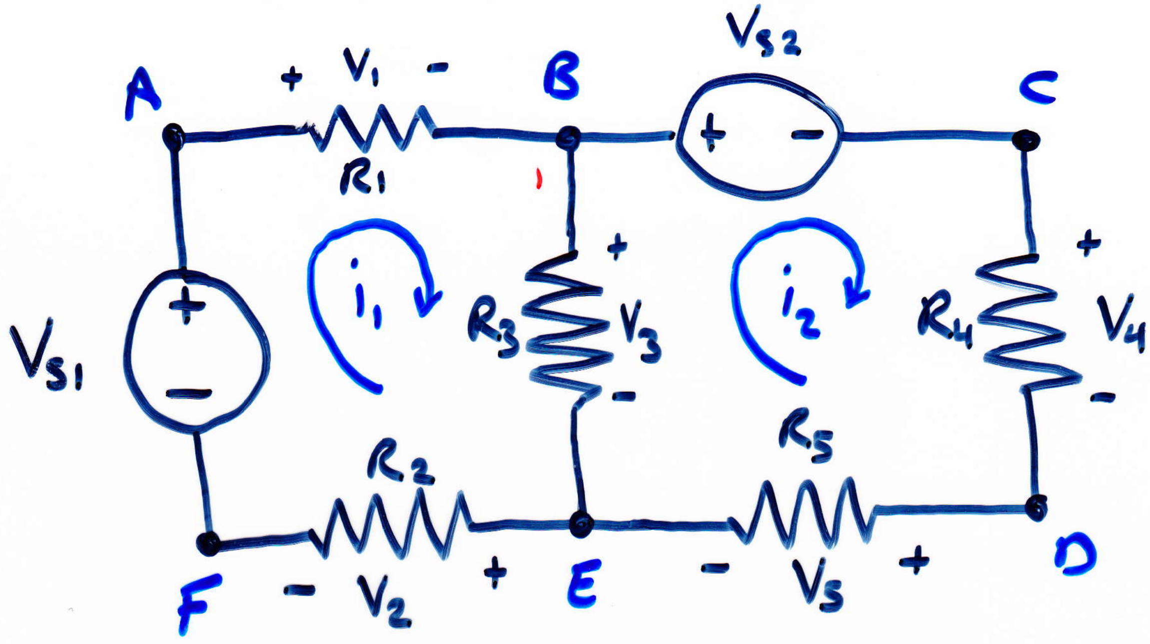
Loop Analysis.

Analysis	Typical Unknowns	Use
Node	node voltages	KCL
Loop	circuit currents	KVL

If the circuit contains N independent loops, then N independent simultaneous equations are required to describe the network.

Consider the approach by example.



Two loops ABEFA and BCDEB

Apply KVL to ABEFA,

$$V_1 + V_3 + V_2 - V_{s1} = 0 \qquad (9.1)$$

then BCDEB,

$$V_{s2} + V_4 + V_5 - V_3 = 0 \qquad (9.2)$$

Where

$V_1 = i_1 R_1$, $V_2 = i_1 R_2$, $V_3 = (i_1 - i_2) R_3$

$V_4 = i_2 R_4$ and $V_5 = i_2 R_5$

Substituting these into 9.1 and 9.2

$$i_1(R_1 + R_2 + R_3) - i_2 R_3 = V_{S1} \qquad (9.3)$$

$$-i_1 R_3 + i_2(R_3 + R_4 + R_5) = -V_{S2} \qquad (9.4)$$

In matrix form

$$\begin{bmatrix} R_1 + R_2 + R_3 & -R_3 \\ -R_3 & R_3 + R_4 + R_5 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} V_{S1} \\ -V_{S2} \end{bmatrix}$$

New term <u>MESH</u>

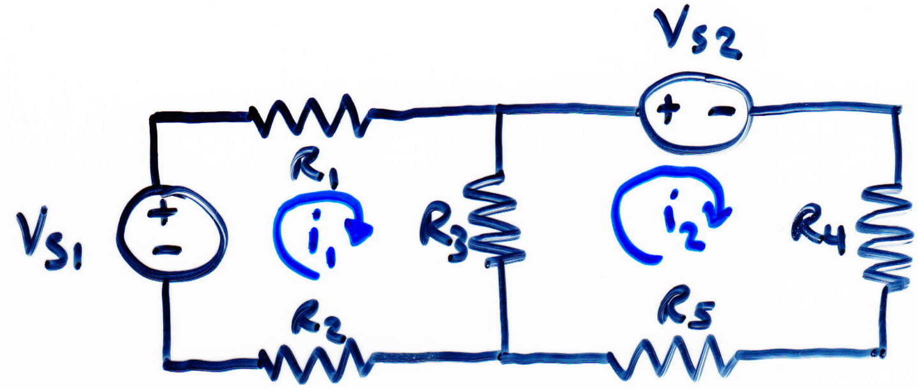
Mesh is a loop that does not contain any loops within it.

ABEFA, BCDEB are meshes

ABCDEFA is <u>not</u> a mesh (but it is a loop).

Applying KVL as we have done is often called <u>mesh analysis</u> and the currents are known as <u>mesh currents</u>

Recall !



$$i_1(R_1+R_2+R_3) - i_2 R_3 = V_{S1}$$

$$-i_1 R_3 + i_2(R_3+R_4+R_3) = -V_{S2}$$

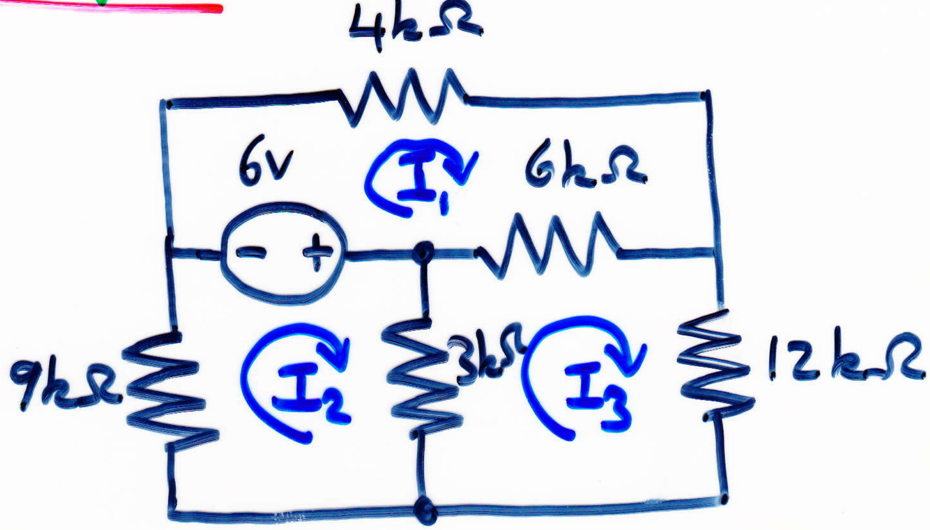
Can see there is some symmetry (similar to what you read in nodal analysis).

Rules for writing equations by inspection only. <u>Only</u> with independant sources & resistors

- First consider all currents to be in same direction.
- Apply KVL to mesh j with mesh current i_j
- Coeff. of i_j is sum of the resistances in mesh j.
- Other coeffs. (e.g. i_{j-1}, i_{j+1}) are the negatives of the resistance common to these meshes and mesh j

- RHS of equation is equal to the algebraic sum of the voltage sources in mesh j. Voltage source is +ve if it aids current i_j and -ve if it opposes.

Example



$$(4k+6k)I_1 - (0)I_2 - 6k\,I_3 = -6$$

$$-(0)I_1 + (9k+3k)I_2 - 3k\,I_3 = 6$$

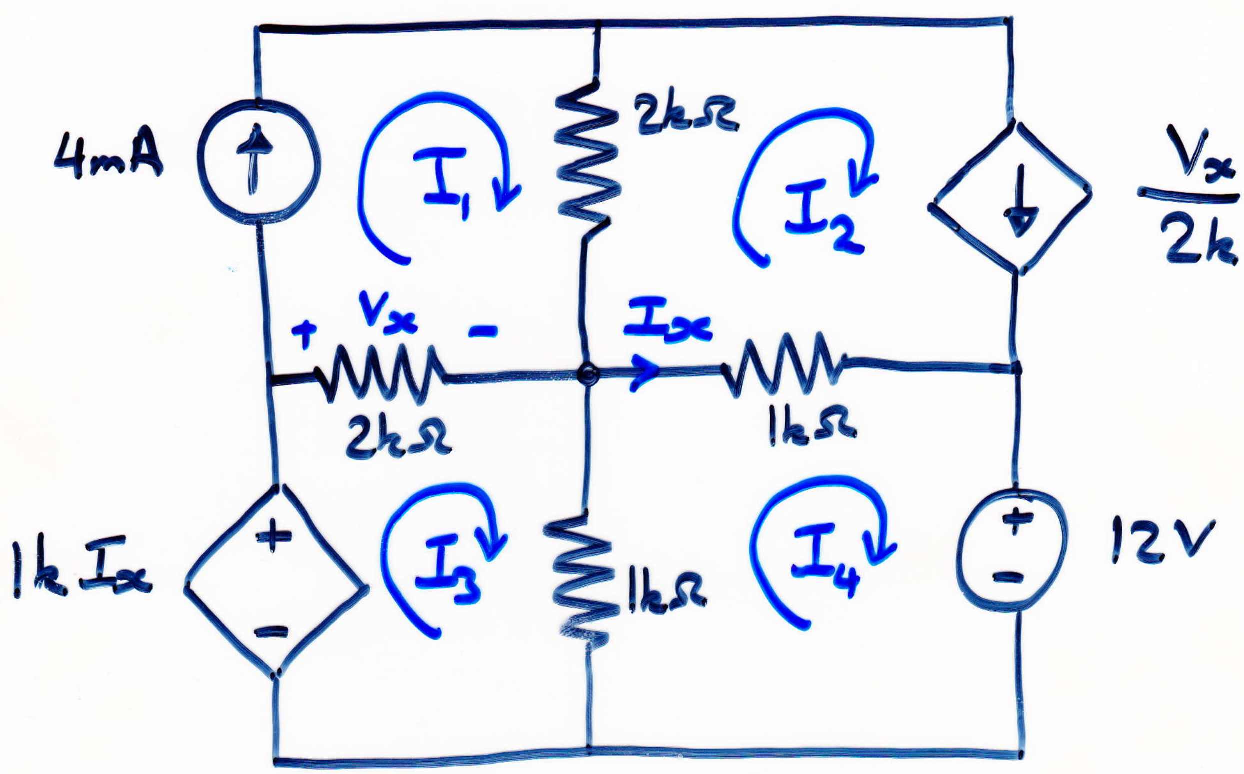
$$-6k\,I_1 - 3k\Omega\,I_2 + (3k+6k+12k)I_3 = 0$$

Matrix form

$$\begin{bmatrix} 10k & 0 & -6k \\ 0 & 12k & -3k \\ -6k & -3k & 21k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 6 \\ 0 \end{bmatrix}$$

Again. The 'rules' given and shown above are only for independent sources and resistors only.

Example when circuit contains dependent source.



$$I_1 = 4mA = \frac{4}{k} A \qquad (1)$$

$$I_2 = \frac{V_x}{2k} \qquad (2)$$

$$-kI_x + 2k(I_3 - I_1) + 1k(I_3 - I_4) = 0 \quad (3)$$

$$12 + 1k(I_4 - I_3) + 1k(I_4 - I_2) = 0 \quad (4)$$

where $V_x = 2k(I_3 - I_1)$ (5)

$$I_x = I_4 - I_2 \quad (6)$$

Combining

From (1) $$I_1 = \frac{4}{k}$$

(2) & (5) $$I_2 = I_3 - I_1$$

(3) & (6) $$kI_2 + 3kI_3 - 2kI_4 - 2kI_1 = 0 \quad (7)$$

(7) & (1) $$kI_2 + 3kI_3 - 2kI_4 = 8$$

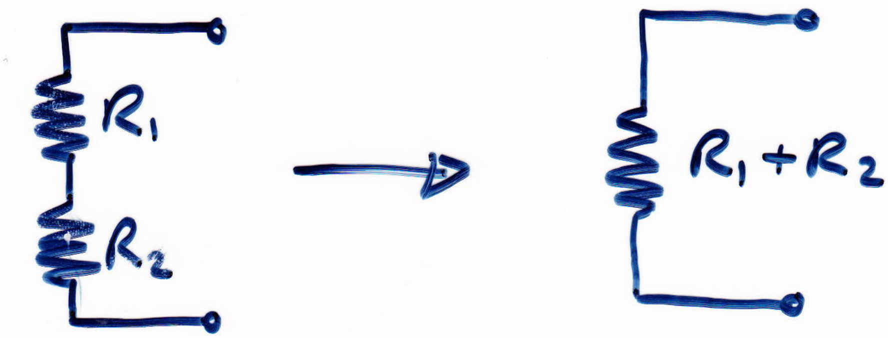
(4) $$kI_2 + kI_3 - 2kI_4 = 12$$

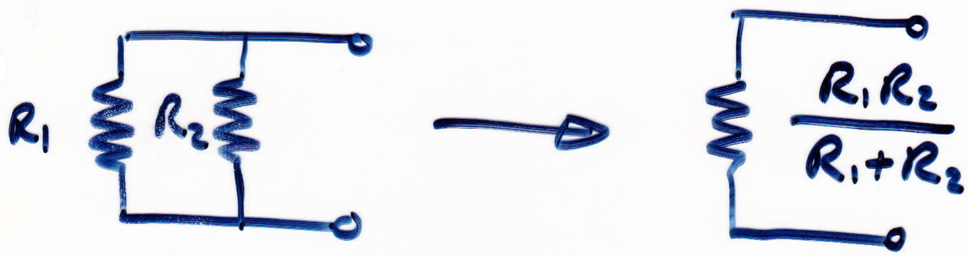
Matrix Form

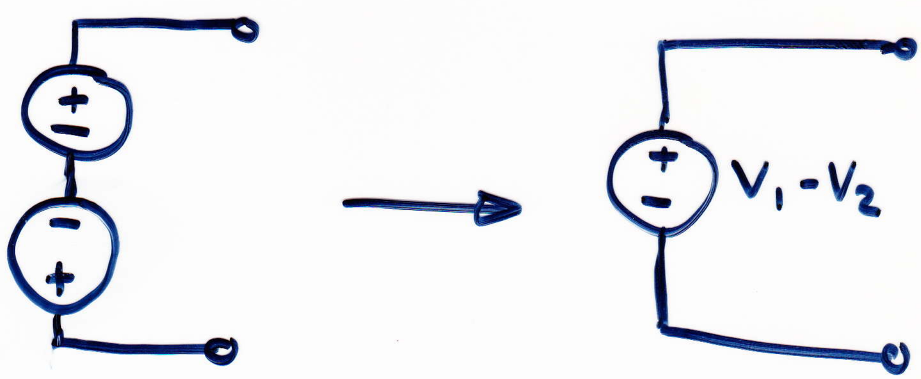
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 \\ 0 & 1k & 3k & -2k \\ 0 & 1k & 1k & -2k \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} 4/k \\ 0 \\ 8 \\ 12 \end{bmatrix}$$

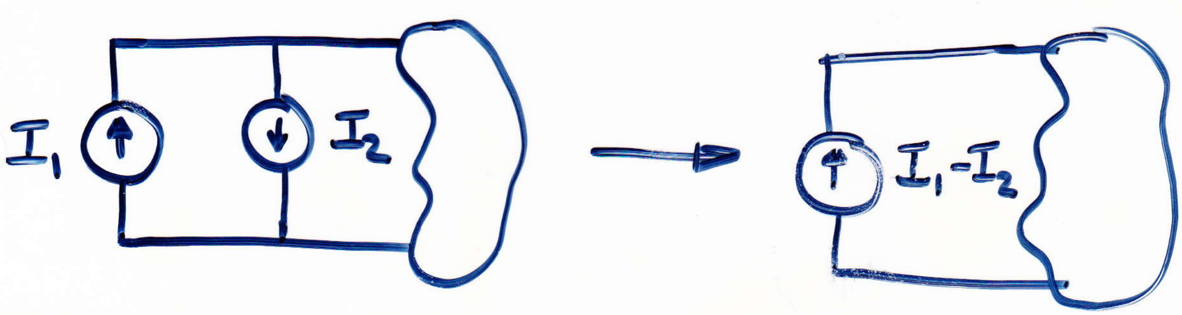
Equivalence, Linearity & Superposition

Equivalence.

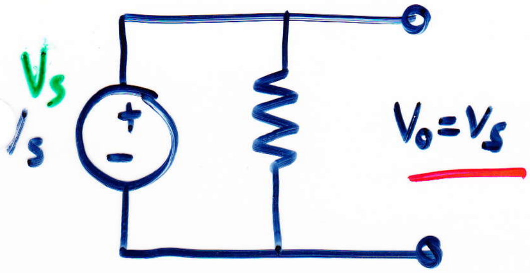


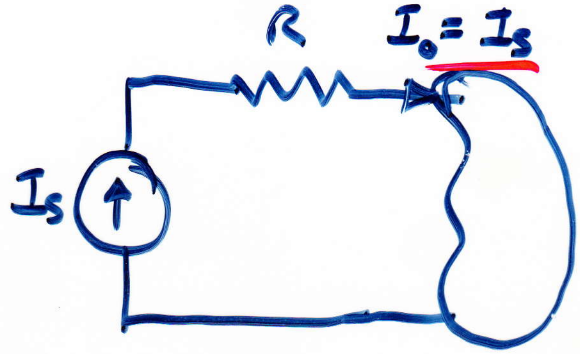






* see note *



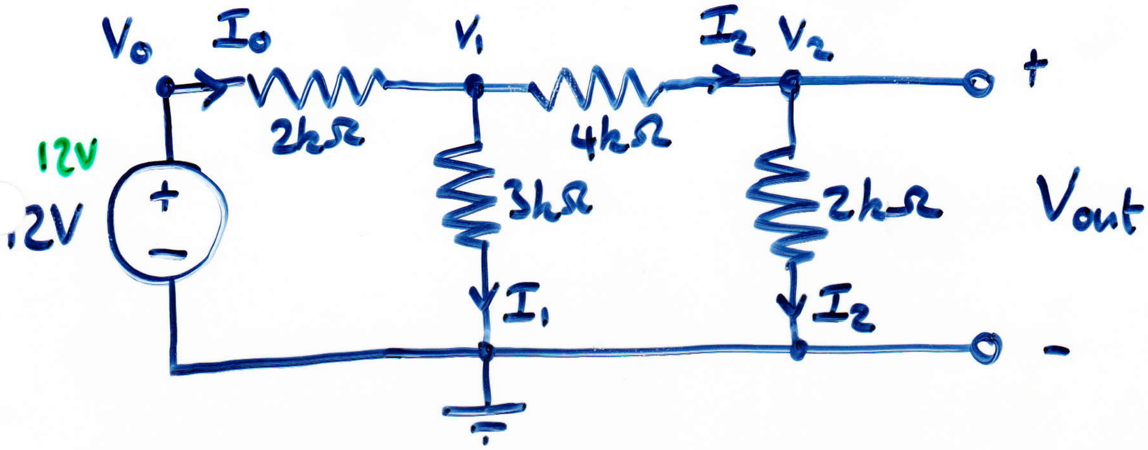


Note: do not consider series connection of current sources or a parallel connection of voltage sources, unless sources are orientated in same direction and have the same values.

Linearity

Majority of circuits we will consider in this course are linear in nature. Linearity has the property of addition and scaling. We can use this feature to our advantage.

10.3



Determine Vout.

Could used techniques like we have already used.

Or could use property of scaling.

To do this assume an output and work back to determine the supply, V_s. (denoted as V_0)

Assume $V_{out} = V_2 = 1V$.

$$\therefore I_2 = \frac{V_2}{2k\Omega} = 0.5mA$$

$$V_1 - V_2 = 4k I_2$$

$$V_1 = 4k I_2 + V_2$$
$$= 3V$$

$$I_1 = \frac{V_1}{3k} = \frac{3}{3k} = 1mA$$

Apply KCL

$$I_0 = I_1 + I_2 = 1.5\,mA$$

$$\therefore V_S = 2k\,I_0 + V_1 = 3+3$$
$$= 6V$$

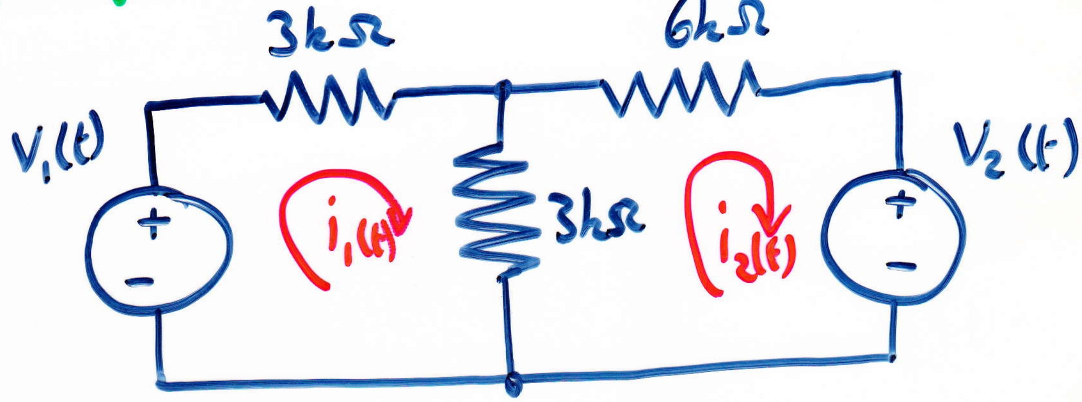
Note this is V_S for our assumed output of 1V.

Our circuit says V_S is 12V.

So scaling factor is 2 and therefore

$$V_{out} = 2 \times 1V = 2V$$

Superposition



Mesh equations

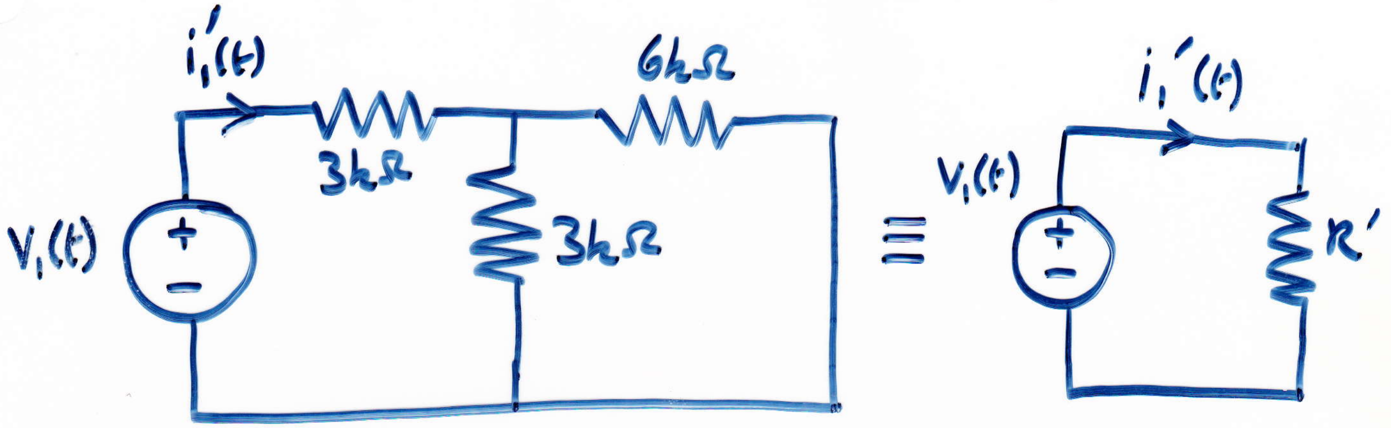
$$6k\, i_1(t) - 3k\, i_2(t) = V_1(t)$$
$$-3k\, i_1(t) + 9k\, i_2(t) = -V_2(t)$$

Solving for $i_1(t)$ gives

$$i_1(t) = \frac{V_1(t)}{5k} - \frac{V_2(t)}{15k} \qquad (10.1)$$

$i_1(t)$ has two components.

Now consider the circuit with only one of the sources present (do twice for each source).

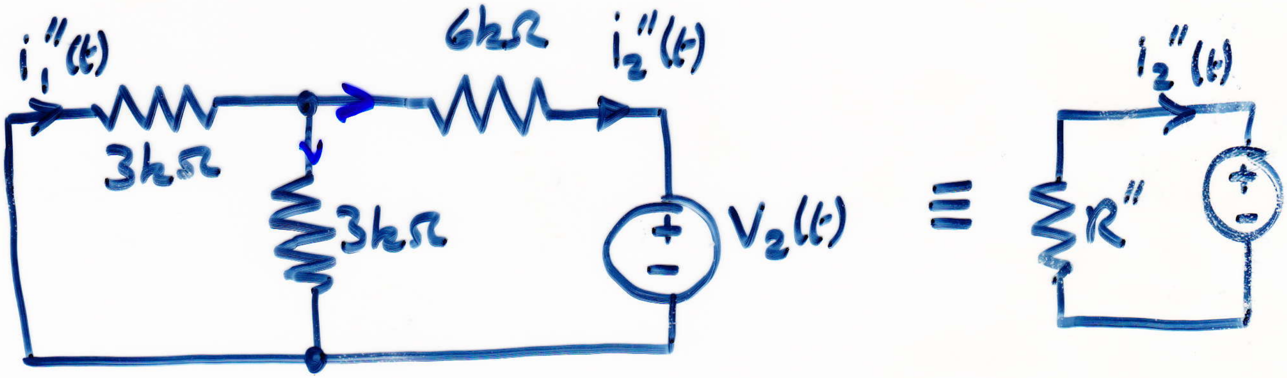


$$R' = 3k + \frac{6 \times 3}{3+6} = 3 + \frac{18}{9}$$

$$= 5k$$

$$\underline{i_1'(t) = V_1(t)/5k.}$$

Also



$$R'' = \left(6 + \frac{3 \times 3}{3+3}\right)k = 6k + \frac{9k}{6} = \frac{45k}{6}$$

$$= \frac{15k}{2}$$

$$i_2''(t) = -\frac{2V_2(t)}{15k}$$

Using current division

$$i_1''(t) = \frac{-2V_2(t)}{15k}\left(\frac{3k}{3k+3k}\right) = \frac{-V_2(t)}{15k}$$

Now add $i_1'(t)$ & $i_1''(t)$

$$i_1(t) = i_1'(t) + i_1''(t) = \frac{V_1(t)}{5k} - \frac{V_2(t)}{15k}$$

Same as (10.1)

Principle of Superposition.

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as the algebraic sum of the individual contributions of each source acting alone.

Note: current sources become open circuits in such analysis.