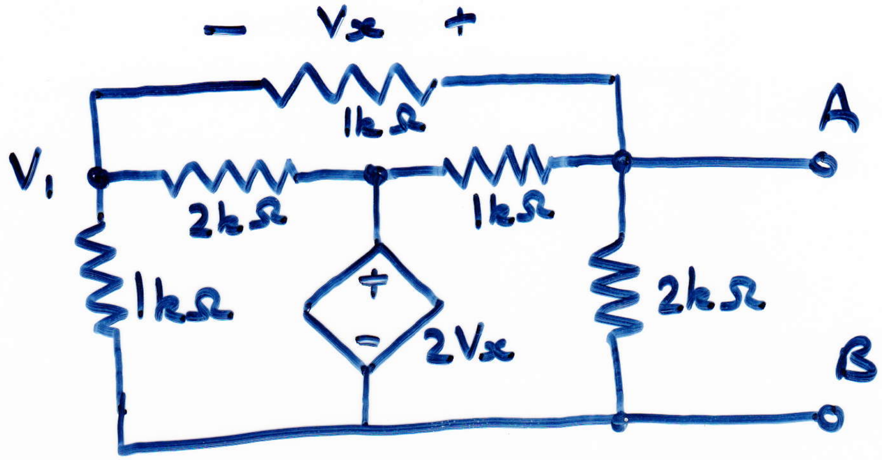
Thevenin & Norton's Theorems II.

Circuits containing only dependent sources

For independent sources can calculate V_{oc} and i_{sc} and then R_{Th}. If circuit contains no independent sources, that is it has only dependent sources, there can be difficulties if introducing open or short circuits.

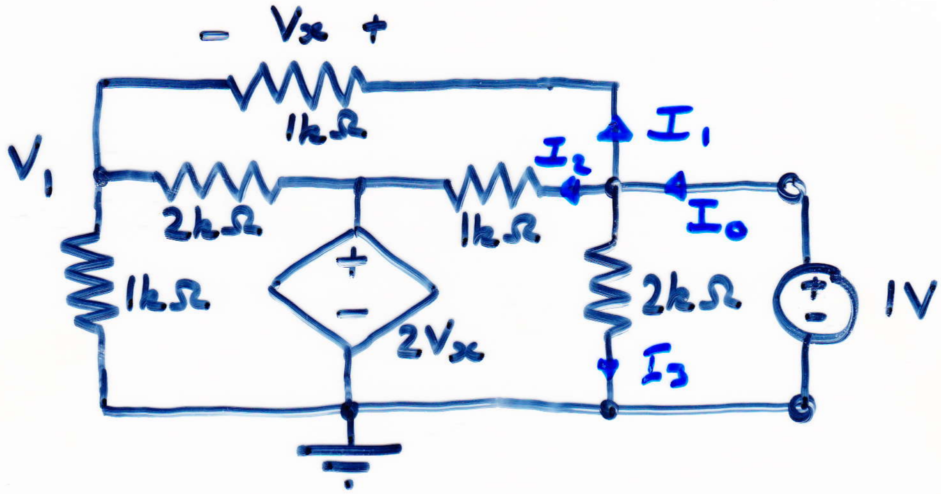
Approach : introduce a test source e.g. V_t or i_t and find either i_t or V_t, then determine $R_{Th} = V_t / i_t$. Easiest is to introduce a test value of 1V or 1A.

Example



Thévenin equivalent at A-B.

Apply 1V across AB



Apply KVL around outer loop

$$V_1 + V_x = 1$$

KCL at node labelled V_1

$$\frac{V_1}{1k} + \frac{V_1 - 2V_x}{2k} + \frac{V_1 - 1}{1k} = 0$$

Two simultaneous eqn. with two unknowns

$$\therefore \quad \frac{1-V_x}{1k} + \frac{1-3V_x}{2k} - \frac{V_x}{1k} = 0$$

$$2 - 2V_x + 1 - 3V_x - 2V_x = 0$$

$$7V_x = 3$$

$$V_x = 3/7 \text{ V}$$

$$\therefore \quad I_1 = \frac{V_x}{1k} = \frac{3}{7} \text{ mA}$$

$$I_2 = \frac{1-2V_x}{1k} = \frac{1}{7} \text{ mA}$$

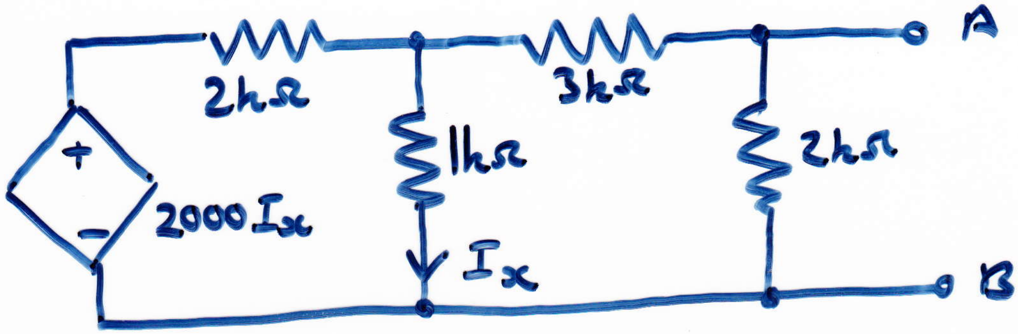
$$I_3 = \frac{1}{2k} = \frac{1}{2} \text{ mA}$$

$$\therefore \quad I_0 = I_1 + I_2 + I_3 = \frac{15}{14} \text{ mA}$$

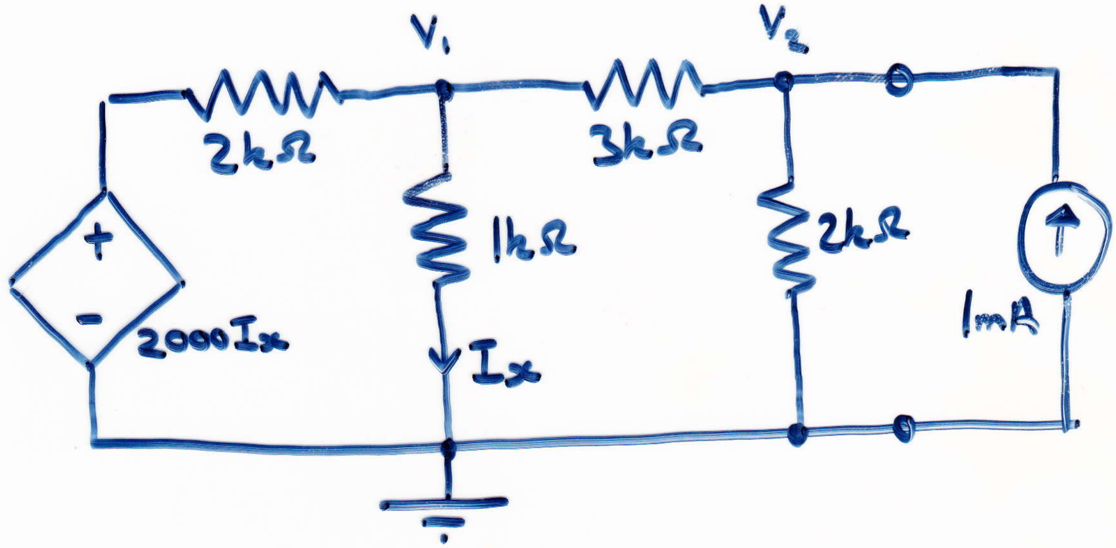
$$R_{Th} = \frac{1}{I_0} = \frac{14}{15} \text{ k}\Omega$$

Example 2

Determine R_{Th} at A-B.



Apply 1mA current source at terminals A-B



Node equations for the network are

$$\frac{V_1 - 2000 I_x}{2k} + \frac{V_1}{1k} + \frac{V_1 - V_2}{3k} = 0 \qquad (1)$$

$$\frac{V_2 - V_1}{3k} + \frac{V_2}{2k} = 1 \times 10^{-3} \qquad (2)$$

$$I_x = \frac{V_1}{1k} \qquad (3)$$

So from (2)
$$\frac{V_2}{3k} + \frac{V_2}{2k} = 1\times10^{-3} + \frac{V_1}{3k}$$

$$\therefore \frac{5}{6k}V_2 = 1\times10^{-3} + \frac{V_1}{3k}$$

$$\text{So } V_2 = \frac{2}{5}V_1 + \frac{6}{5}$$

So (1) becomes

$$\frac{V_1 - 2000\,(V_1/1k)}{2\cancel{k}} + \frac{V_1}{1\cancel{k}} + \frac{V_1 - (2V_1+6)/5}{3\cancel{k}} = 0$$

$$\frac{V_1}{2} - \cancel{V_1} + \cancel{V_1} + \frac{V_1}{3} - \frac{2V_1}{15} - \frac{\cancel{6}}{\cancel{15}}\frac{2}{5} = 0$$

$$V_1\left(\frac{1}{2} + \frac{1}{3} - \frac{2}{15}\right) = \frac{2}{5}$$

$$V_1\left(\frac{15+10-4}{30}\right) = \frac{2}{5}$$

$$V_1\left(\frac{\cancel{21}\,7}{\cancel{30}\,10}\right) = \frac{2}{5}$$

$$V_1 = \frac{2}{\cancel{5}} \times \frac{\cancel{10}^{2}}{7} = \frac{4}{7}\ V$$

So $V_2 = \frac{2}{5} \times \frac{4}{7} + \frac{6}{5}$

$$= \frac{8}{35} + \frac{6}{5} = \frac{8 + 42}{35}$$

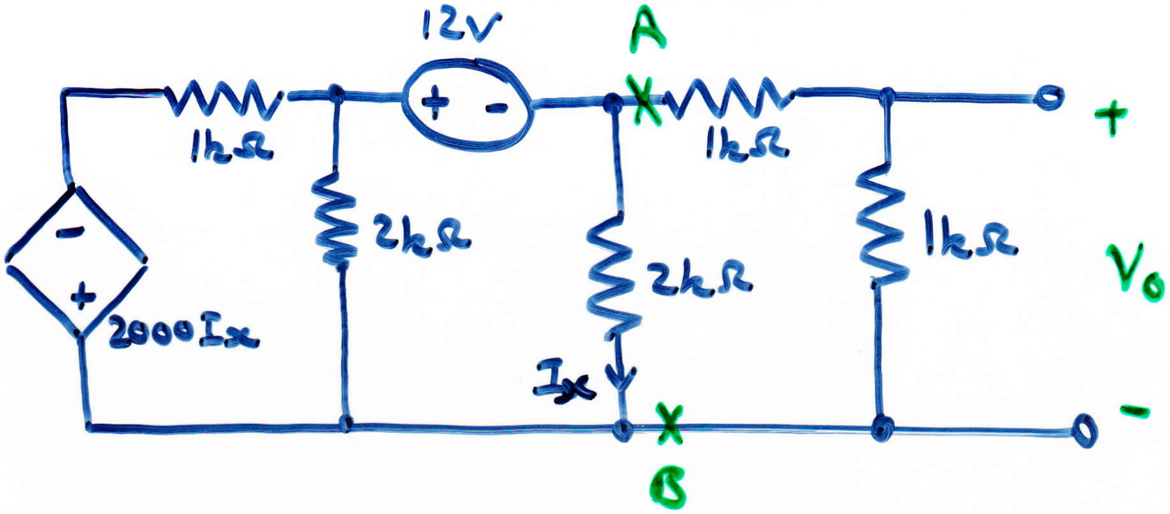
$$= \frac{50}{35} = \frac{10}{7}\ V$$

$$\therefore R_{Th} = \left(\tfrac{10}{7}\right) / 1 \times 10^{-3} = \tfrac{10}{7}\ k\Omega$$

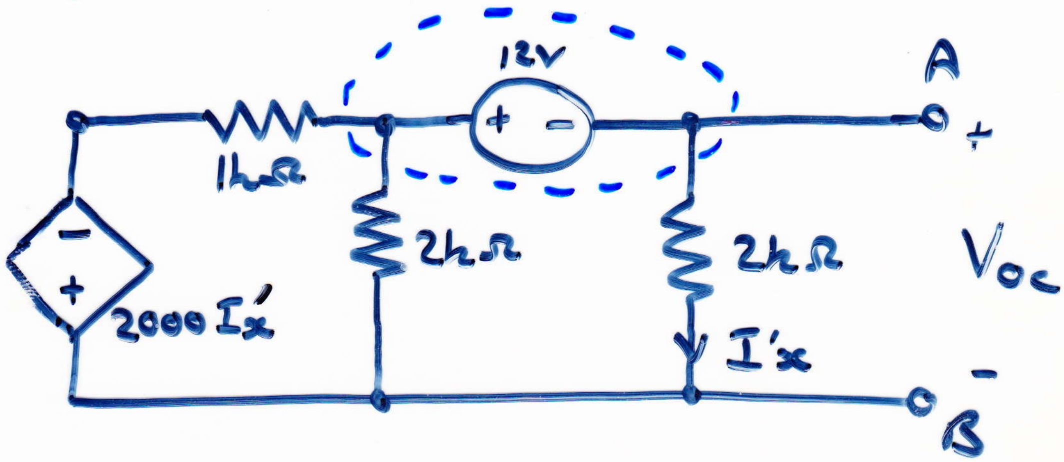
Circuits with independent & dependent sources.

Calc. both the open-circuit voltage and short-circuit current, then calc. R_{Th}.

Cannot split the dependent source and its controlling variable when finding T & N equivalent.



Break circuit at A B



KCL for supernode around 12V source

$$\frac{(V_{oc}+12)-(-2000I_x')}{1k}+\frac{V_{oc}+12}{2k}+\frac{V_{oc}}{2k}=0$$

Where $I_x' = \frac{V_{oc}}{2k}$

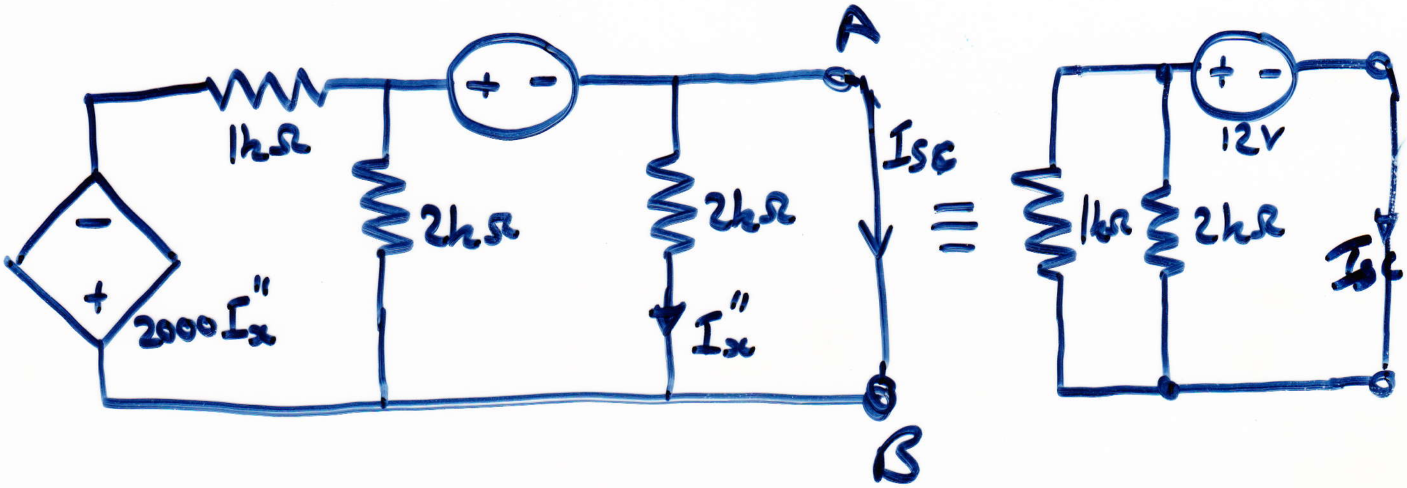
$$\frac{(V_{oc}+12)+(\cancel{2000}\,V_{oc}/\cancel{2k})}{1k}+\frac{V_{oc}+12}{2k}+\frac{V_{oc}}{2k}=0$$

$$2V_{oc}+12+\frac{V_{oc}}{2}+6+\frac{V_{oc}}{2}=0$$

$$3V_{oc} = -18$$

$$V_{oc} = -6V$$

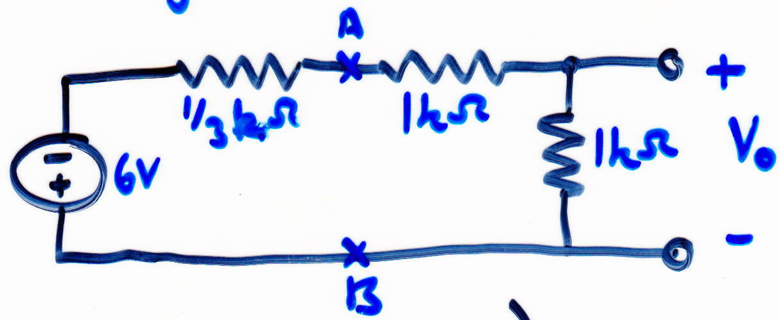
What about the short circuit current, I_{sc}?



$$I_{sc} = \frac{-12}{2/3k} = -18mA$$

$$\therefore R_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{-6}{-18\times10^{-3}} = \frac{1}{3}k\Omega$$

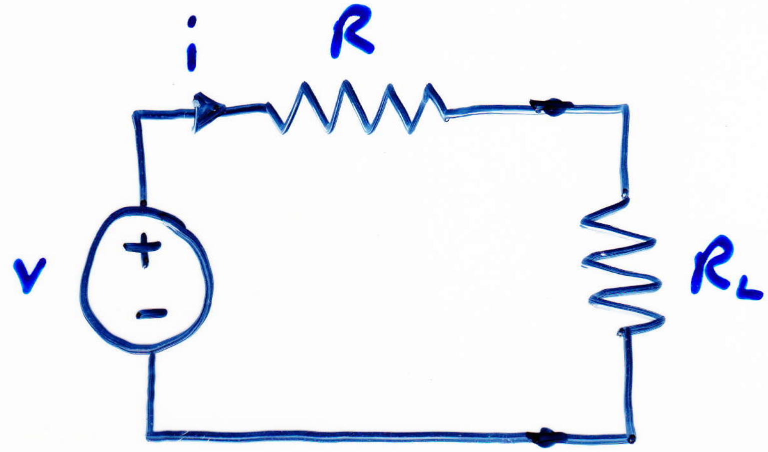
So connecting in the Thevenin eq. to the remainder of the network we have



$$V_o = (-6)\left(\frac{1k}{1/3k + 1k + 1k}\right) = -6\times\frac{1}{7/3}$$

$$= -\frac{18V}{7}$$

Maximum Power Transfer.



$$P_{Load} = i^2 R_L = \left(\frac{V}{R+R_L}\right)^2 R_L$$

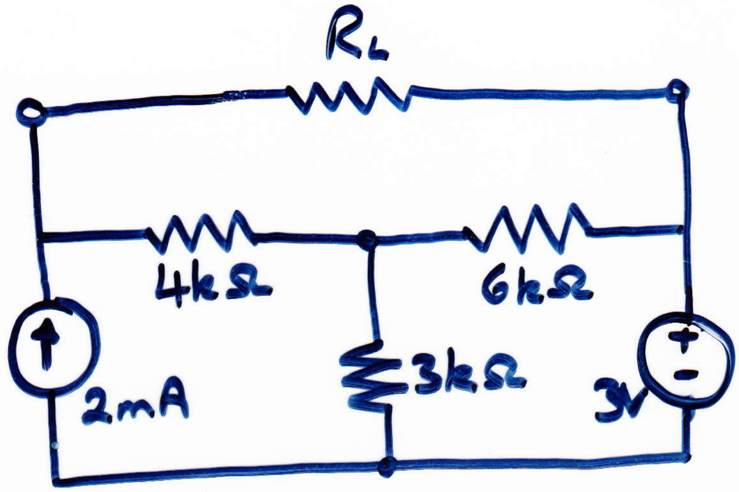
What is the value of R_L that maximizes P_{Load}?

$$\frac{dP_{Load}}{dR_L} = \frac{(R+R_L)^2 V^2 - 2V^2 R_L (R+R_L)}{(R+R_L)^4} = 0$$

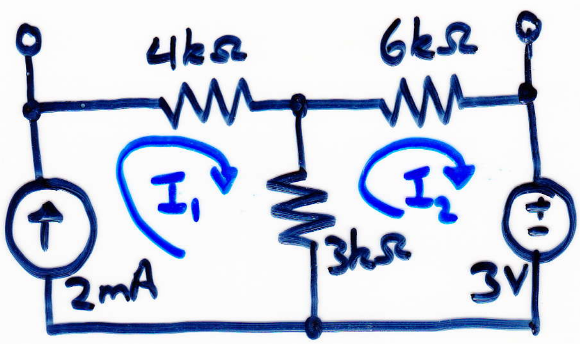
Gives $R_L = R$

The above circuit is a very simple network. However, R and v represents the Thévenin equivalent circuit for a linear network.

Example



Determine V_{oc}



$$I_1 = 2\times 10^{-3}\,A$$

$$3k(I_2 - I_1) + 6kI_2 + 3 = 0$$

$$\therefore\ 3kI_2 - 6 + 6kI_2 + 3 = 0$$

$$9kI_2 = 3$$

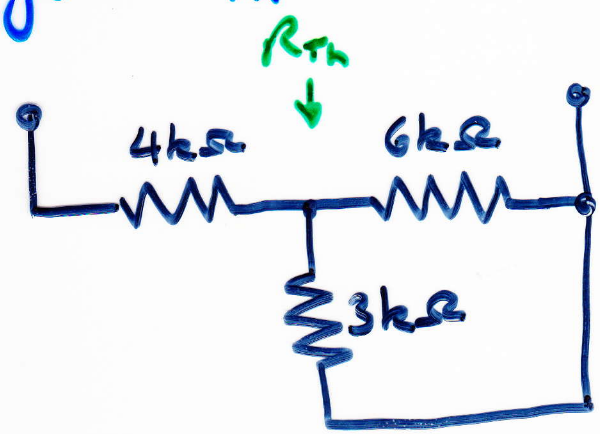
$$I_2 = \frac{3}{9k} = \frac{1}{3}\,mA$$

$$V_{oc} = 4k I_1 + 6k I_2$$

$$= 8 + 2$$

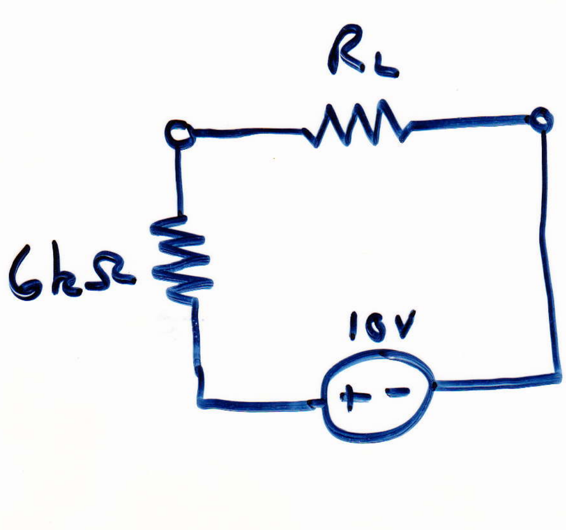
$$= 10\,V$$

Now for R_{Th}



$$R_{Th} = 4k + \frac{6 \times 3k}{(6+3)}$$

$$= 6k\Omega$$



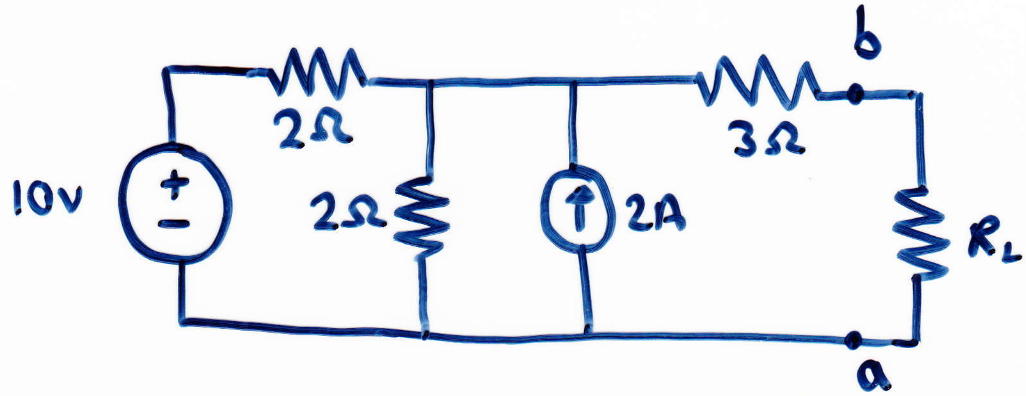
Where $R_L = R_{Th}$ for max. power transfer.

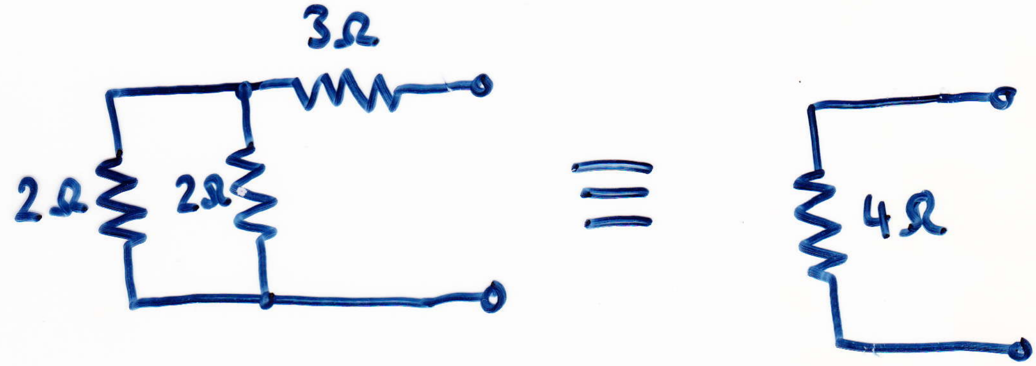
$$\therefore P_L = \left(\frac{10}{12k}\right)^2 (6k)$$

$$= \frac{25}{6}\,mW$$

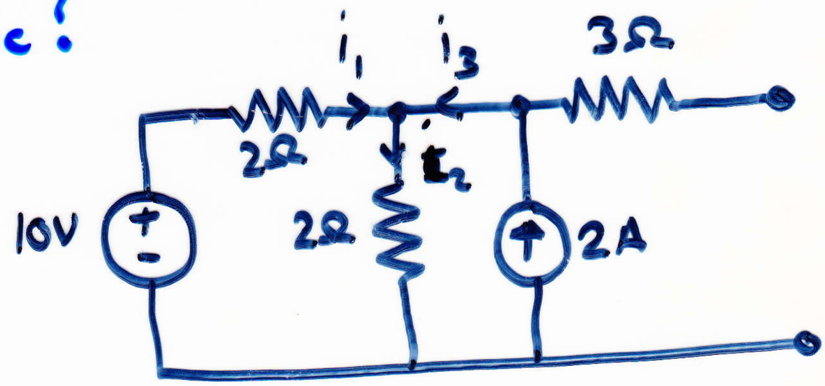
Example

Determine the value of the load resistance to achieve max. power transfer.





V_{oc}?



$$i_3 = 2A$$

$$-10 + i_1 \times 2 + i_2 \times 2 = 0$$

$$i_1 - i_2 + 2 = 0$$

$$i_1 = i_2 - 2$$

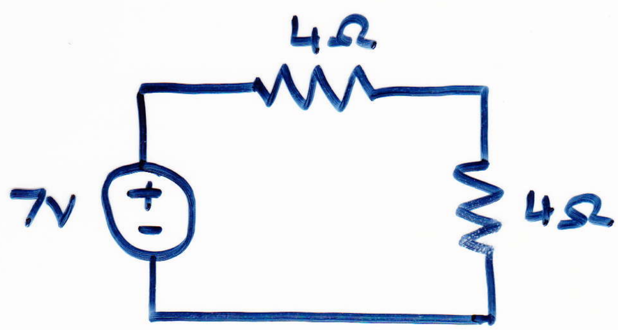
$$-10 + (i_2 - 2) \times 2 + i_2 \times 2 = 0$$

$$2i_2 - 4 + 2i_2 = 10$$

$$4i_2 = 14$$

$$i_2 = \frac{14}{4} A$$

$$\therefore \; V_{oc} = \left(\frac{14}{4}\right) 2 = 7V$$



$\therefore$ Max power:

$$i = 7/8$$

$$i^2 = 49/64$$

$$P_{max} = i^2 R_L = \frac{49}{64} \times 4 = \frac{49}{16} W.$$