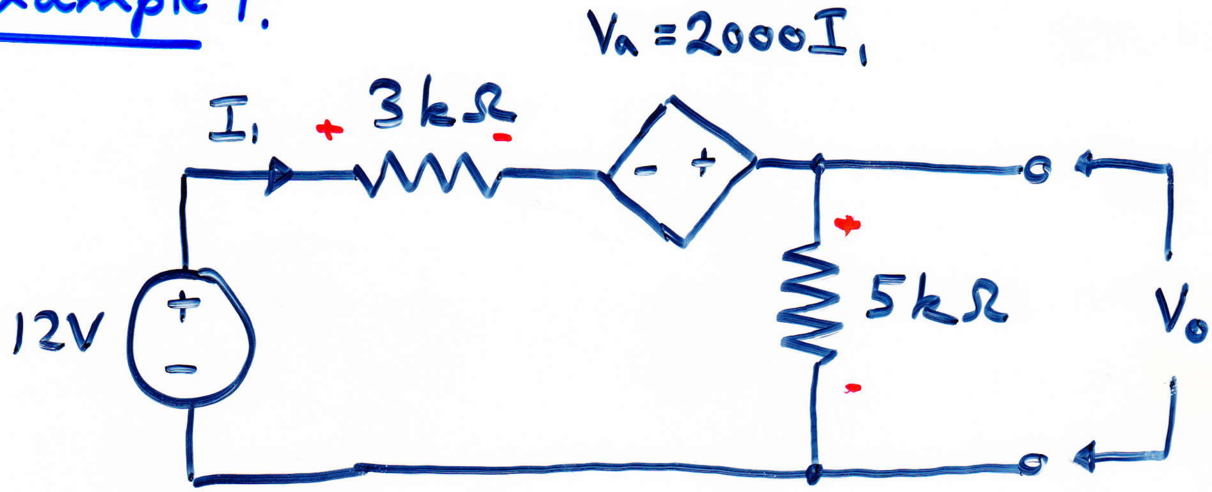
Circuits with Dependent Sources

Example 1.



Determine V_o.

Apply KVL

$$-12 + I_1 3\times10^3 - V_a + 5\times10^3 I_1 = 0$$

$$-12 + I_1 3\times10^3 - 2000 I_1 + 5\times10^3 I_1 = 0$$

$$\therefore\ I_1 (3\times10^3 - 2\times10^3 + 5\times10^3) = 12$$

$$I_1 = \frac{12}{6\times10^3} = \underline{2\,mA}$$

$$\therefore\ V_o = 2\times10^{-3} \times 5\times10^3$$

$$= \underline{\underline{10V}}$$

Strategy to Problem Solving:

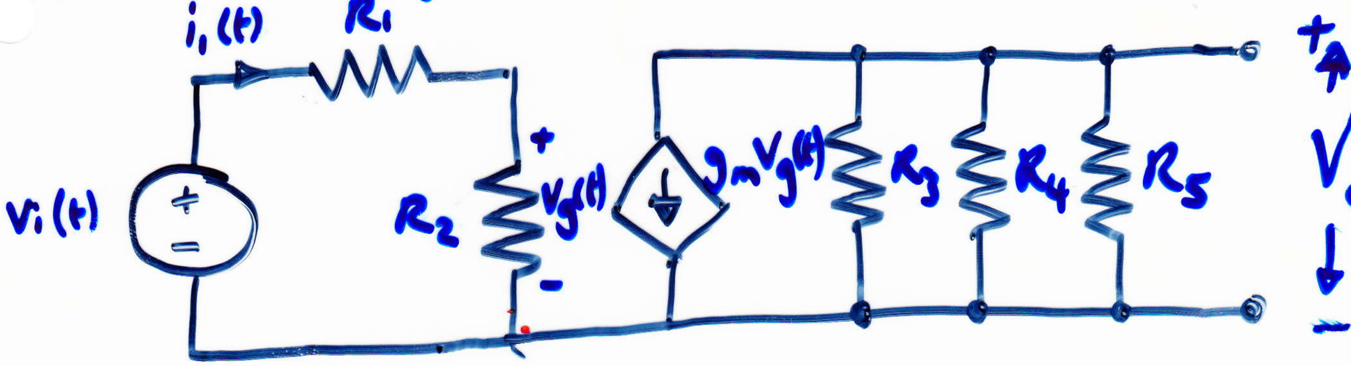
1. Write KVL and/or KCL equations, treating dependent sources as independent sources!

2. Write equation that specifies the relationship of the dependent source to the controlling parameter.

3. Solve equations for unknown.

Example 2

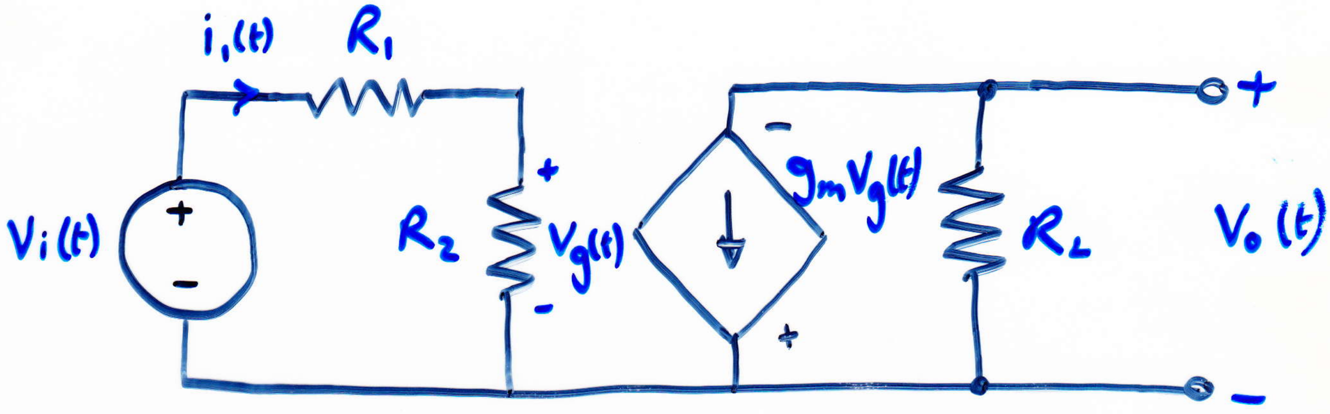
$g_m V_g(t)$



Equivalent circuit for FET common-source amp or BJT common-emitter amp.

Determine an expression for the gain of the amp (i.e. Vout/Vin).

Combine R_3, R_4 & R_5 into an equivalent resistance, often called a <u>load</u> resistance.



So $\dfrac{1}{R_L} = \dfrac{1}{R_3} + \dfrac{1}{R_4} + \dfrac{1}{R_5}$

Apply KVL to input section of the circuit

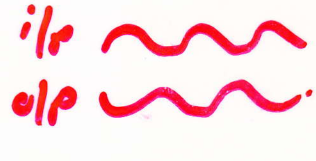
$$V_i(t) = i_1(t)R_1 + i_1(t)R_2 = i_1(t)(R_1 + R_2)$$
$$= i_1(t)R_1 + V_g(t)$$

But $i_1(t) = V_i(t)/(R_1 + R_2)$

$$\therefore \; V_g(t) = \frac{R_2}{(R_1+R_2)} V_i(t) \quad \text{— (7.1)}$$

At the output side

$$V_o = -g_m V_g(t) R_L$$



Substituting in (7.1) gives

$$V_o = -g_m \frac{R_2 V_i(t)}{(R_1 + R_2)} R_L$$

$$\therefore \frac{V_o(t)}{V_i(t)} = -\frac{g_m R_L R_2}{(R_1+R_2)}$$

Considering some values

$R_1 = 100\,\Omega$, $R_2 = 1\,k\Omega$, $g_m = 0.045\,S$

$R_3 = 50\,k\Omega$, $R_4 = R_5 = 10\,k\Omega$

$$\frac{1}{R_L} = \frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5} = \frac{1}{50k} + \frac{1}{10k} + \frac{1}{10k}$$

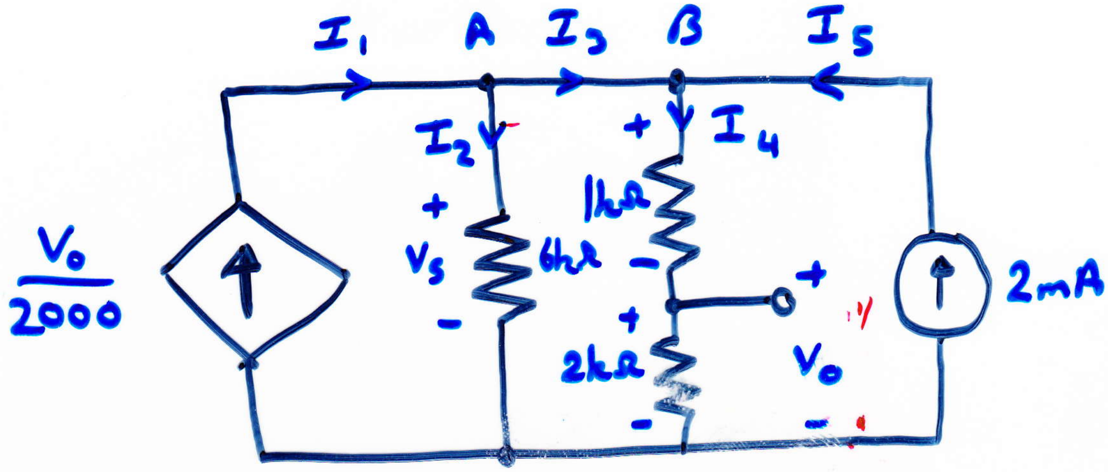
$$= \frac{1+5+5}{50k}$$

$$R_L = \frac{50}{11}k = 4.55\,k\Omega$$

$$\text{So } \frac{V_o(t)}{V_i(t)} = \frac{-0.045 \times 4.55 \times 10^3 \times 1 \times 10^3}{(100 + 1000)} = \frac{-0.045 \times 4.55 \times 10^3}{1.1}$$

$$= -165.45 \quad V/V$$

Example (Irwin extension) E2.18



$$I_1 = V_0/2000 \ \text{A}$$

$$I_5 = 2\text{mA}$$

Apply KCL at node A.

$$V_0/2000 = I_2 + I_3 \quad -(1)$$

KCL at node B

$$2\text{mA} + I_3 = I_4 \quad -(2)$$

Also

$$I_4 = V_0/2000 \quad -(3)$$

$$V_5 = I_2 \times 6\text{k}\Omega$$

$$V_0 = \frac{2}{3} V_s$$

Sub (3) into (2)

$$2\times10^{-3} + I_3 = V_0/2000$$

Compare this with (1)

Can see

$$I_2 = 2 \times 10^{-3}\ A$$

$$\therefore\ V_s = 6\,k\Omega \times 2\,mA$$
$$= 12V$$

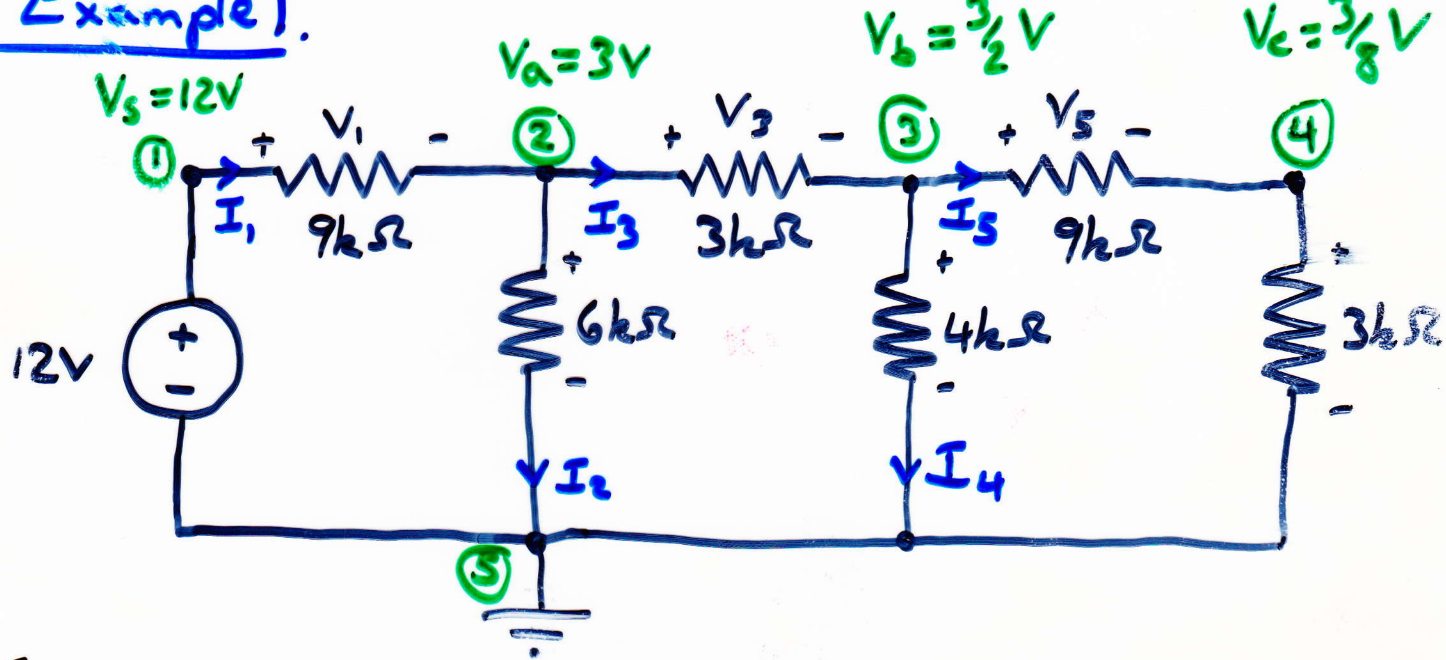
So $V_o = \frac{2}{3} V_s = \frac{2}{3} \times 12$

$$\therefore\ \underline{V_o = 8V}$$

Nodal Analysis.

In this type of analysis the variables are the node voltages, with one node being selected as a reference node. The other nodes in the circuit are defined with respect to this node.

Example 1.



(Circuit was examined previously in lecture ~~6~~ 4 - series and parallel resistors).

V_s, V_a, V_b and V_c are measured w.r.t. node 5, which is labeled the ground.

Knowing the nodal voltages we can determine the branch currents and hence the power in each element.

For example consider the left most $9k\Omega$ resistor

$$V_1 = V_s - V_a$$
$$= 12V - 3V$$
$$= 9V$$

$$\therefore \quad I_1 = \frac{V_s - V_a}{9k\Omega} = \frac{9}{9k}$$

$$= \underline{1mA}.$$

Similarly,

$$I_2 = \frac{V_a - 0}{6k} = \frac{3}{6k} = \underline{\tfrac{1}{2}mA}$$

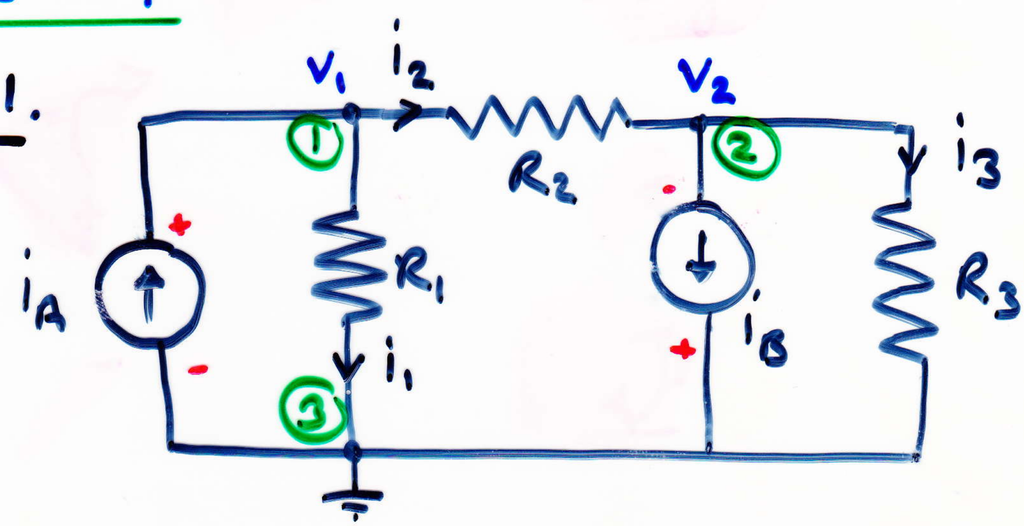
$$I_3 = \frac{V_a - V_b}{3k\Omega} = \frac{3 - 3/2}{3k} = \frac{3/2}{3k} = \underline{\tfrac{1}{2}mA}$$

$$I_4 = \frac{V_b - 0}{4k} = \frac{\frac{3}{2} - 0}{4k} = \underline{\frac{3}{8}\,mA}$$

$$I_5 = \frac{V_b - V_c}{9k} = \frac{\frac{3}{2} - \frac{3}{8}}{9k} = \frac{\frac{9}{8}}{9k} = \underline{\frac{1}{8}\,mA}$$

Example 2.

Example 3.1.



Applying KCL at node 1

$$-i_A + i_1 + i_2 = 0$$

Using Ohm's law, $i = GV$

$$-i_A + G_1(v_1 - 0) + G_2(v_1 - v_2) = 0$$

$$V_1(G_1 + G_2) - G_2 V_2 = i_A \quad *$$

KCL at node 2 gives

$$-i_2 + i_B + i_3 = 0$$

Again using Ohm's Law

$$-G_2(V_1 - V_2) + i_B + G_3(V_2 - 0) = 0$$

$$-G_2 V_1 + (G_2 + G_3) V_2 = -i_B \quad *$$

So we have 2 equations for two ~~unkown~~ unkown voltages.

$$V_1(G_1 + G_2) - G_2 V_2 = i_A$$

$$-V_1 G_2 + (G_2 + G_3) V_2 = -i_B$$

Solution of the simultaneous equations can be done by Gaussian elimination or matrix methods. By way of example let us consider the following values.

$I_A = 1mA$, $R_1 = 12k\Omega$, $R_2 = 6k\Omega$, $I_B = 4mA$ & $R_3 = 6k\Omega$

Gaussian elimination

$$V_1\left(\frac{1}{12k}+\frac{1}{6k}\right)-V_2\left(\frac{1}{6k}\right)=1\times10^{-3}$$

$$-V_1\left(\frac{1}{6k}\right)+V_2\left(\frac{1}{6k}+\frac{1}{6k}\right)=-4\times10^{-3}$$

So have:

$$\frac{V_1}{4k}-\frac{V_2}{6k}=1\times10^{-3}$$

$$-\frac{V_1}{6k}+\frac{V_2}{3k}=-4\times10^{-3}$$

Now

$$V_1=V_2\left(\frac{2}{3}\right)+4 \qquad -(8.1)$$

$$\therefore\ \frac{-1}{6k}\left(\frac{2}{3}V_2+4\right)+\frac{V_2}{3k}=-4\times10^{-3}$$

Or $\underline{V_2=-15\,V}$

Sub. back into (8.1) gives

$$V_1=-15\left(2/3\right)+4$$

$$\underline{V_1=-6V}$$

Using matrix equations

$$GV = I$$

$$G = \begin{bmatrix} 1/4k & -1/6k \\ -1/6k & 1/3k \end{bmatrix}, \quad V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \;\&\; I = \begin{bmatrix} 1\times10^{-3} \\ -4\times10^{-3} \end{bmatrix}$$

Matrix solution

$$V = G^{-1} I$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 1/4k & -1/6k \\ -1/6k & 1/3k \end{bmatrix}^{-1} \begin{bmatrix} 1\times10^{-3} \\ -4\times10^{-3} \end{bmatrix}$$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Need G^{-1}.

$$\text{Adj } G = \begin{bmatrix} 1/3k & 1/6k \\ 1/6k & 1/4k \end{bmatrix}$$

$\text{Adj} A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

and determinant.

$$|G| = \left(\frac{1}{3k}\right)\left(\frac{1}{4k}\right) - \left(-\frac{1}{6k}\right)\left(-\frac{1}{6k}\right)$$

$$= \frac{1}{18k^2}$$

$$\therefore \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 18k^2 \begin{bmatrix} 1/3k & 1/6k \\ 1/6k & 1/4k \end{bmatrix} \begin{bmatrix} 1 \times 10^{-3} \\ -4 \times 10^{-3} \end{bmatrix}$$

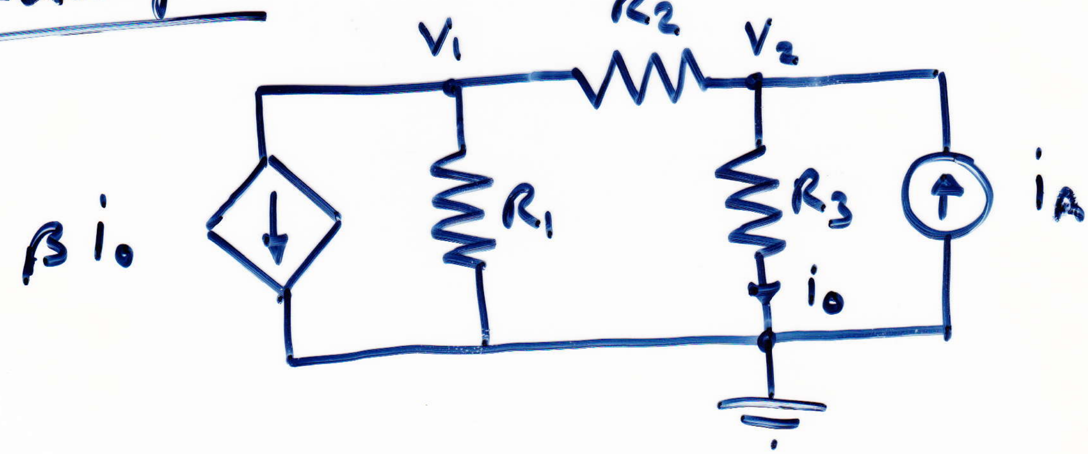
$\frac{Adj(G)}{|G|}$

$$= 18k^2 \begin{bmatrix} \frac{1}{3k^2} & -\frac{4}{6k^2} \\ \frac{1}{6k^2} & -\frac{1}{k^2} \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ -15 \end{bmatrix}$$

Could use Matlab, see Irwin p ~~70~~ 101

Example



Equations for non-reference nodes:

$$\beta i_o + \frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} = 0$$

$$\frac{V_2 - V_1}{R_2} + i_o - i_A = 0$$

Where $i_o = V_2 / R_3$

Simplifying

$$(G_1 + G_2)V_1 - (G_2 - \beta G_3)V_2 = 0 \quad (8.2)$$

$$-G_2 V_1 + (G_2 + G_3)V_2 = i_A \quad (8.3)$$

If $\beta = 2$, $R_2 = 6k\Omega$, $i_A = 2mA$, $R_1 = 12k\Omega$ and $R_3 = 3k\Omega$

Then 8.2 becomes

$$\left(\frac{1}{12k} + \frac{1}{6k}\right)V_1 - \left(\frac{1}{6k} - \frac{2 \times 1}{3k}\right)V_2 = 0$$

$$\frac{1}{4k}V_1 + \frac{1}{2k}V_2 = 0 \quad (8.4)$$

And 8.3 becomes

$$-\frac{1}{6k}V_1 + \left(\frac{1}{6k} + \frac{1}{3k}\right)V_2 = 2\times10^{-3}$$

$$-\frac{1}{6k}V_1 + \frac{1}{2k}V_2 = 2\times10^{-3} \qquad (8.5)$$

Subtract (8.5) from (8.4)

$$V_1\left(\frac{1}{4k} + \frac{1}{6k}\right) = -2\times10^{-3}$$

$$V_1\left(\frac{5}{12k}\right) = -2\times10^{-3}$$

$$V_1 = -\frac{24}{5}\,V$$

Sub. for V_1 into (8.4)

$$-\frac{1}{4k}\times\frac{24}{5} + \frac{1}{2k}V_2 = 0$$

$$V_2 = \frac{12}{5}\,V$$