Thévenin & Norton's Theorems.

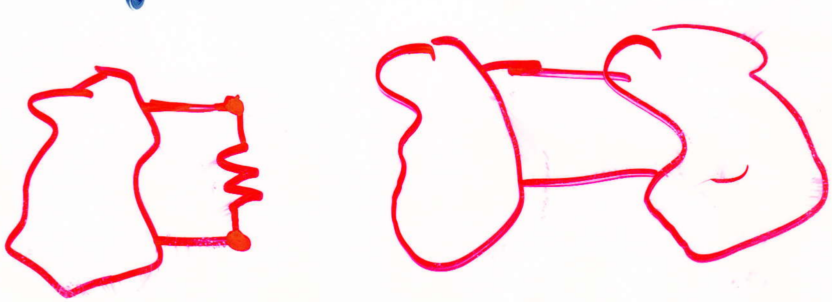
Introduction to the theorems.

Two powerful and useful techniques for circuit analysis.

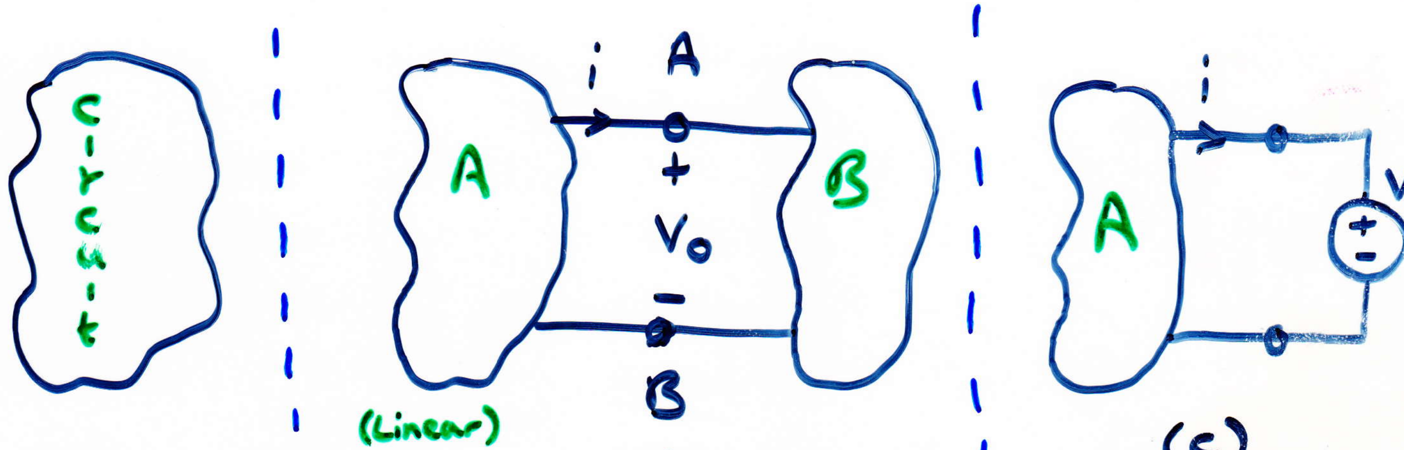
Principles are:

Thévenin's theorem:- can replace an entire network, exclusive of the load, by an equivalent circuit containing only an independent voltage source in series with a resistor in such a way that the current-voltage relationship at the load is unchanged.

Norton's theorem:- identical to above but the equivalent circuit is an independent current source in parallel with a resistor.



Consider ...



(a) (b) (c)

Applying principle of superposition with (c)

$$i = i_o + i_{sc}$$

i_o = current due to V_o

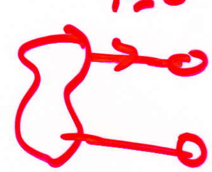
i_{sc} = short-circuited current due to all sources in circuit A. (V_o replaced by short-circuit)

Also $$i_o = \frac{-V_o}{R_{Th}}$$

R_{Th} = equivalent resistance looking back in circuit A from terminals A-B (all independent sources in A made zero).

$$\therefore \quad i = \frac{-V_o}{R_{Th}} + i_{sc} \qquad (11.1)$$

Consider specific case of $i=0$ and so V_o is the open-circuit voltage.



$$i = 0 = -\frac{V_{oc}}{R_{Th}} + i_{sc}$$

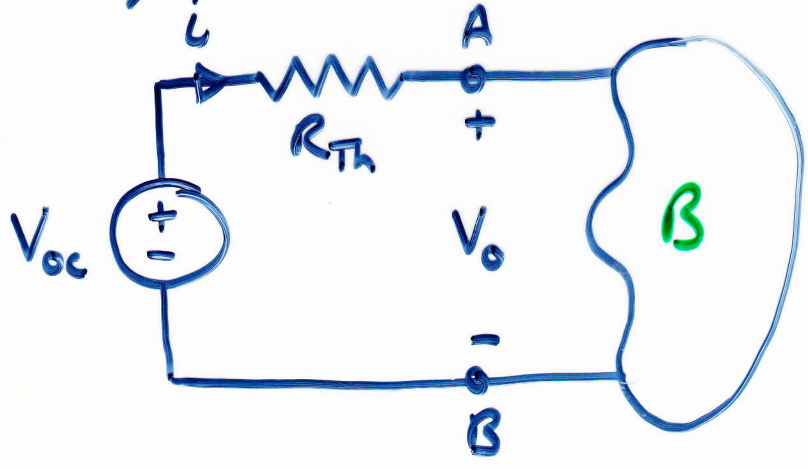
$$\therefore \quad V_{oc} = R_{Th}\, i_{sc}$$

So can now write

$$i = \frac{-V_o}{R_{Th}} + \frac{V_{oc}}{R_{Th}}$$

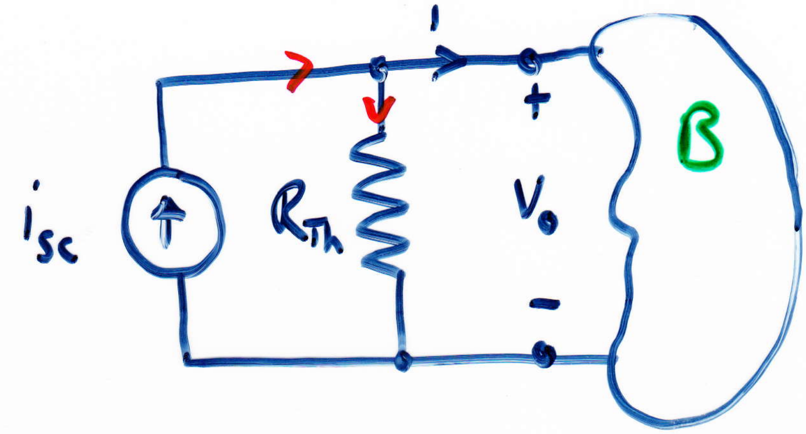
$$\therefore \quad \underline{V_o = V_{oc} - R_{Th}\, i}$$

This equation equates to

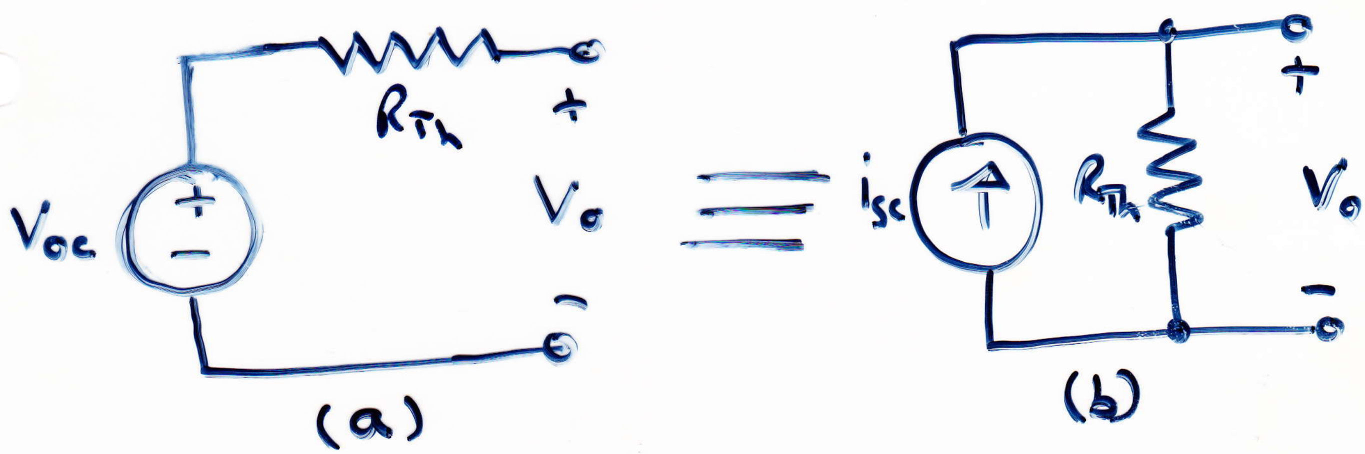


Look at (11.1) can see we have

$$\left\{ i = \frac{-V_o}{R_{Th}} + i_{sc} \quad (11.1) \right\}$$



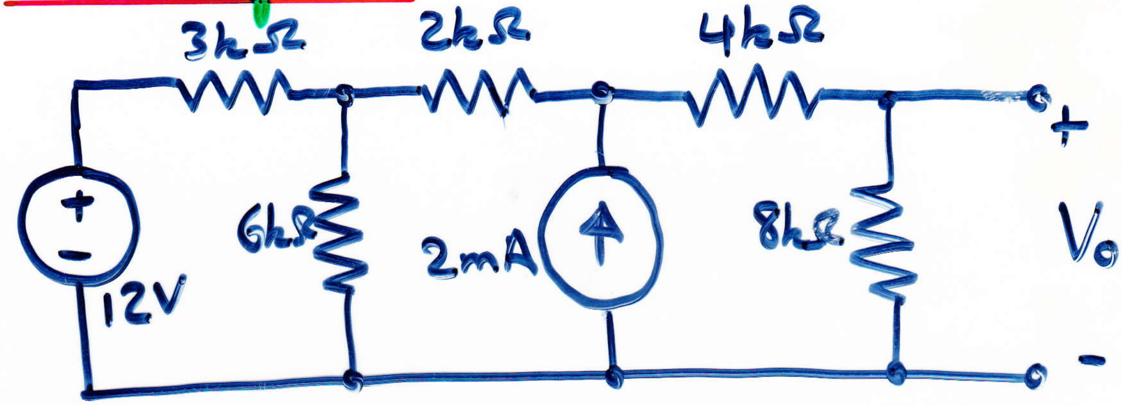
Note the interchangeability ...



This equivalence is <u>ONLY</u> at the external nodes, e.g. (b) dissipate power, (a) does not.

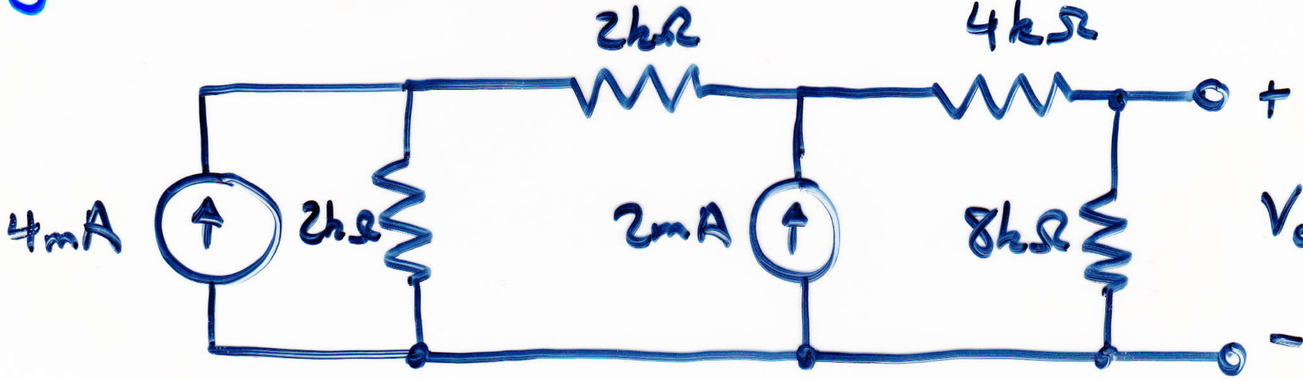
Examples

1.

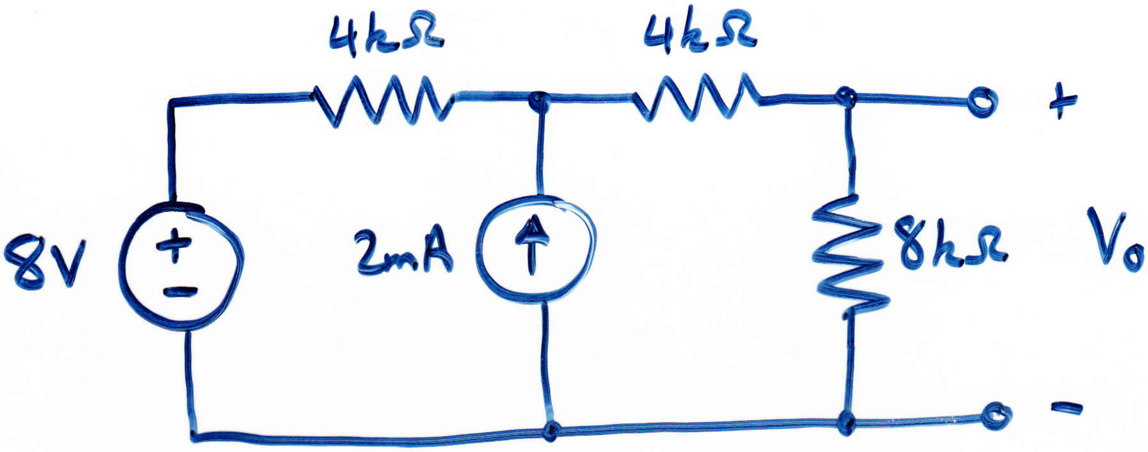


Simplify L.H.S.

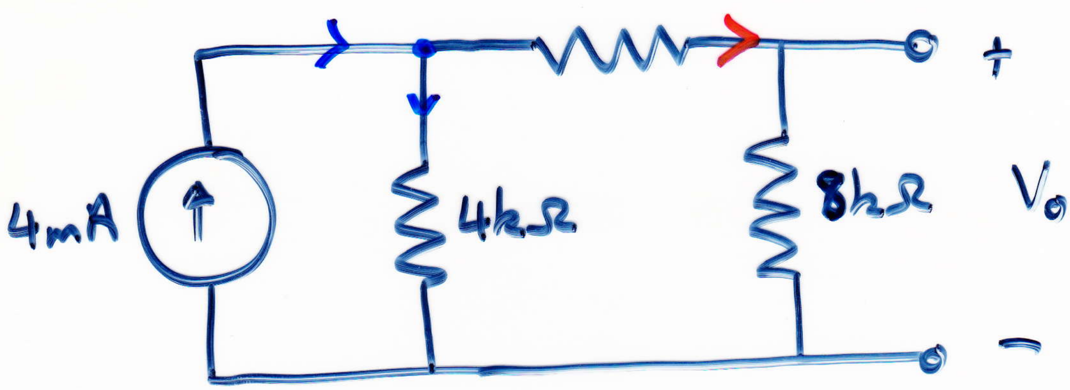
3kΩ resistor and 12V supply give 4mA current source and 3kΩ resistor in parallel. Combining 3kΩ and 6kΩ in parallel we get...



Convert current source to voltage source. Combine the resulting series resistors.



Converting again <u>AND</u> combining current sources.



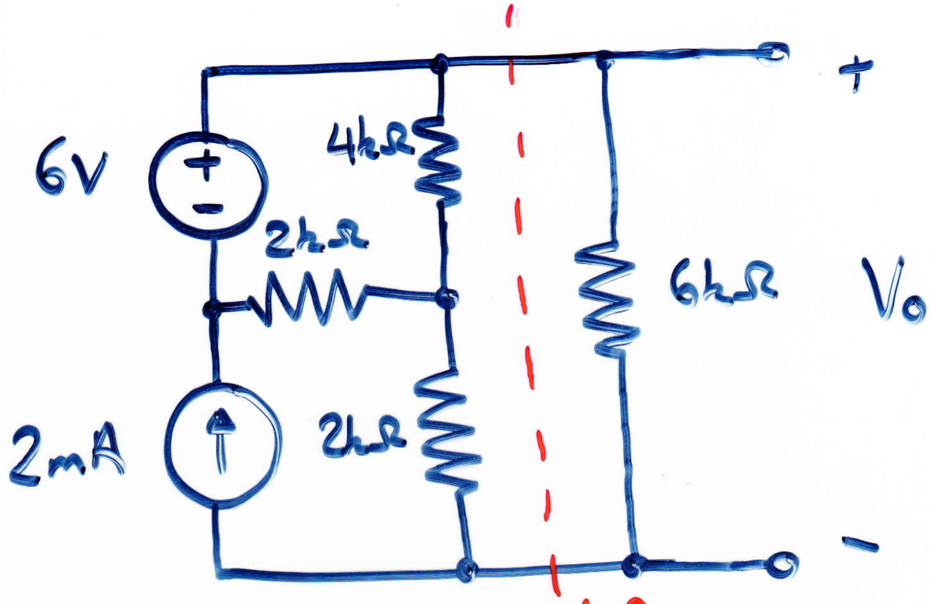
To find I_o use current division

$$I_o = 4\,mA \times \left(\frac{4k}{4k + (4k + 8k)} \right)$$

$$\underline{I_o = 1\,mA}$$

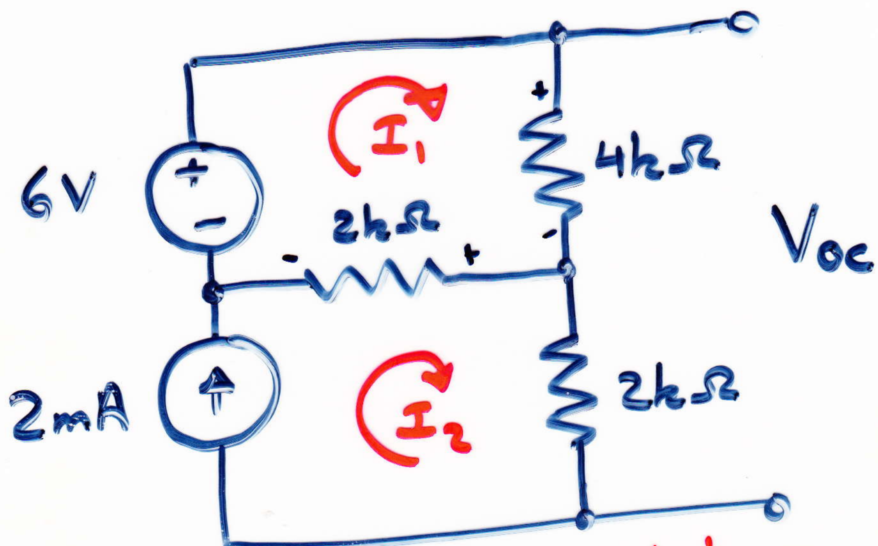
$$\therefore V_o = 1 \times 10^{-3} \times 8k = \underline{8V}$$

2



Circuit a

Consider L.H.S of dashed line.



Circuit b

$$I_2 = 2mA$$

$$-6 + 4k I_1 + 2k(I_1 - I_2) = 0$$

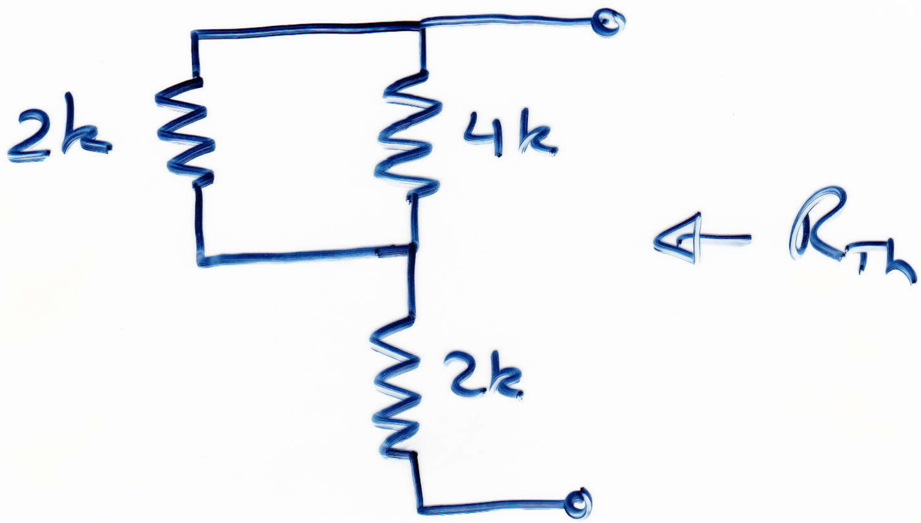
$$\therefore \quad 6k I_1 = 6 + 4$$

$$I_1 = 5/3 \text{ mA.}$$

$$V_{oc} = 4k \times \frac{5}{3} \times 10^{-3} + 2k \times 2 \times 10^{-3}$$

$$= \frac{20}{3} + 4 = \frac{32}{3}\ V$$

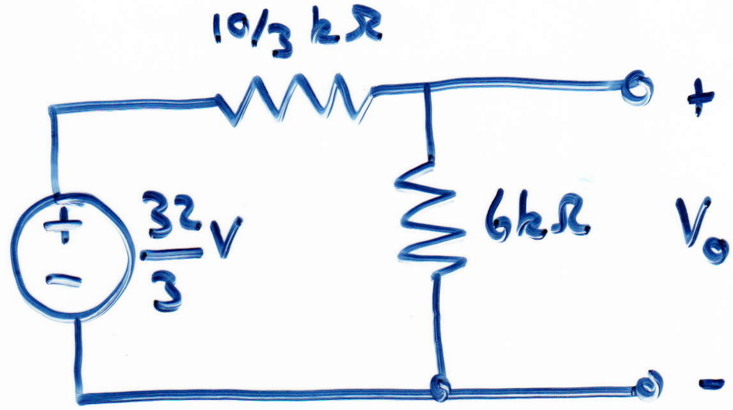
The Thévenin resistance of circuit b is



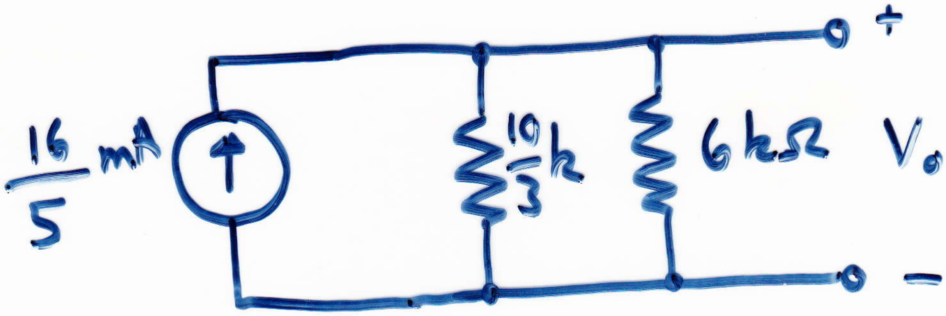
$$R_{Th} = \frac{2 \times 4}{4 + 2}k + 2k = \frac{8k}{6} + 2k$$

$$= \frac{10}{3}\ k\Omega$$

Know voltage and R_{TH}, so redrawing original circuit (circuit a)



Alternatively



$$V_o = \frac{32}{3}\left(\frac{6}{6+\frac{10}{3}}\right) = \frac{32}{3}\left(\frac{6}{\frac{28}{3}}\right)$$

$$= \overset{8}{\cancel{32}} \times \frac{6}{\underset{7}{\cancel{28}}}$$

$$= \frac{48}{7}\text{ V}$$