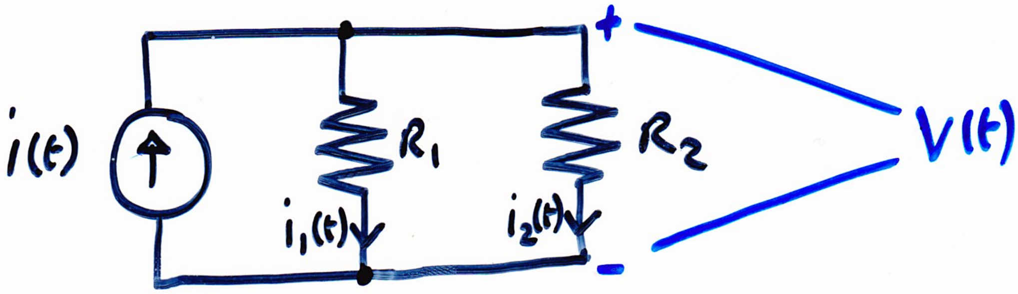
Single-Node Pair Circuit.



Apply KCL

$$i(t) = i_1(t) + i_2(t)$$

Using Ohm's law

$$i(t) = \frac{V(t)}{R_1} + \frac{V(t)}{R_2}$$

$$= \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V(t)$$

$$\therefore \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$

$\equiv$

$$R_p = \frac{R_1 R_2}{R_1 + R_2}$$

What about size of R_p compared to R_1 & R_2

Let $R_1 = 1\Omega$, $R_2 = 2\Omega$ so $R_p =$ ______

c.f. R_1 & R_2

What about currents?

$$i_1(t) = \frac{V(t)}{R_1}$$

But $V(t) = R_p\, i(t) = \frac{R_1 R_2}{R_1 + R_2}\, i(t)$

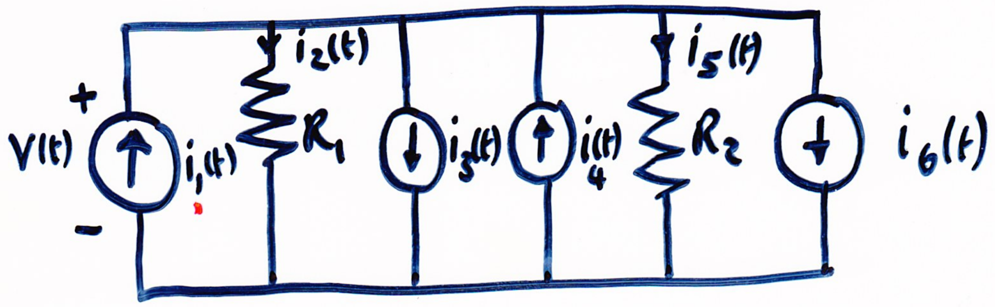
$$i_1(t) = \frac{R_2}{R_1 + R_2}\, i(t)$$

Similarly

$$i_2(t) = \frac{R_1}{R_1 + R_2}\, i(t)$$

S.

Multiple Source Networks

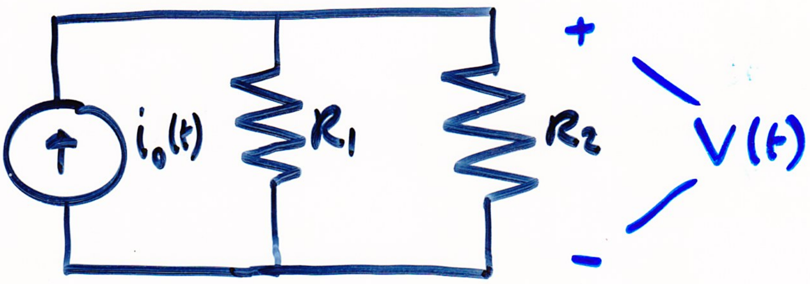


Apply KCL

$$i_1(t) - i_2(t) - i_3(t) + i_4(t) - i_5(t) - i_6(t) = 0$$

$$i_1(t) - i_3(t) + i_4(t) - i_6(t) = i_2(t) + i_5(t) = i_0($$

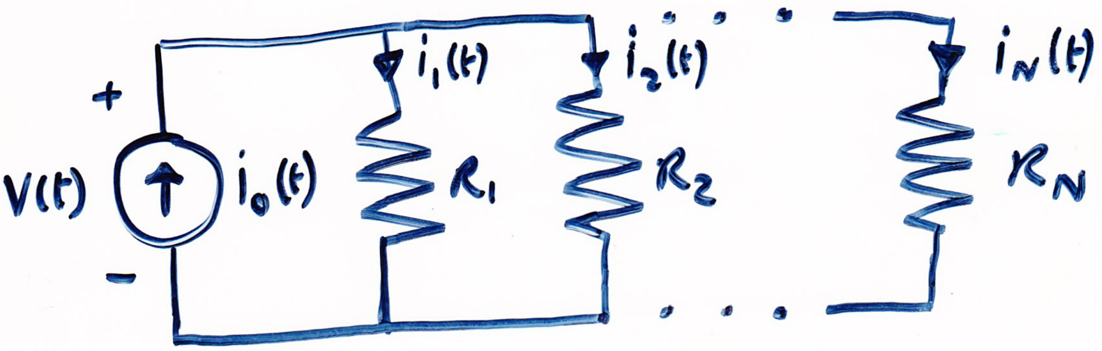
Above circuit is equivalent to



$$\therefore \quad i_0(t) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V(t)$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

Multiple Resistors



Applying KCL

$$i_0(t) = i_1(t) + i_2(t) + \dots + i_N(t)$$
$$= \left(\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_N}\right) V(t)$$

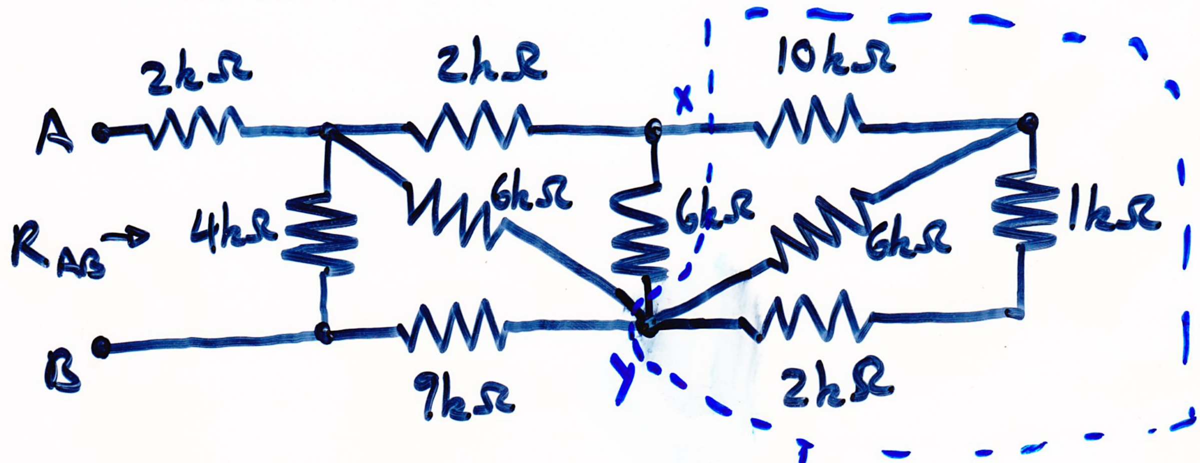
$$\therefore \quad i_0(t) = \frac{V(t)}{R_p}$$

where $\dfrac{1}{R_p} = \displaystyle\sum_{i=1}^{N} \frac{1}{R_i}$

$$i_j(t) = \frac{V(t)}{R_j}$$

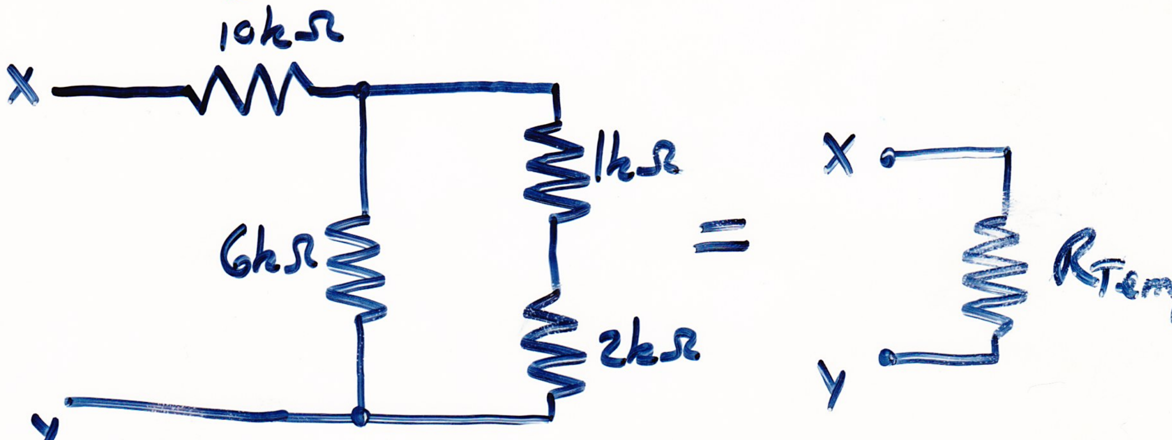
$$i_j(t) = \frac{R_p}{R_j} i_0(t)$$

Series & Parallel Resistor Combination



Circuit A

Simplify

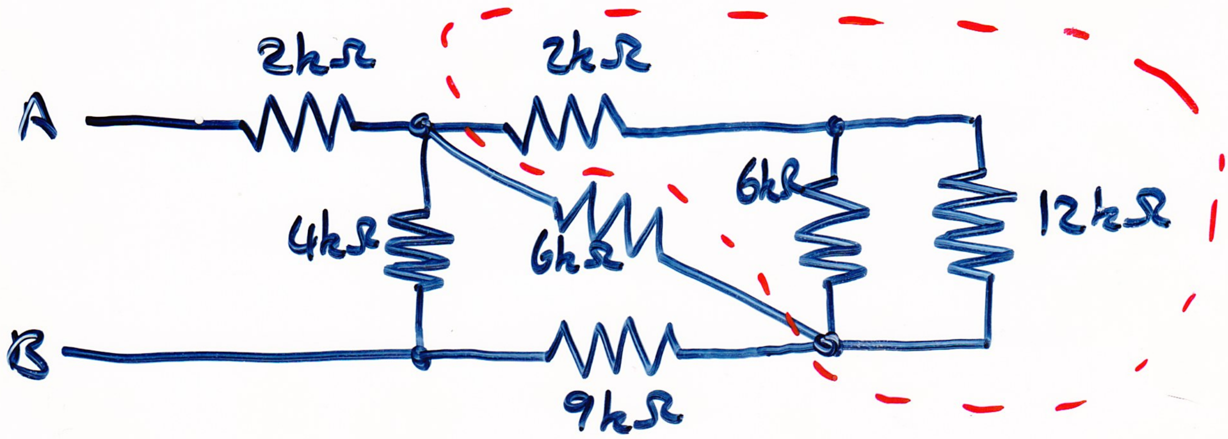


$$R_{Temp} = 10\,k\Omega + \frac{3 \times 6}{3 + 6}\,k\Omega$$

$$= 10\,k\Omega + 2\,k\Omega$$

$$= 12\,k\Omega$$

Circuit A can be drawn as:

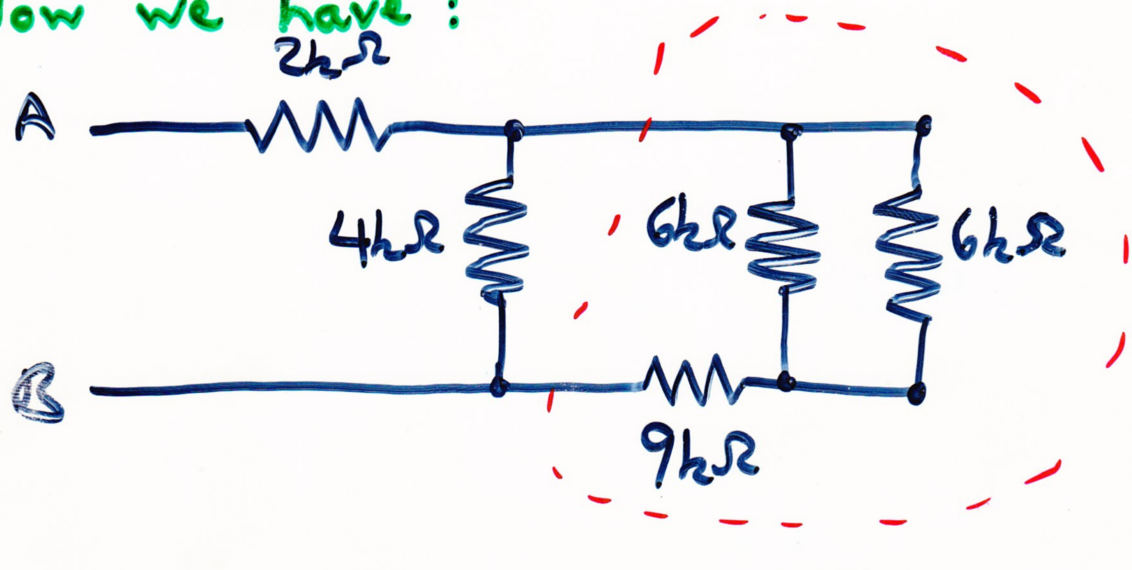


$$R_{Temp} = 2k\Omega + \frac{6 \times 12}{6+12} k\Omega$$

$$= 2k\Omega + \frac{72}{18} k\Omega = 2k\Omega + 4k\Omega$$

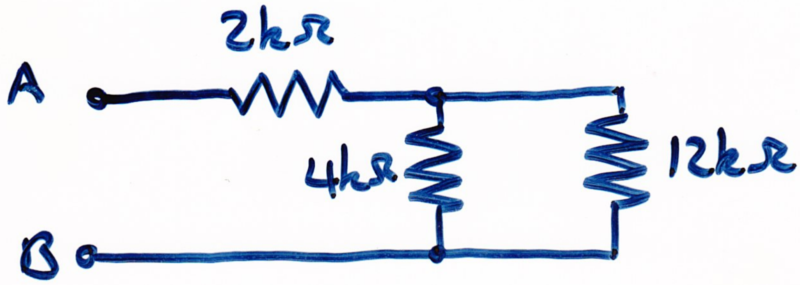
$$= 6k\Omega$$

Now we have:



$$R_{Temp} = 9k\Omega + \frac{6 \times 6}{6+6} k\Omega = 9k\Omega + 3k\Omega$$

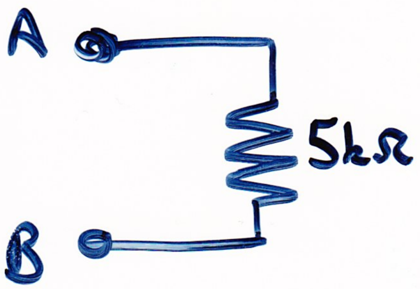
$$= 12k\Omega$$



$$R = 2k\Omega + \frac{4 \times 12}{4+12} k\Omega$$

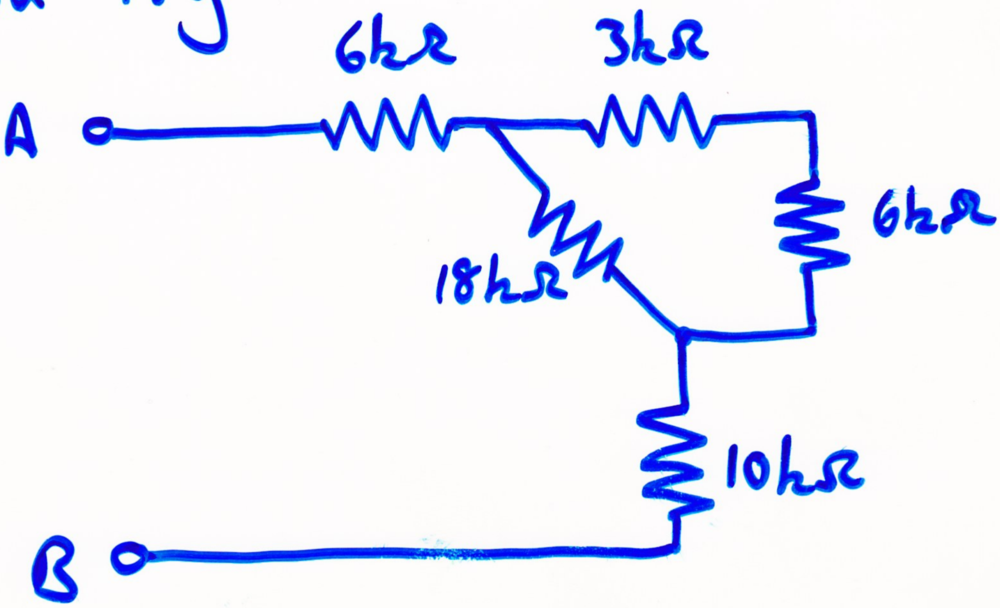
$$= 2k\Omega + 3k\Omega$$

$$= 5k\Omega$$



≡ Circuit A

You try



Ans ?

Circuits with Series-Parallel Combination of Resistors.

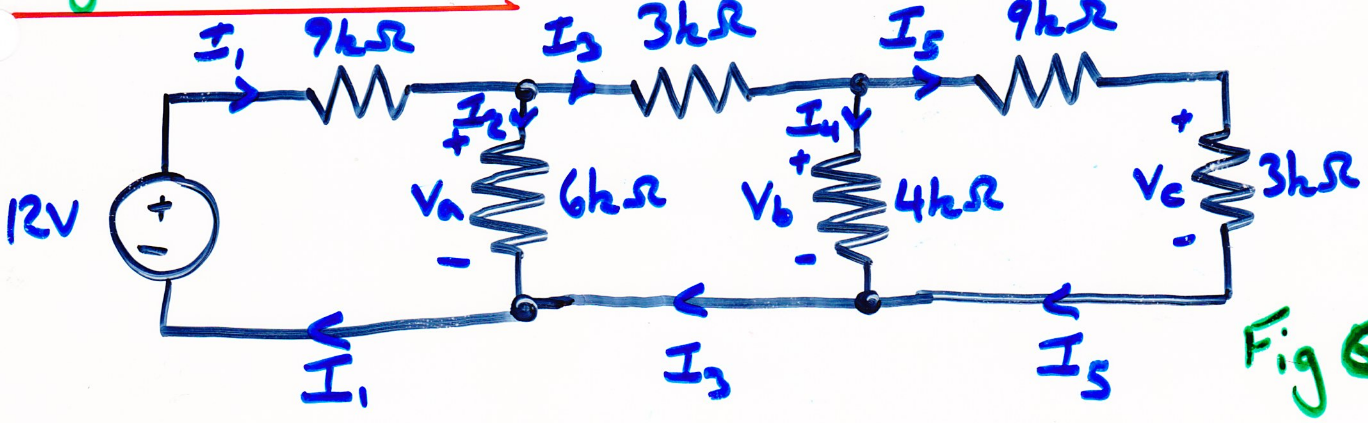


Fig 6A.

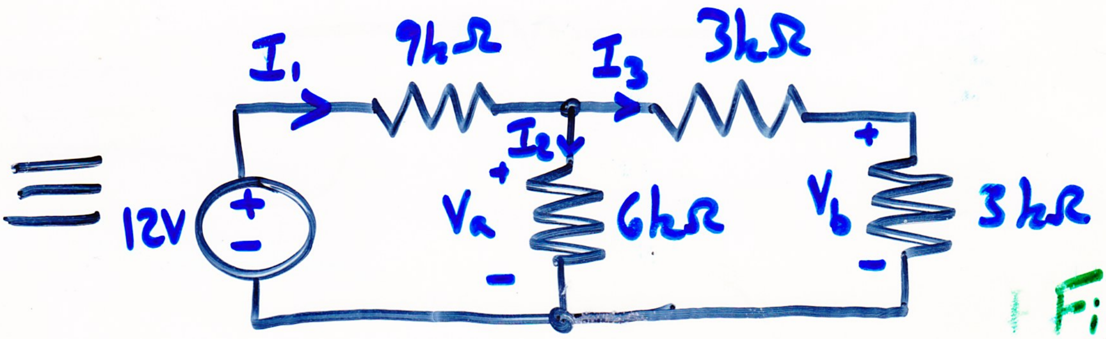


Fig 6B

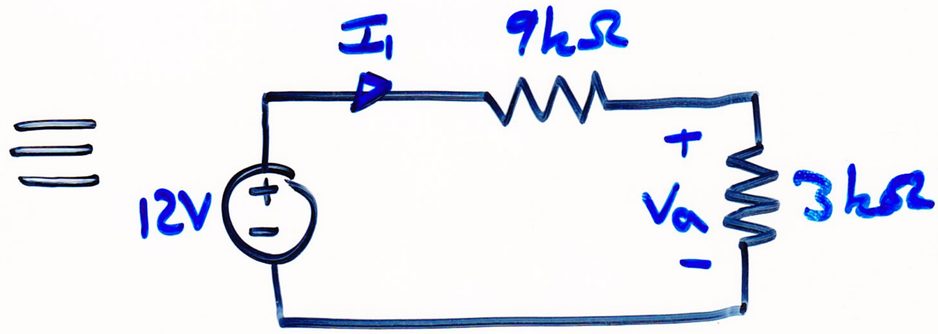


Fig 6c

Using Fig. 6C

$$I_1 = 12/(9+3)k = \underline{\underline{1mA}}$$

$$V_a = 1\times10^{-3} \times 3\times10^{3} = \underline{\underline{3V}}$$

$$I_2 = 3/6k = \underline{\underline{0.5mA}}$$

Using Fig 6B and KCL

$$I_3 = 1mA - 0.5mA = 0.5mA$$

$$V_b = 0.5 \times 10^{-3} \times 3 \times 10^{3} = 1.5V$$

Using fig 6A and Ohm's Law

$$I_4 = 1.5/4k$$

$$= 3/8 \, mA$$

Using KCL

$$I_5 = \frac{1}{2} - \frac{3}{8} \, mA$$

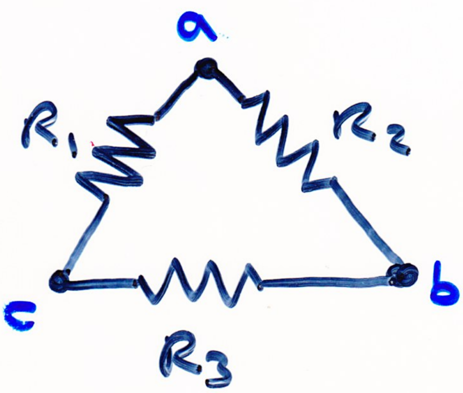
$$= 1/8 \, mA$$

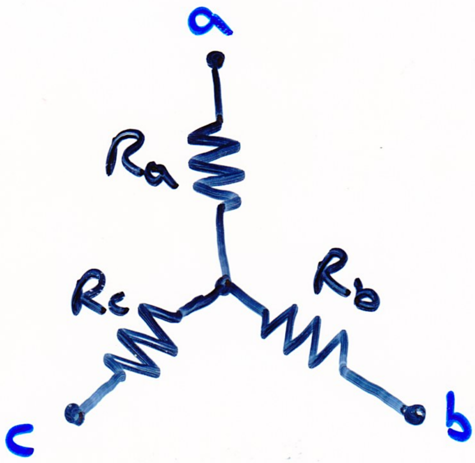
$$\therefore \quad V_c = 1/8 \, mA \times 3 k\Omega$$

$$= 3/8 \, V$$

$Y \rightleftharpoons \Delta$ Transformations

Here exploring relationships/equivalency between circuits of the form.





$$R_{ab} = R_a + R_b = \frac{R_2(R_1 + R_3)}{R_2 + R_1 + R_3}$$

$$R_{bc} = R_b + R_c = \frac{R_3(R_1 + R_2)}{R_3 + R_1 + R_2}$$

$$R_{ca} = R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

Solving the simultaneous equations

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_1 R_3}{R_1 + R_2 + R_3}$$

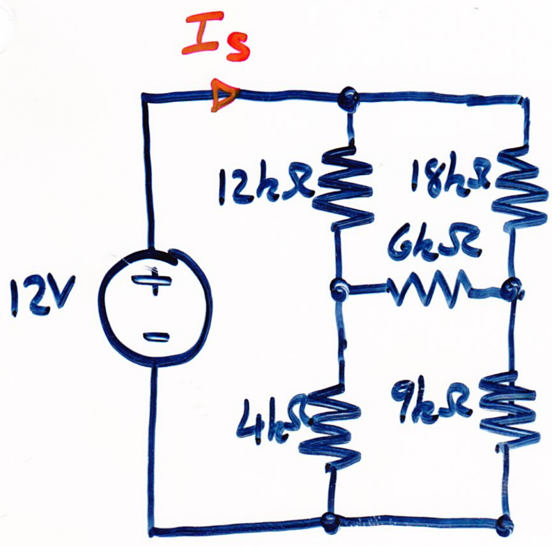
Similarly

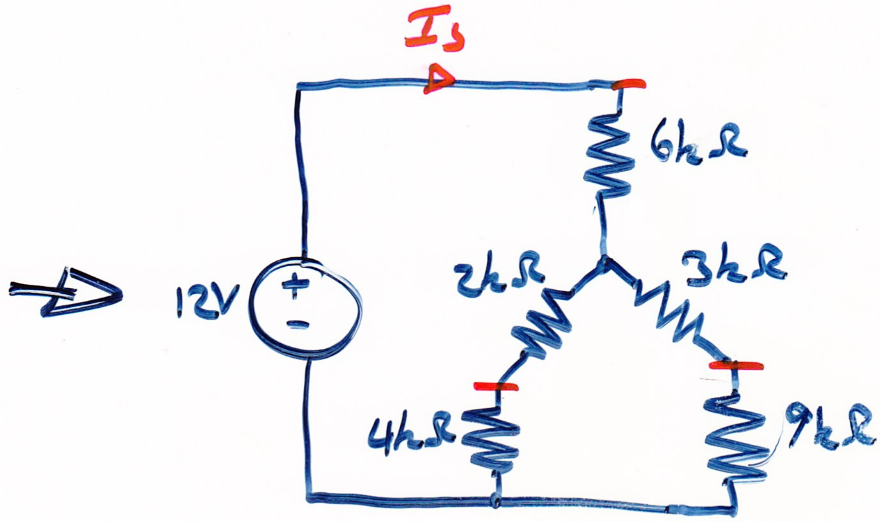
$$R_1 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_a R_c}{R_a}$$

Example (Irwin e.g. 2.22) [2.26]





Have 6kΩ and 12kΩ in parallel.

$$R_{para} = \frac{12 \times 6}{6 + 12} k\Omega = \frac{12}{3} k\Omega = 4k\Omega$$

$$\therefore I_s = 12/(6+4)k$$

$$= 1.2 mA.$$