MATH 100 (A01 - A05 sections),
Midterm # 1 — September 19th, 2013
Time: 2h

Trivia: On September 19th, 1783 the brothers Montgolfier repeated their experiment of 4 June 1783, in the presence of Louis XVI at Versailles. At one o'clock the crowd went wild as the hot air balloon soared with the first passengers: a rooster, a sheep, and a duck.

Last name: Solutions

First name: Instructor

Student number: A

Tutorial section number: __________

Questions	Score	Out of
2 to 15		28
16		8
17		6
18		3
Total		**45**

- The only calculators allowed on any examination are Sharp EL-510R and Sharp EL-510RNB.
- This test consists of 17 questions (numbered 2 through 18) and has 12 pages (including this cover).
  - Questions 2 through 15 are multiple-choice. **Enter your final answer in the bubble sheet and mark them in this paper as well. You need to show your work for all answers, as we may disallow any answer which is not properly justified.**
  - Questions 16 through 18 are long-answer. Write your full answer in this booklet as indicated.
- For the multiple-choice questions, select the numerical answer closest to yours. If the answer is equidistant from two nearest choices, select the largest of the two choices.
- Before starting your test enter your name, student number, and tutorial section number on this page and on the bubble sheet. **On the name field on your bubble sheet, enter your last name as one word, followed by one blank space, followed by your first name as one word, followed by nothing else.**
- At the end of your test, turn in both this booklet and the bubble sheet.
- Enter "A" as your answer to Question 1 now.

1. **Enter "A" as your answer to Question 1 now.**

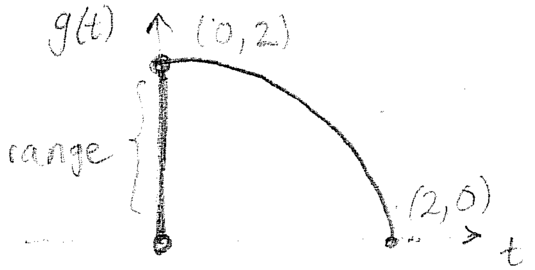
2. *[2 points]* Solve the equation $e^x = 1 + 6e^{-x}$ for x.
If there are multiple solutions for x, select the smallest one.

(A) -3.0; (B) -2.1; (C) -1.6; (D) -1.1; (E) -0.5;
(F) 0.5; (G) 1.1; (H) 1.6; (I) 2.1; (J) 3.0

$$e^x - 1 - 6e^{-x} = 0$$
$$(e^x)^2 - e^x - 6 = 0$$
$$(e^x - 3)(e^x + 2) = 0$$
$$e^x = 3 \text{ or } \cancel{-2}$$
$$x = \ln 3 \approx 1.10 \quad (\text{since } \ln(-2) \text{ is undefined})$$

3. *[2 points]* Determine the range of the function $g(t) = \sqrt{4 - t^2}$.

(A) $(-4, 4)$; (B) $(-2, 2)$; (C) $(0, 4)$; (D) $(0, 2)$; (E) $(-4, 0)$;
(F) $[-4, 4]$; (G) $[-2, 2]$; (H) $[0, 4]$; (I) $[0, 2]$; (J) $[-4, 0]$;



first quadrant of the circle of radius 2 centered at (0,0)

4. *[2 points]* From the listed choices, select the closest approximation to the value of $\log_4 \frac{1}{10}$

(A) -2.3; (B) -1.6; (C) -1.3; (D) -1.0; (E) -0.6;
(F) 0.6; (G) 1.0; (H) 1.3; (I) 1.6; (J) 2.3

$$\log_4 \frac{1}{10} = \frac{\ln\left(\frac{1}{10}\right)}{\ln 4} = \frac{-\ln 10}{\ln 4} \approx -1.66$$

5. *[2 points]* Find the equation of the line that passes through the point $(2, 1)$, and has y–intercept equal to 9.

(A) $y = -2x + 1$; (B) $y = 9x - 17$; (C) $y = -\frac{2}{9}$; (D) $y = \frac{1}{2}x$;

(E) $y = \frac{9}{2}x - 1$; (F) $y = -4x + 9$; (G) $y = 2x + 9$; (H) $y = -9x$;

(I) $y = -2x + 2$; (J) $y = -\frac{1}{4}x + 2$.

y-intercept

$y = mx + 9$

$(x, y) = (2, 1)$ satisfies this equation

so $1 = m \cdot 2 + 9$

$-8 = 2m$

$m = -4$

so equation is $y = -4x + 9$

6. *[2 points]* Calculate $\lim_{x \to 1+} \frac{3}{x - 1}$.

(A) $-\infty$; (B) -7; (C) -5; (D) -3; (E) 0;
(F) 3; (G) 5; (H) 7; (I) $+\infty$;
(J) The left-side limit is different from the right-side limit.

$x - 1$ is positive and small (in absolute value) for $x \to 1^+$

so $\frac{3}{x-1} \to +\infty$

7. *[2 points]* Solve $\sqrt{x+4}+\sqrt{x-1}=5$. If there are multiple solutions, select the largest solution of all solutions. From the multiple choice options select the answer closest to yours.

(A) 2.0 (B) 2.4 (C) 2.8 (D) 3.3 (E) 3.7

(F) 4.2 (G) 4.6 (H) 5.1 (I) 5.5 (J) 6.0

$$\sqrt{x+4} = 5-\sqrt{x-1}$$
$$x+4 = (5-\sqrt{x-1})^2$$
$$\cancel{x}+4 = 25-10\sqrt{x-1}+\cancel{x}-1$$
$$-20 = -10\sqrt{x-1}$$
$$\sqrt{x-1} = 2$$
$$x-1 = 4$$
$$x = 5$$

8. *[2 points]* Calculate $\lim_{b\to 3} \sqrt[3]{10-b^2}$.

(A) $-\infty$; (B) -2; (C) -1; (D) -0.5; (E) 0;
(F) 0.5; (G) 1; (H) 2; (I) $+\infty$;
(J) The left-side limit is different from the right-side limit.

$$\lim_{b\to 3} \sqrt[3]{10-b^2} = \sqrt[3]{10-3^2} = \sqrt[3]{10-9}$$
$$= \sqrt[3]{1}$$
$$= 1$$

9. *[2 points]* Calculate $\lim_{x \to \pi/4} \dfrac{\sin x}{x}$.

(A) $+\infty$; (B) 1; (C) 0.9; (D) 0.8; (E) 0;
(F) -0.8; (G) -0.9; (H) -1; (I) $-\infty$;

(J) The left-side limit is different from the right-side limit.

$$\lim_{x \to \pi/4} \frac{\sin x}{x} = \frac{\sin\left(\frac{\pi}{4}\right)}{\frac{\pi}{4}}$$

$$= \frac{1/\sqrt{2}}{\pi/4}$$

$$= \frac{4}{\pi\sqrt{2}} \approx 0.900$$

10. *[2 points]* Calculate $\lim_{x \to 0} \dfrac{\sin(x)}{5x^3 - 4x}$.

(A) -1.25; (B) -1.00; (C) -0.75; (D) -0.50; (E) -0.25;
(F) 0.25; (G) 0.50; (H) 0.75; (I) 1.00;

(J) Does not exist.

$$\lim_{x \to 0} \frac{\sin x}{5x^3 - 4x} = \lim_{x \to 0} \frac{\sin x}{x(5x^2 - 4)}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1}{5x^2 - 4}$$

$$= 1 \cdot \frac{1}{5 \cdot 0^2 - 4}$$

$$= \frac{-1}{4}$$

11. *[2 points]* Calculate $\lim_{x\to 2} \frac{1-2/x}{1-4/x^2}$.

(A) $-\infty$; (B) -2; (C) -1; (D) -0.5; (E) 0;
(F) 0.5; (G) 1; (H) 2; (I) $+\infty$;
(J) The left-side limit is different from the right-side limit.

$$\lim_{x\to 2} \frac{1-2/x}{1-4/x^2} = \lim_{x\to 2} \frac{x^2-2x}{x^2-4}$$

$$= \lim_{x\to 2} \frac{x\cancel{(x-2)}}{\cancel{(x-2)}(x+2)}$$

$$= \lim_{x\to 2} \frac{x}{x+2}$$

$$= \frac{2}{4} = \frac{1}{2}$$

12. *[2 points]* Calculate $\lim_{x\to 4} \frac{|4-x|}{(4-x)^2}$.

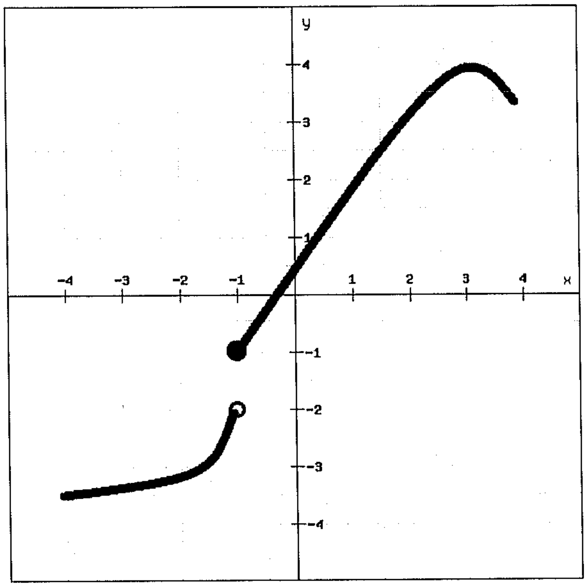
(A) $-\infty$; (B) -2; (C) -1; (D) -0.5; (E) 0;
(F) 0.5; (G) 1; (H) 2; (I) $+\infty$;
(J) The left-side limit is different from the right-side limit.

$$(4-x)^2 = |4-x|^2$$

so $$\lim_{x\to 4} \frac{|4-x|}{(4-x)^2} = \lim_{x\to 4} \frac{|4-x|}{|4-x|^2}$$

$$= \lim_{x\to 4} \frac{1}{|4-x|}$$

For questions 13–15, consider the function g defined by the graph below:



$y = g(x)$

13. *[2 points]* Calculate $\lim_{x \to -1^-} g(x)$.

(A) -4; (B) -3; (C) -2; (D) -1; (E) 0;
(F) 1; (G) 2; (H) 3; (I) 4; (J) Does not exist

14. *[2 points]* Calculate $\lim_{x \to -1} g(x^2)$. $= g\left(\lim_{x \to -1} x^2\right) = g(1) = 2$

(A) -4; (B) -3; (C) -2; (D) -1; (E) 0;
(F) 1; (G) 2; (H) 3; (I) 4; (J) Does not exist

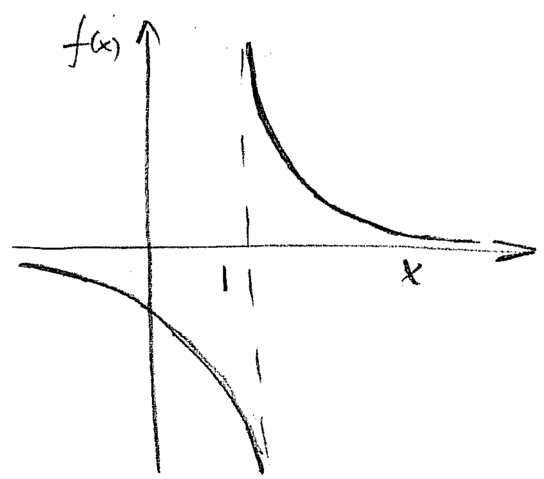
15. *[2 points]* Calculate $\lim_{x \to -1^+} [g(x)]^2$. $= \left[\lim_{x \to -1^+} g(x)\right]^2 = (-1)^2 = 1$

(A) -4; (B) -3; (C) -2; (D) -1; (E) 0;
(F) 1; (G) 2; (H) 3; (I) 4; (J) Does not exist

16. *[8 points].* In each of the following questions, we ask you to give us one example of a function satisfying a certain property. There are many different correct answers to each question (including some easy ones). **Write your final answer to each question in the box and sketch its graph in the space underneath.**

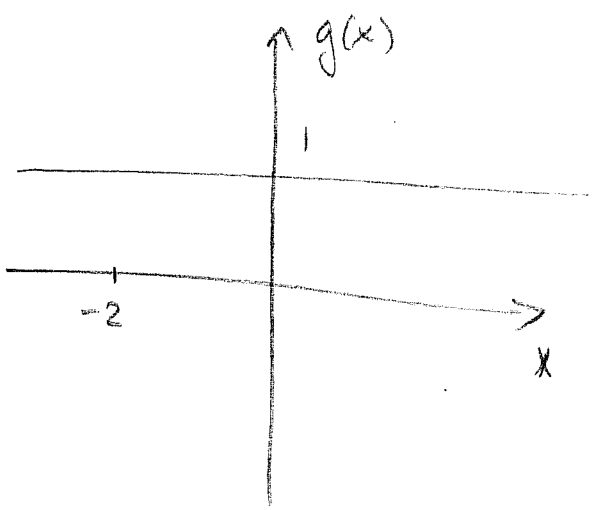
(a) Give us a function f that is continuous at every real number, except at $x = 1$, where it has a discontinuity that is not removable.

Your answer: $f(x) = \dfrac{1}{x-1}$



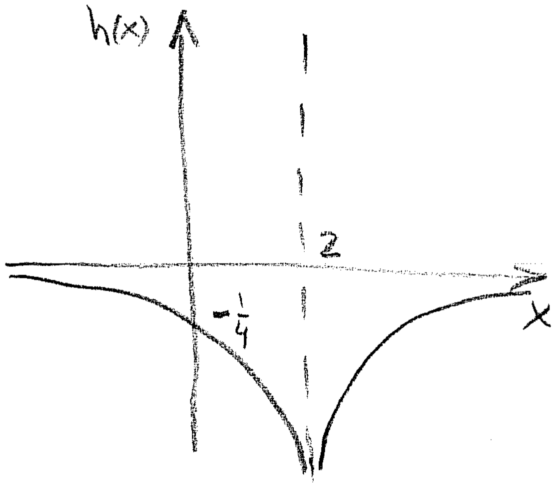
(b) Give us a function g such that it is continuous at every real number and has $\lim_{x \to -2} g(x) = 1$.

Your answer: $g(x) = 1$



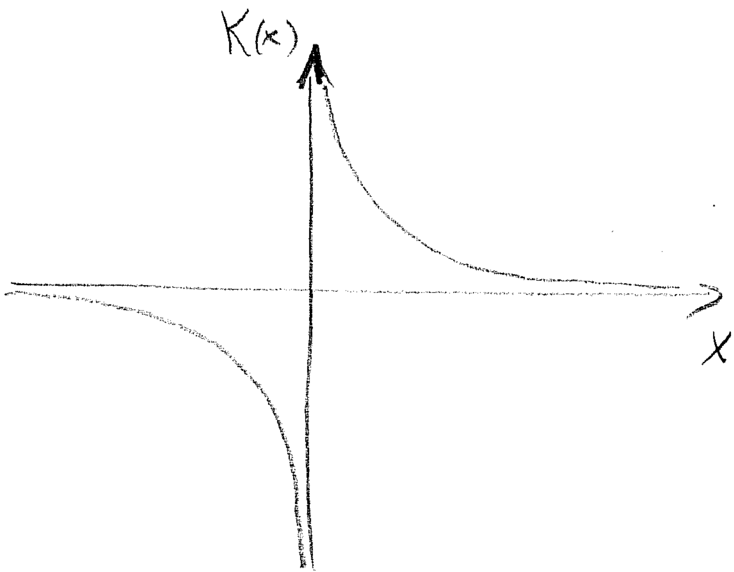
(c) Give us a function h such that $\lim_{x \to 2} h(x) = -\infty$.

Your answer: $h(x) = -\dfrac{1}{(x-2)^2}$



(d) Give us a function K such that $\lim_{x \to 0^+} K(x) = +\infty$, and $\lim_{x \to 0} K(x) = -\infty$.

Your answer: $K(x) = \dfrac{1}{x}$



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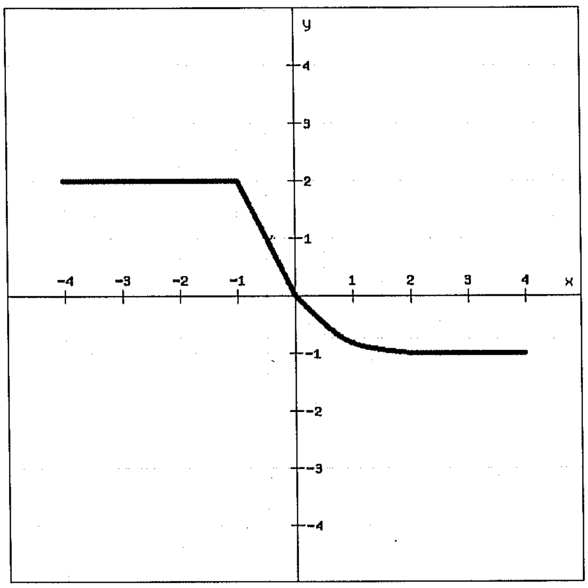
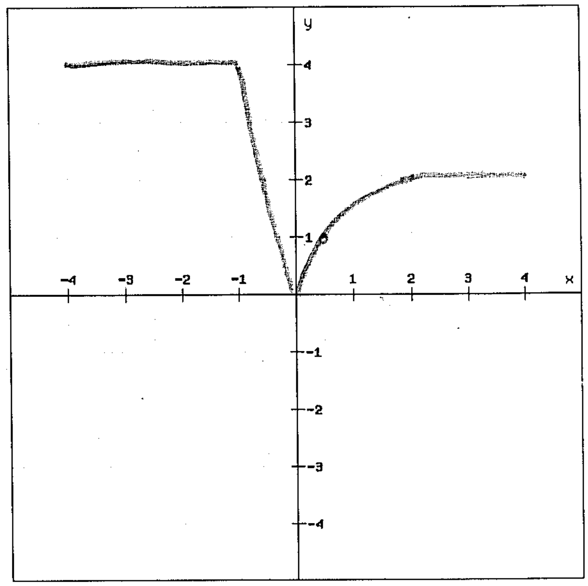
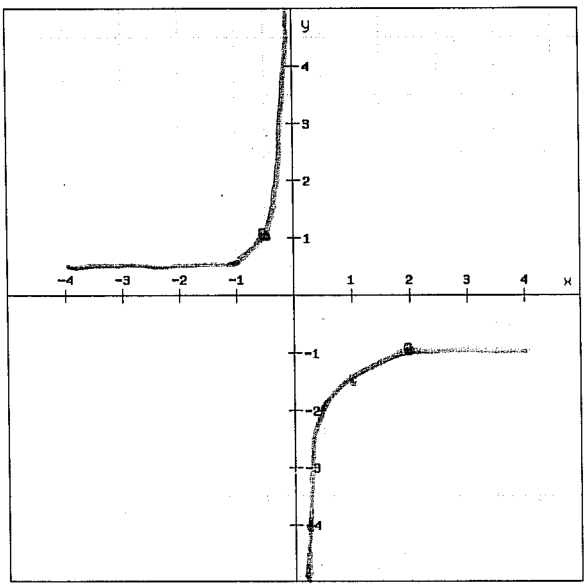
17. *[6 points]* The first grid contains the graph of a function h. In the other three grids draw the graphs indicated.



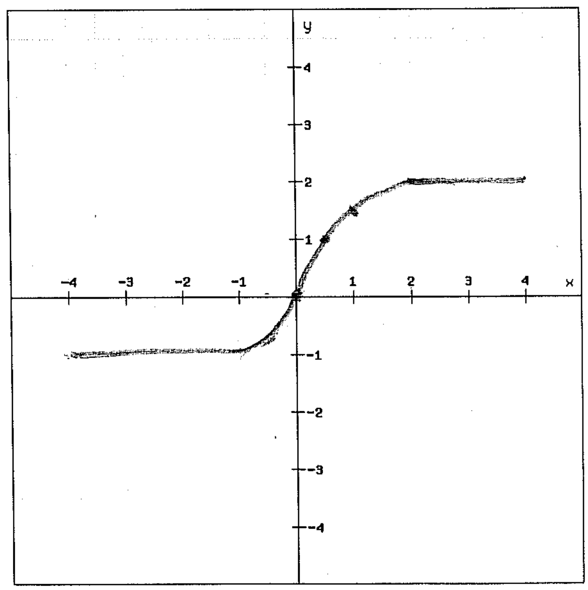
$y = h(x)$



$y = |2h(x)|$



$y = \dfrac{1}{h(x)}$



$y = h(h(x))$

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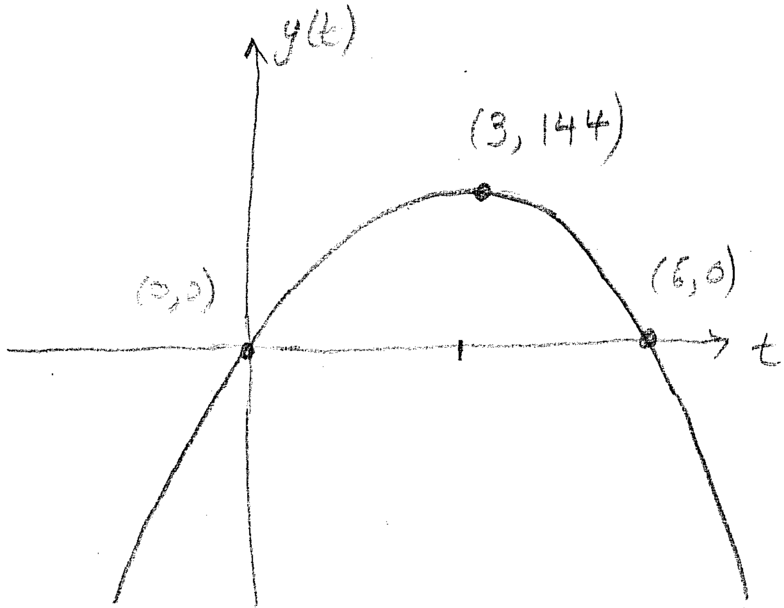
18. *[3 points]* If a ball is thrown straight upward with initial velocity 96 ft/s, then is height t seconds later is $y(t) = 96t - 16t^2$ feet.

(a) Sketch the graph of $y(t)$.

(b) Determine the slope of the secant line that passes through points (2, 128) and (5, 80).

(c) Determine the slope of the tangent line that passes through point (2, 128). (Note: For this part of the question you must use limit. Do not use the differentation rules.)

(a) $y(t) = 16(6t - t^2)$
$= -16(t^2 - 6t)$
$= -16[(t-3)^2 - 9]$
$= -16(t-3)^2 + 144$



(b) slope (of secant line) $= \dfrac{\Delta y}{\Delta x} = \dfrac{128-80}{2-5} = \dfrac{48}{-3} = \boxed{-16 \ (ft/s)}$

(c) $m(2) = \lim\limits_{h \to 0} \dfrac{y(2+h) - y(2)}{h}$

$= \lim\limits_{h \to 0} \dfrac{[96(2+h) - 16(2+h)^2] - 128}{h}$

$= \lim\limits_{h \to 0} \dfrac{[192 + 96h - 16(4 + 4h + h^2)] - 128}{h}$

$= \lim\limits_{h \to 0} \dfrac{-16h^2 + 32h}{h} = \lim\limits_{h \to 0} (-16h + 32)$

$= \boxed{32 \ (ft/s)}$