Lecture 1 Signals
ELEC2501

- Course outline
- Introduction to signals
- Definition of signals
- Signal types
- Periodic signals
- Describing periodic signals
- Examples
- Properties of signals
  - Instantaneous values
  - Peak values
  - Peak to peak
  - Average
  - Root mean squared (RMS)
- Examples

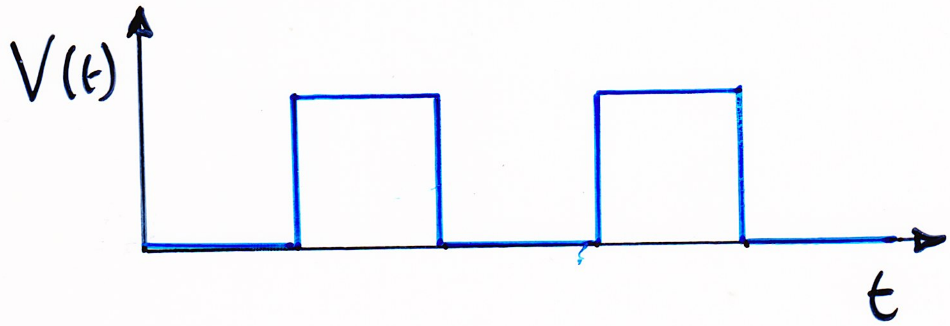
Definition of Signals

- What is a signal?

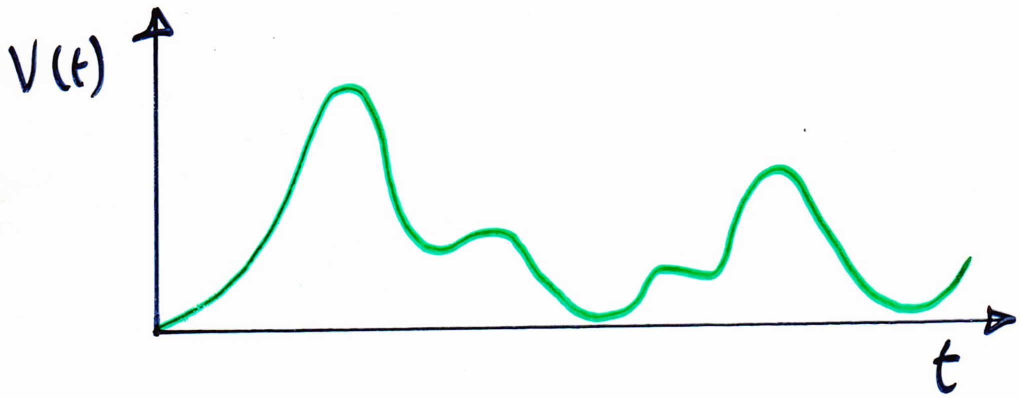
Characteristics

- Convey information
- Need a source
- Medium of propagation
- Destination

Signal types: Periodic & non-periodic

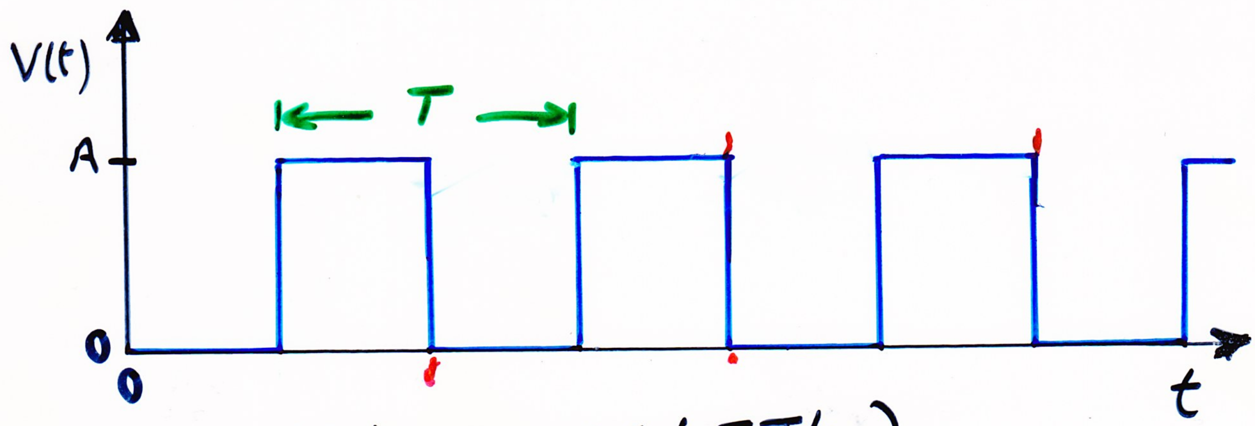


Periodic



Non-periodic
(aperiodic)

Considering periodic signal



$$V(t) = 0 \quad 0 \le t < T/2$$
$$= A \quad T/2 \le t < T$$
Repeats with period, T.

What is the frequency of the signal?

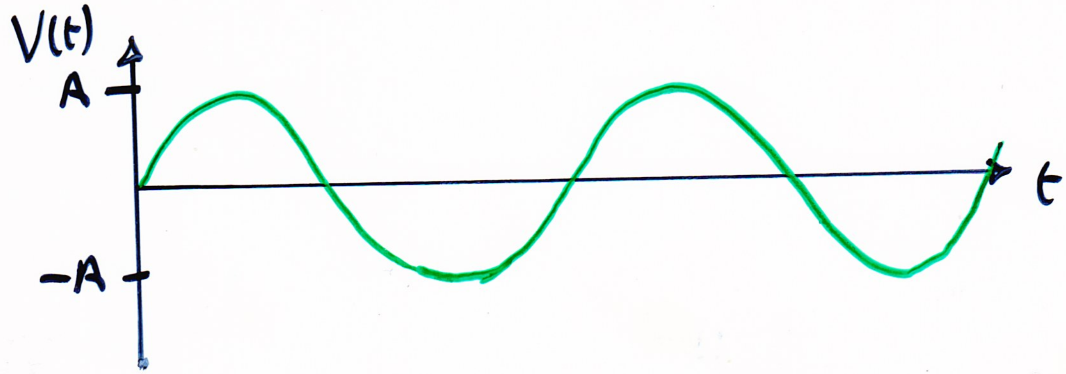
$$f = \frac{1}{T}$$

Units are Hertz, Hz, if T is in seconds.

What about a non-periodic signal?

Some other periodic signals

Sine Wave



$$v(t) = A\sin\omega t$$

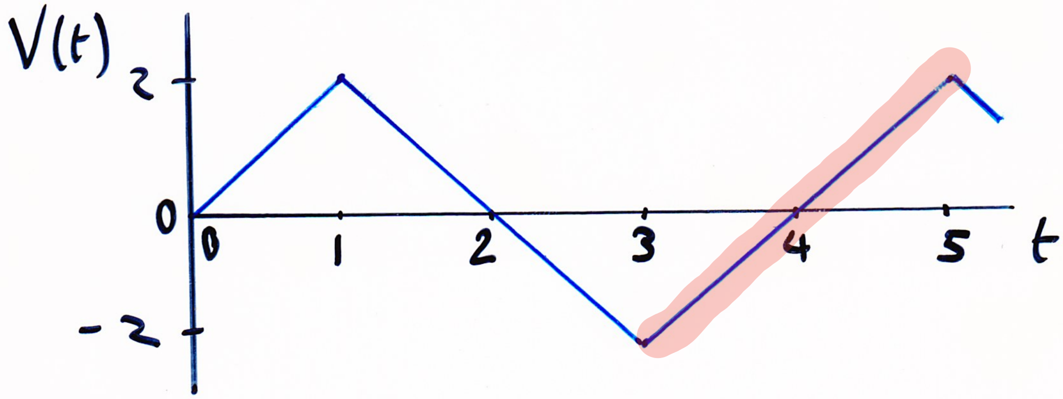
ω is angular frequency (rad/s)

$$\omega = 2\pi f$$

$$\therefore\ v(t) = A\sin 2\pi f t = A\sin\frac{2\pi t}{T}$$

Sine (cosine) wave is the simplest form of wave. All other periodic waves can be constructed from the addition of a series of sine (cosine) waves of different amplitudes, frequency and relative phase difference - see Fourier Series later in the course

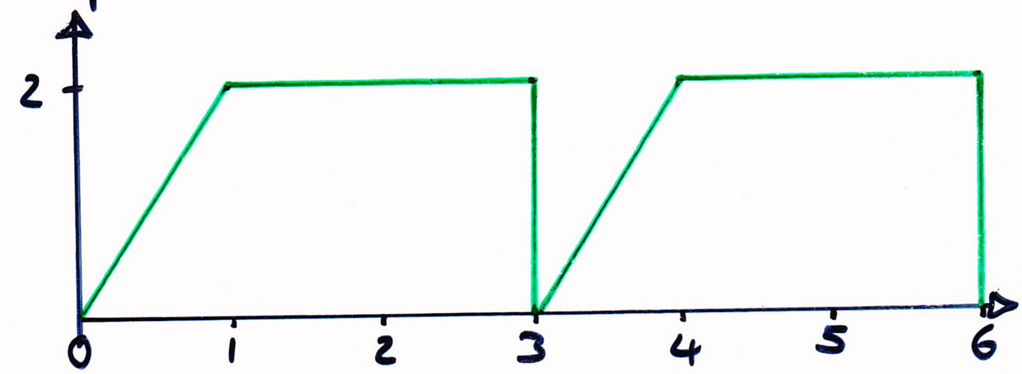
$$A\sin\omega t + \frac{A}{3}\sin 3\omega t + \frac{A}{5}\sin 5\omega t + \frac{A}{7}\sin 7\omega t + \ldots$$



$$V(t) = 2t \qquad 0 \le t < 1$$
$$= -2t + 4 \qquad 1 \le t < 3$$
$$= 2t - 8 \qquad 3 \le t < 5$$

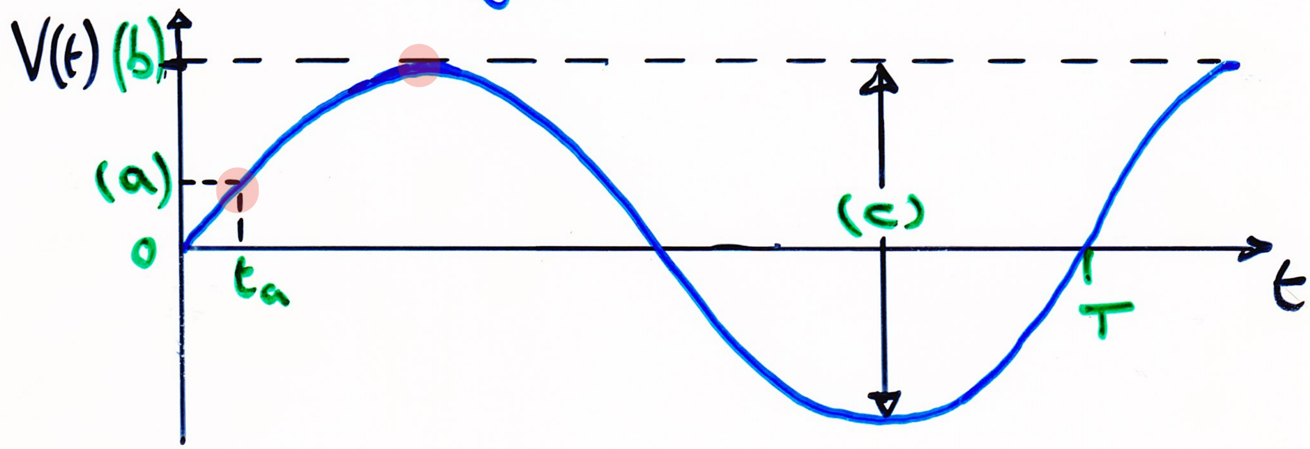
Period ?

Example



$$V(t) = 2t \qquad 0 \le t < 1$$
$$= 2 \qquad 1 \le t < 3$$

Signal Properties



(a) Instantaneous value - value at any given instant.

(b) Peak value - highest value.

(c) Peak to Peak value - difference between max. & min. values.

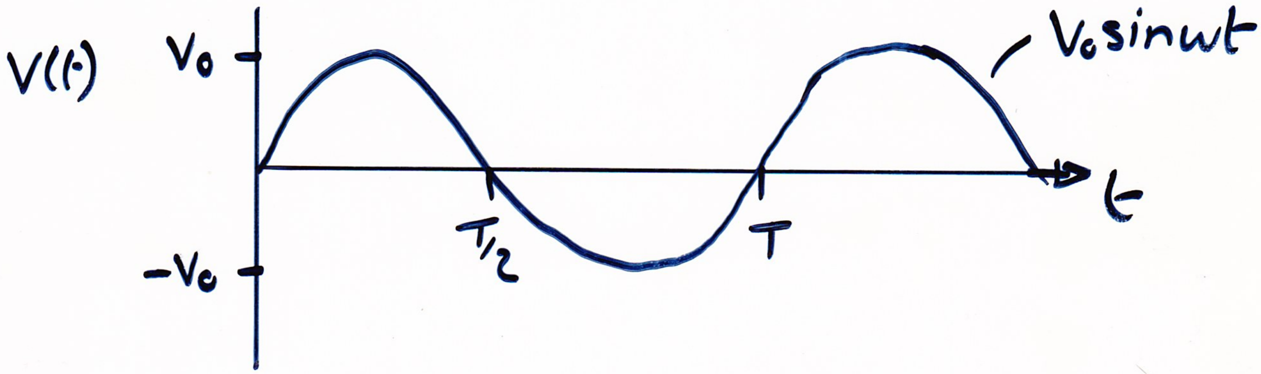
Average value

$$V_{avg} = \frac{1}{T}\int_0^T v(t)\,dt$$

Root Mean Square (RMS)

$$V_{RMS} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}$$

Examples.



$$V_{avg} = 0$$

Proof.

$$V_{avg} = \frac{1}{T}\int_0^T V_0 \sin \omega t \, dt$$

$$= \frac{V_0}{T}\int_0^T \sin \omega t \, dt$$

$$= \frac{V_0}{\omega T}\Big[-\cos \omega t\Big]_0^T$$

$$= \frac{V_0}{\omega T}\left(-\cos \frac{2\pi \cancel{T}}{\cancel{T}} - (-\cos 0)\right)$$

$$= \frac{V_0}{\omega T}\left(-1-(-1)\right)$$

$$= 0$$

$$V_{Rms} = \sqrt{\frac{1}{T}\int_0^T v^2(t)\,dt}$$

$$\frac{1}{T}\int_0^T v^2(t)\,dt = \frac{V_0^2}{T}\int_0^T \sin^2 \omega t\,dt = \frac{V_0^2}{T}\int_0^T \frac{1}{2}(1-\cos 2\omega t)\,dt$$

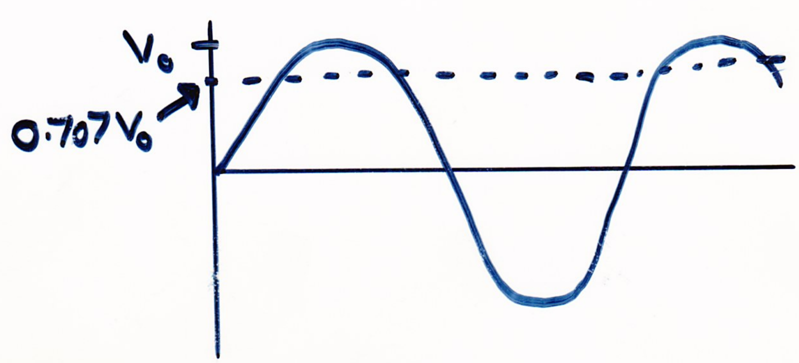
$$= \frac{V_0^2}{2T}\left[t - \frac{\sin 2\omega t}{2\omega}\right]_0^T$$

$$= \frac{V_0^2}{2T}\left(T - \frac{\sin 4\pi}{2\omega} - \left(0 - \frac{\sin 0}{2\omega}\right)\right)$$

$$= V_0^2/2$$

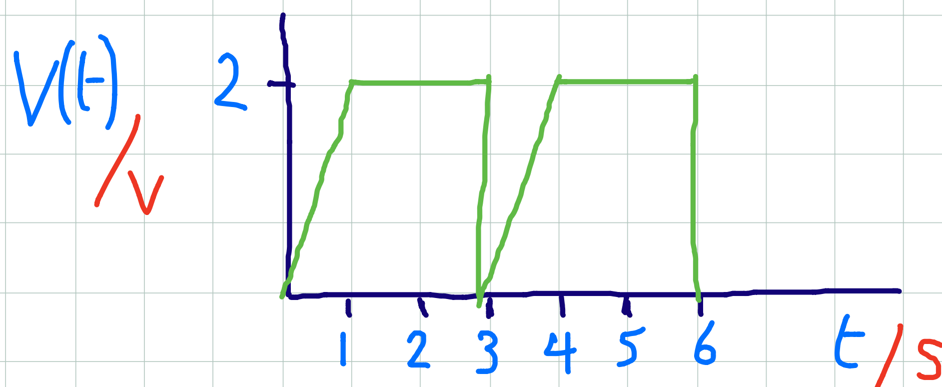
$$V_{Rms} = \sqrt{V_0^2/2} = V_0/\sqrt{2}$$

$$= 0.707\,V_0$$



Example 2

Consider the earlier example



$$V(t) = 2t \qquad 0 \leq t < 1$$

$$\quad\;\; = 2 \qquad\;\; 1 \leq t < 3$$

$$V_{avg} = \frac{1}{3}\left(\int_0^1 2t\,dt + \int_1^3 2\,dt \right)$$

$$= \frac{1}{3}\left\{ \left[t^2\right]_0^1 + \left[2t\right]_1^3 \right\}$$

$$= \frac{1}{3}\left(1 + 6 - 2\right)$$

$$V_{avg} = 5/3\,V$$

$$V_{rms} = \sqrt{\frac{1}{T}\int_0^T V^2(t)\,dt}$$

$$\int_0^T V^2(t)\,dt = \int_0^1 4t^2\,dt + \int_1^3 4\,dt$$

$$= \left[\frac{4}{3}t^3\right]_0^1 + \left[4t\right]_1^3$$

$$= \left(\frac{4}{3} + 12 - 4\right)$$

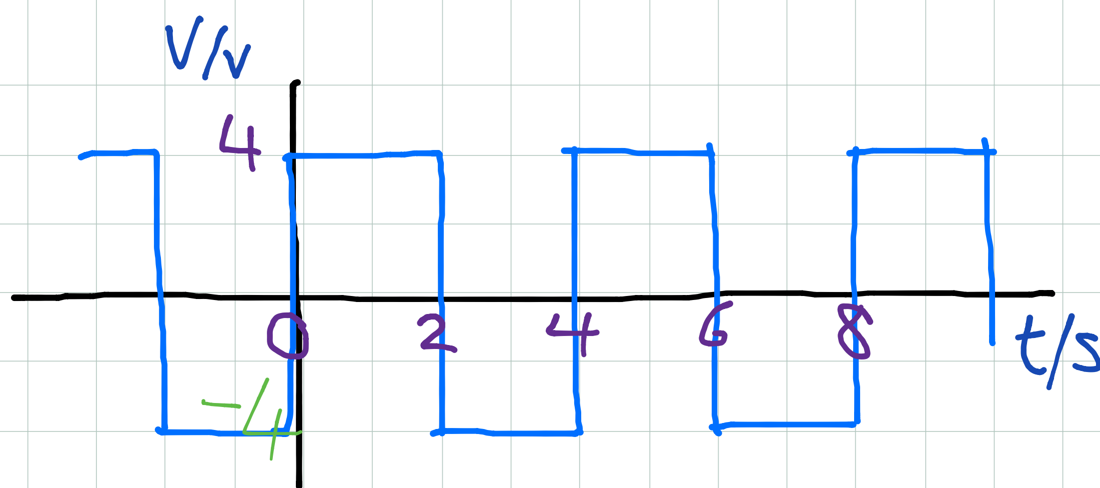
$$= 9\tfrac{1}{3}$$

$$\therefore\; V_{rms} = \sqrt{\tfrac{1}{3}\left(9\tfrac{1}{3}\right)}$$

$$= \sqrt{3\tfrac{1}{9}}$$

$V_{rms} = 1{\cdot}76\,V$

For you to try:



a) What is the frequency of the signal?

b) What is the V_{RMS}?