Problem 1: The solution set is $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \text{Span} \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$. It's not a subspace.

Problem 2: No, because $2\begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$ and $2\begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix} \neq \begin{pmatrix} 1 \\ 6 \\ 3 \end{pmatrix}$.

Problem 3: $\det A = -16$

Problem 4: There is a unique matrix satisfying the desired properties, as Av_1 is equal to $2v_1$, Av_2 is equal to $5v_2$ and v_1, v_2 form a basis of $\mathbb{R}^2$. This matrix is equal to PDP^{-1}, where P is the matrix $\begin{pmatrix} 1 & 1 \\ 2 & 5 \end{pmatrix}$ and D is equal to $\begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix}$. After computation, we find $A = \begin{pmatrix} 0 & 1 \\ -10 & 7 \end{pmatrix}$

Problem 5: No, because 1 is an eigenvalue with multiplicity 2 and the corresponding eigenspace is of dimension 1.

Problem 6: By using the projection formula and the fact that v_1, v_2 and v_3 form an orthogonal basis of $\mathbb{R}^3$, we find :

$$w_0 = \frac{9}{14}v_1 + \frac{1}{14}v_2$$

It follows that

$$w_n = A^n w_0 = \frac{9}{14}v_1 + \frac{1}{14(2^n)}v_2$$

which, in turn, gives the formulas for a_n, b_n and c_n.

Problem 7: $W^\perp$ is the line spanned by the vector $\begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}$. The vector of W closest to y is $\begin{pmatrix} 0 \\ 2 \\ -1 \end{pmatrix}$.

Problem 8: The desired parabola is

$$y = \frac{35 - 21x^2}{26}.$$

Problem 9: (B), (E), (C).

Problem 10: T, F, T , F , F , T.