

Lecture 3: Loops and Nodes

Combination of Resistors

ELEC2501 Circuits and Signals

- Ohm's Law
- Kirchhoff laws
 - KCL
 - KVL
- Single Loop - Voltage Division
 - Multiple sources
 - Multiple resistors

Ohm's Law.

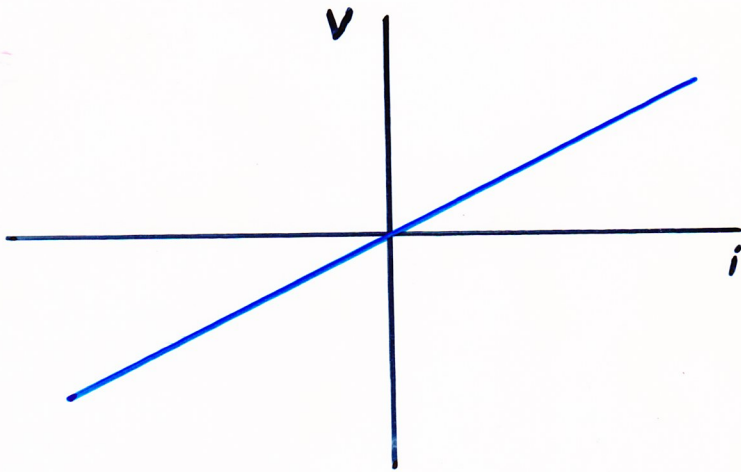
Voltage across a resistance is directly proportional to the current flowing through it.

$$V(t) \propto i(t)$$

$$V(t) = R i(t)$$

R , constant of proportionality, measured in ohms

$$1 \Omega = 1 \text{ V/A.}$$



Considering power

$$p(t) = V(t) i(t)$$

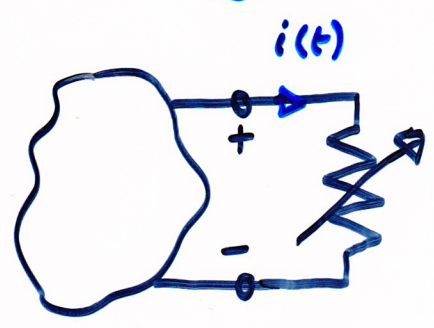
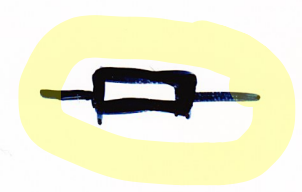
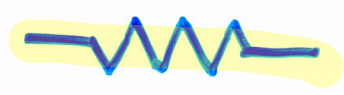
$$\therefore p(t) = R i^2(t) = \frac{V^2(t)}{R}$$

Reciprocal of resistance is conductance, G .

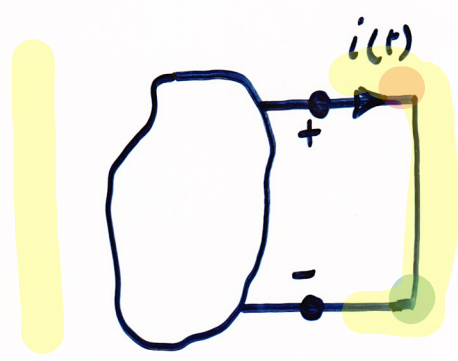
$$G = \frac{1}{R}$$

Units are Siemens ($1S = 1A/V$). (mho's)

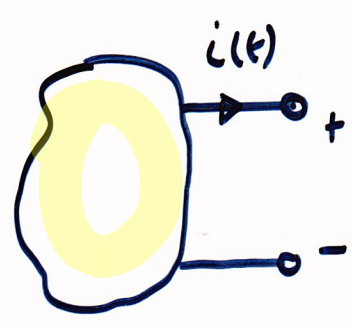
Resistor symbol



Variable resistance.



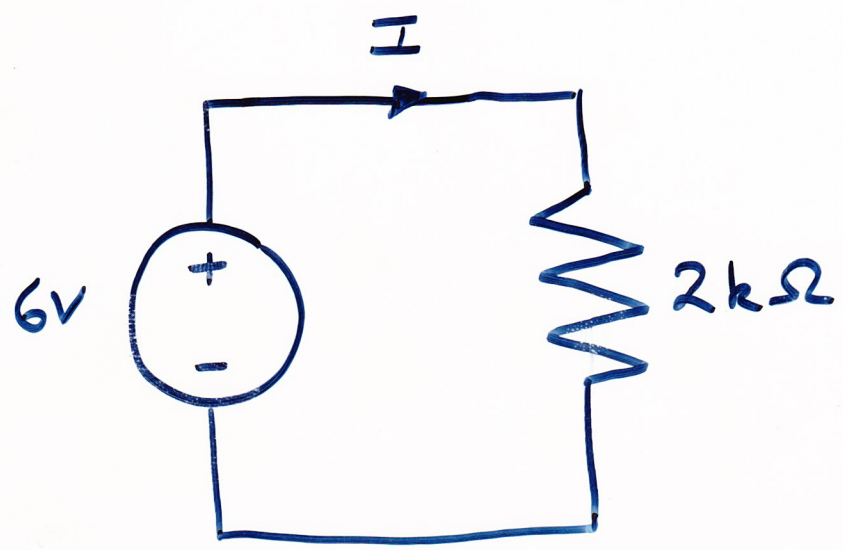
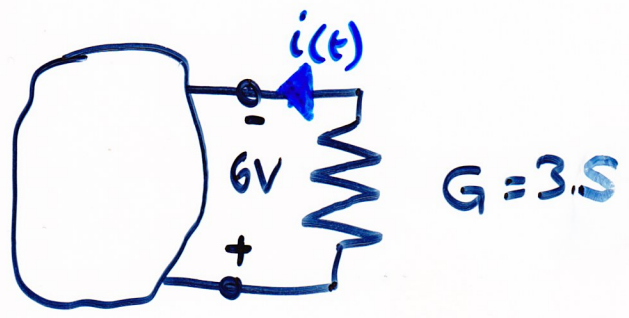
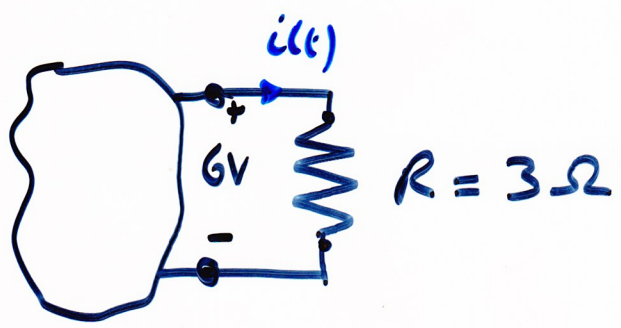
Resistance is zero
What is $v(t)$?



Resistance is ∞
What is $i(t)$?

Examples

$i(t)$?



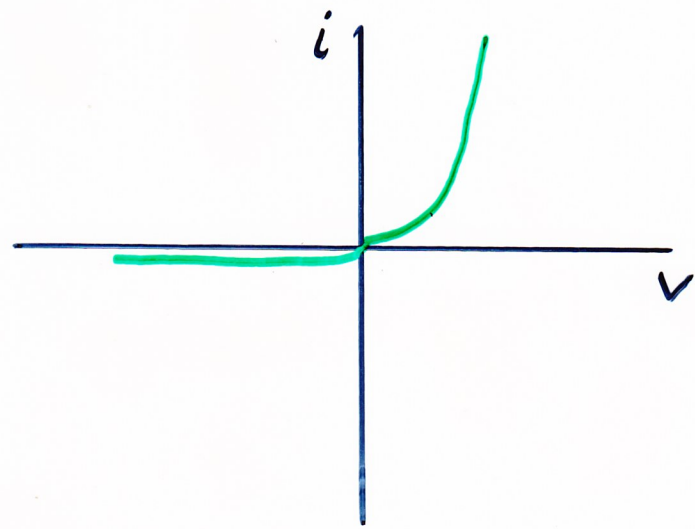
What are the power and current ?

$$I = V/R = 6/2 \times 10^3 = 3 \text{ mA}$$

$$P = V^2/R = 36/2 \times 10^3$$

$$= \underline{\underline{18 \text{ mW}}}$$

Note. Not all circuit elements obey Ohm's law. Consider a diode.

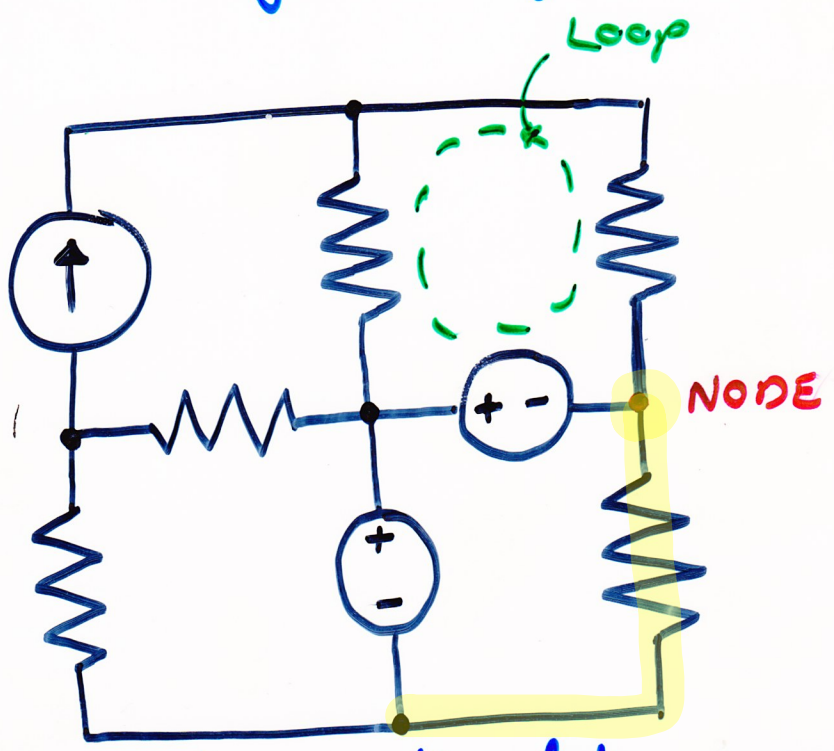


Kirchhoff's Laws.

So far looked at single resistor circuits. Need to be able to analyze multiple resistors in various arrangements. To aid us we can use two important laws known as Kirchhoff's laws.

(We make assumption that connecting wires have zero resistance).

First, some definitions of terms.



- NODE:** point of connection of two or more circuit elements
- LOOP:** any closed path through the circuit in which no node is encountered more than once.
- BRANCH:** portion of a circuit containing only a single element and the nodes at each end of the element.

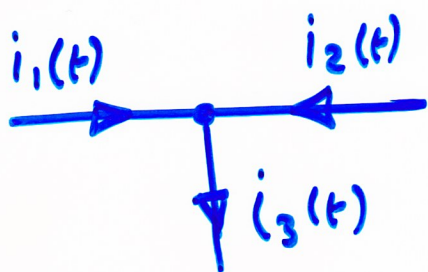
Kirchhoff's Current Law.

The algebraic sum of the currents entering any node is zero.

$$\sum_{j=1}^N i_j(t) = 0$$

$i_j(t)$ is the j th current entering the node through branch j .

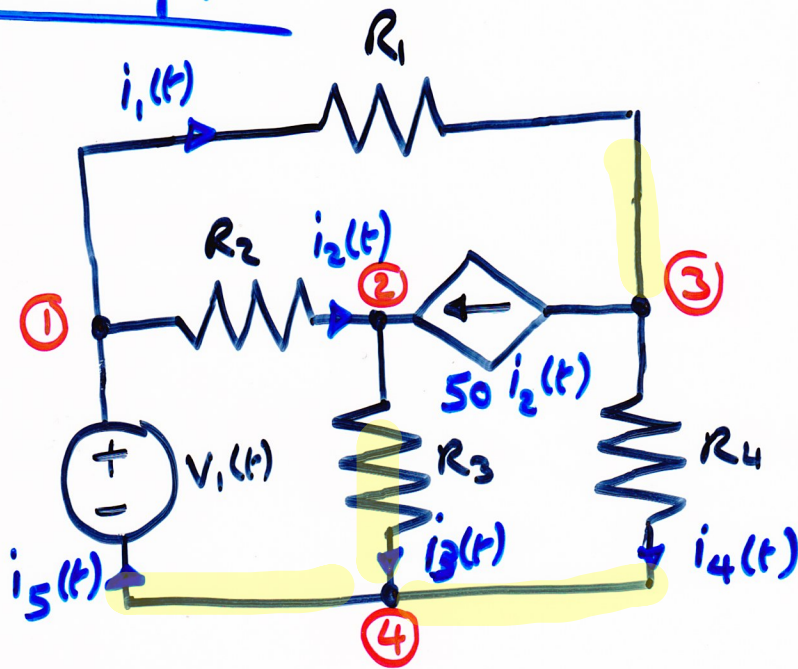
N is the number of branches.



$$i_1(t) + i_2(t) - i_3(t) = 0$$

N.B. KCL applies not just to a node but to any closed surface.

Example



Apply KCL to each node (in order)

$$i_1(t) + i_2(t) - i_5(t) = 0$$

Node 2 $-i_2(t) + i_3(t) - 50 i_2(t) = 0$

$$-i_1(t) + 50 i_2(t) + i_4(t) = 0$$

$$i_5(t) - i_3(t) - i_4(t) = 0$$

Kirchhoff's Voltage Law.

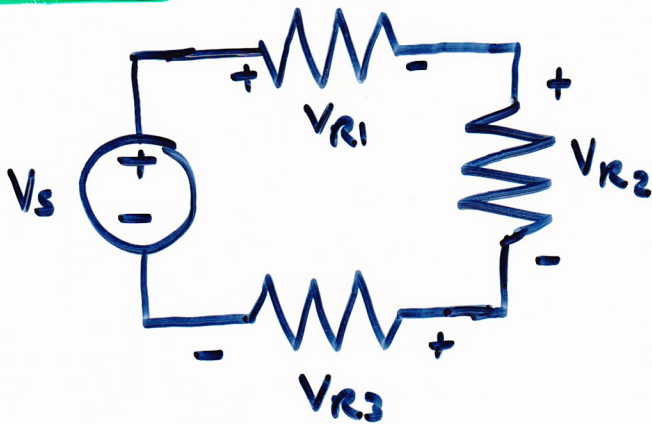
The algebraic sum of the voltages around any loop is zero.

$$\sum_{j=1}^N V_j(t) = 0$$

V_j voltage across j th branch (with proper reference direction)

In \oint loop containing N voltages.

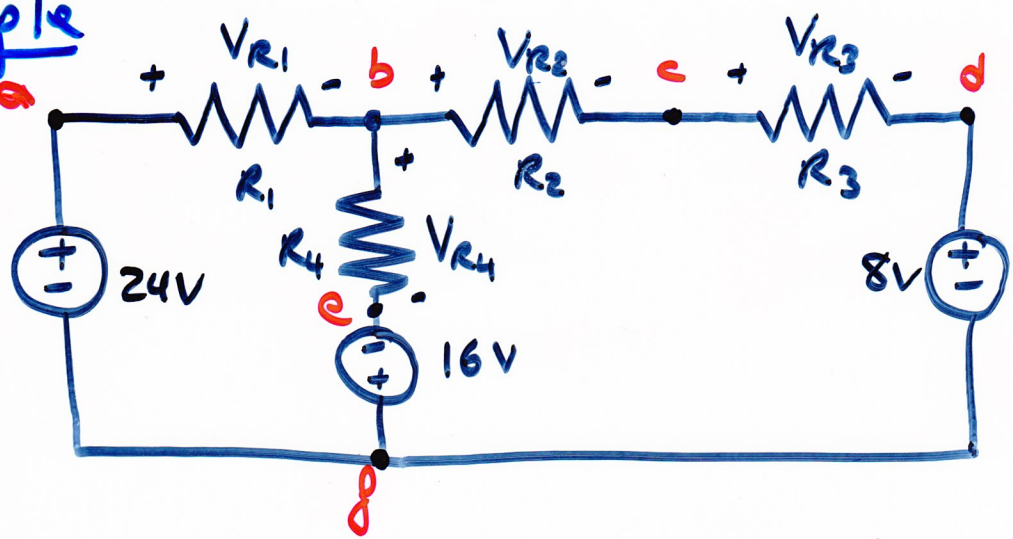
Example



$$-V_s + V_{R1} + V_{R2} + V_{R3} = 0$$

Example

(2.8) a
[2.10]



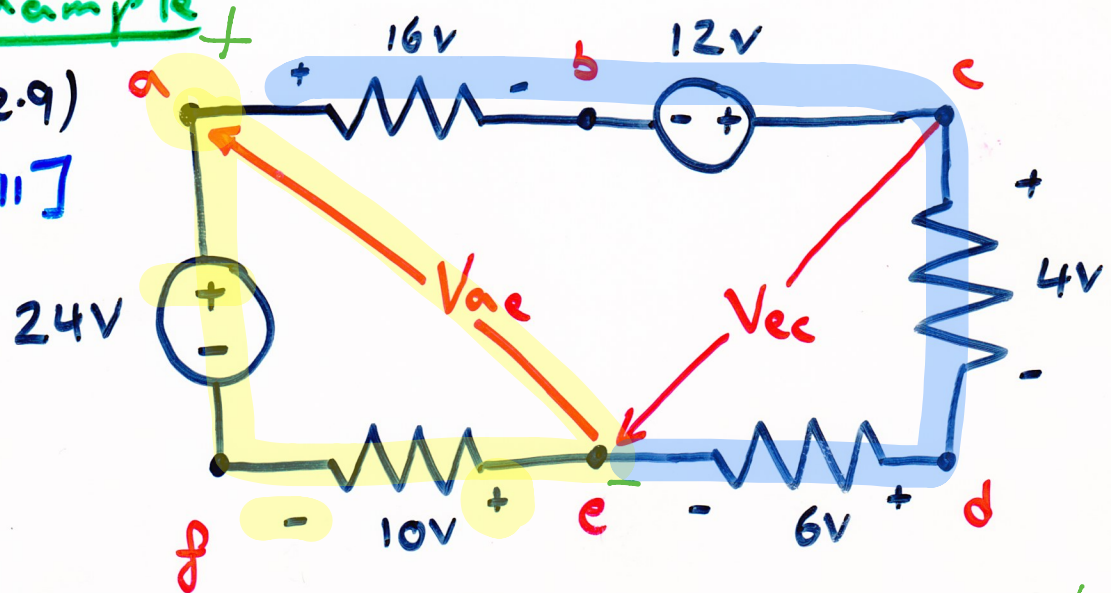
Loop abefa $V_{R1} + V_{R4} - 24 - 16 = 0$

Loop bedfeb $V_{R2} + V_{R3} + 8 + 16 - V_{R4} = 0$

Loop abcdfa

Example

(Eg 2.9)
[2.11]



Find V_{ae} and V_{ec} .

V_{ec} ?

$V_{ae} - 24V + 10V = 0$

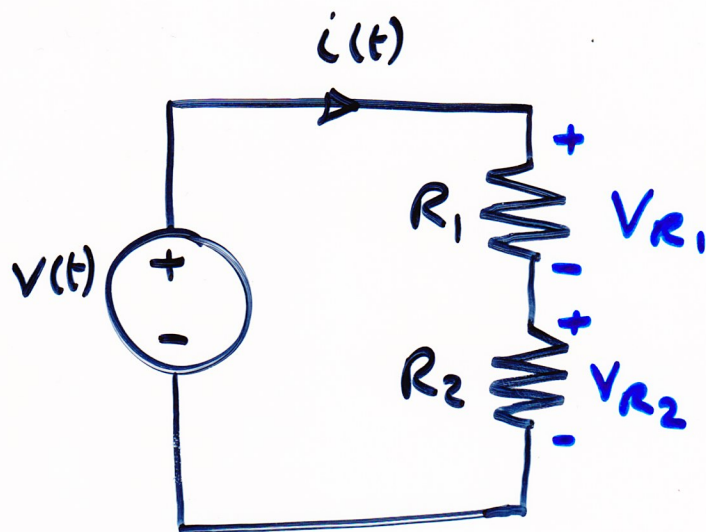
$V_{ae} = 24V - 10V$

$V_{ae} - 16V + 12V - 4V - 6V = 0$

Single Loop & Single Node Circuits.

Single Loop - Voltage Division.

Let us consider the following circuit and analyse it with what we have learned.



Can we obtain a relationship for V_{R1} & V_{R2} ?

Apply KVL

$$-V(t) + V_{R1} + V_{R2} = 0$$

$$V(t) = V_{R1} + V_{R2}$$

But

$$V_{R_1} = R_1 i(t) \quad (1)$$

$$V_{R_2} = R_2 i(t) \quad (2)$$

$$\therefore V(t) = R_1 i(t) + R_2 i(t) \quad (3)$$

$$\text{and } i(t) = \frac{V(t)}{R_1 + R_2} \quad (4)$$

Substituting (4) into (1) gives

$$V_{R_1} = R_1 \left(\frac{V(t)}{R_1 + R_2} \right)$$

Similarly

$$V_{R_2} = \frac{R_2}{R_1 + R_2} V(t)$$

So voltage is divided across the resistors in proportion to their resistance.

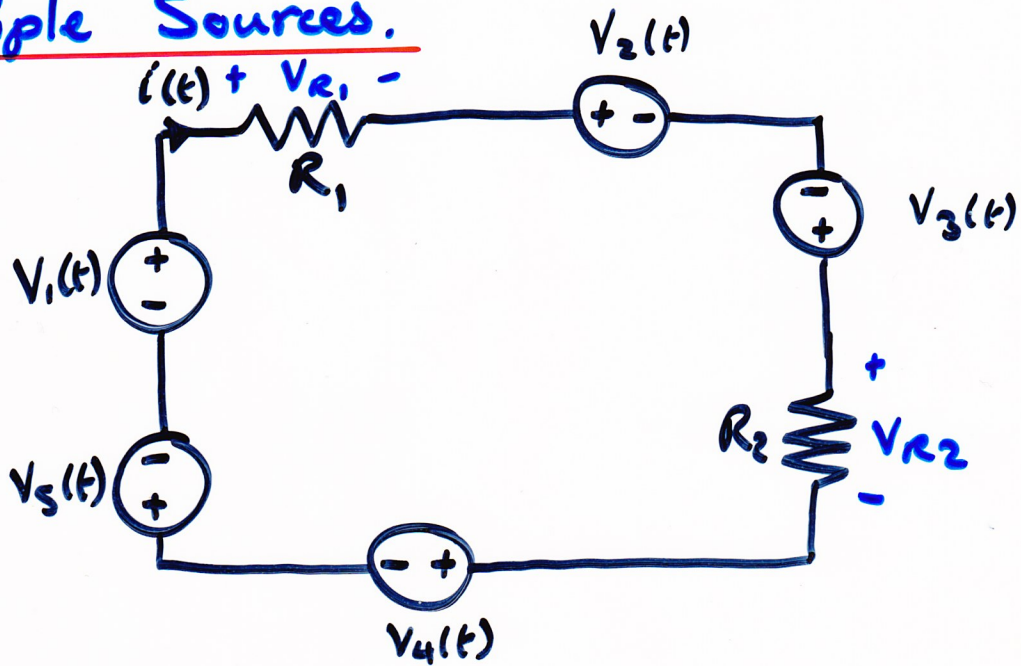
CHECK is KVL satisfied?

$$-V(t) + \frac{R_1}{R_1 + R_2} V(t) + \frac{R_2}{R_1 + R_2} V(t) = 0$$



What if we had multiple sources or multiple resistors?

Multiple Sources.



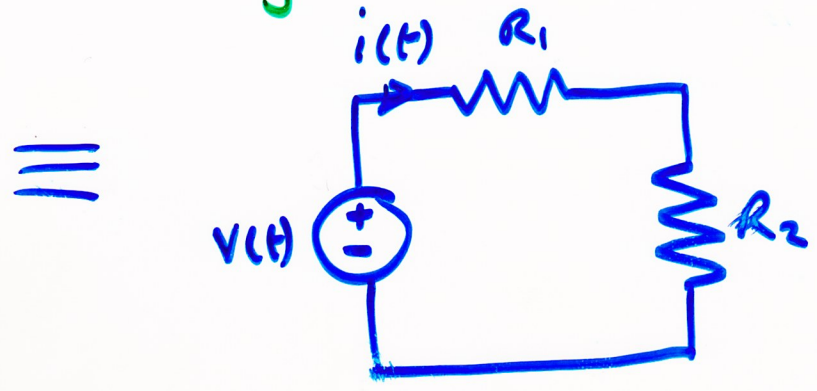
Apply KVL.

$$+V_{R1} + V_2(t) - V_3(t) + V_{R2} + V_4(t) + V_5(t) - V_1(t) = 0$$

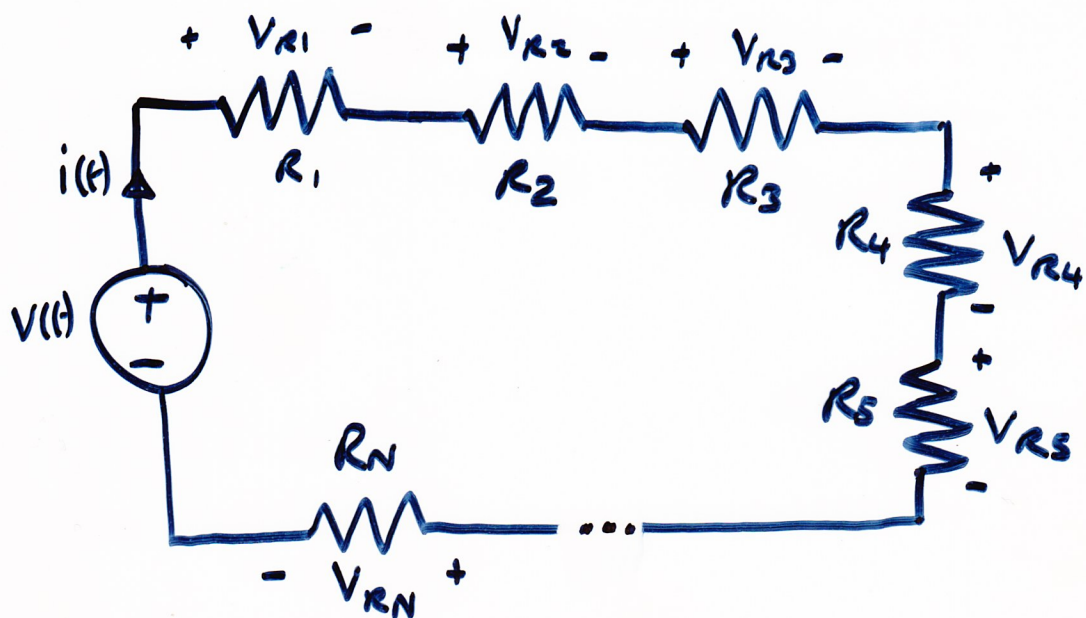
Rearranging and using Ohm's law.

$$\underline{i(R_1 + R_2)} = V_1(t) - V_5(t) - V_4(t) + V_3(t) - V_2(t) = \underline{v(t)}$$

Redrawing the circuit.



Multiple Resistors.



Apply KVL

$$\begin{aligned} V(t) &= V_{R1} + V_{R2} + \dots + V_{RN} \\ &= R_1 i(t) + R_2 i(t) + \dots + R_N i(t) \\ &\equiv R_S i(t) \end{aligned}$$

$$\underline{R_S = R_1 + R_2 + R_3 + \dots + R_N}$$

Also $i(t) = V(t) / R_S$

$$V_{Ri} = \frac{R_i}{R_S} V(t)$$