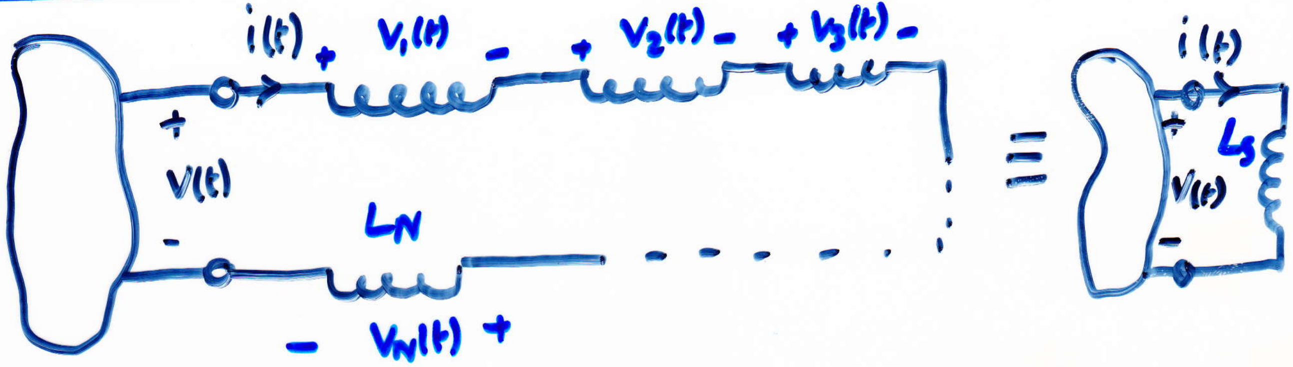
$\therefore$ $C_{234} = 2\mu F$

Combine C_{234} with $C_5 = 2\mu F + 2\mu F = 4\mu F$

Have



Series Inductors



$$v(t) = v_1(t) + v_2(t) + v_3(t) + \ldots + v_N(t)$$

$$v(t) = L_1 \frac{di(t)}{dt} + L_2 \frac{di(t)}{dt} + L_3 \frac{di(t)}{dt} + \ldots + L_N \frac{di(t)}{dt}$$

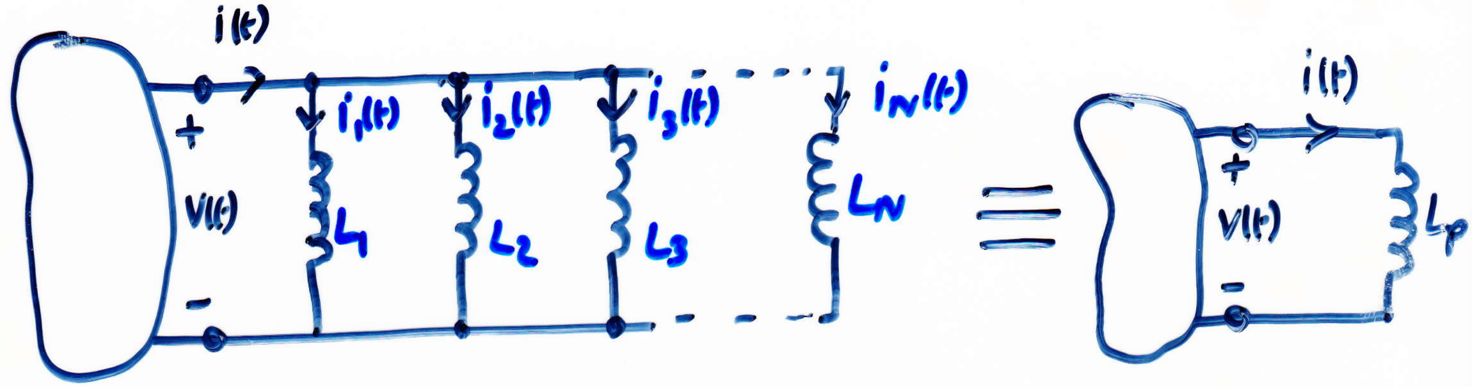
$$= \left(\sum_{i=1}^{N} L_i \right) \frac{di(t)}{dt}$$

$$= L_s \frac{di(t)}{dt}$$

where

$$L_s = \sum_{i=1}^{N} L_i = L_1 + L_2 + \ldots + L_N$$

Parallel Inductors.



$$i(t) = i_1(t) + i_2(t) + i_3(t) + \dots + i_N(t)$$

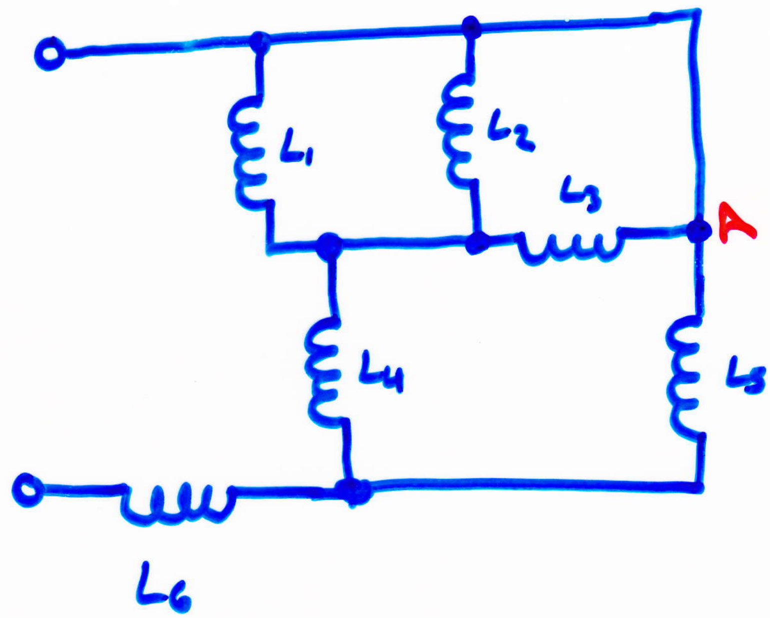
$$i_j(t) = \frac{1}{L_j}\int_{t_0}^{t} v(x)\,dx + i_j(t_0)$$

So $$i(t) = \left(\sum_{j=1}^{N}\frac{1}{L_j}\right)\int_{t_0}^{t} v(x)\,dx + \sum_{j=1}^{N} i_j(t_0)$$

$$= \frac{1}{L_p}\int_{t_0}^{t} v(x)\,dx + i(t_0)$$

where $$\frac{1}{L_p} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_N}$$

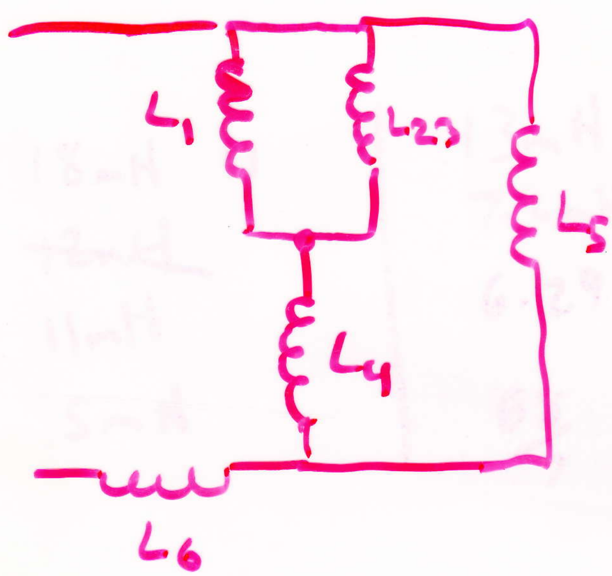
Example



All inductors are 6mH.

What is the value of an equivalent single inductor?

E6.8



9.429 mH.

Irwin E5.8