MATH 1119B: Test 3

Monday, October 31, 2011, 09:35-10:25

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Total marks: / 45

Authorized Devices: Non-programmable calculators. No lame costumes allowed.

Name: Student number:

[14] 1. Let $C = \begin{bmatrix} -1 & 2 & 6 & 5 & 4 \\ -1 & 4 & 12 & -5 & 4 \\ 2 & 2 & 1 & -2 & 8 \\ 0 & 2 & 6 & -11 & 0 \end{bmatrix}$.

(a) Give a basis of Col(C). 7 (5 for row reducing, 2 for answer)

(b) Give two bases for Row(C). Your second basis must contain at least 8 zeroes between all of the vectors. 4

1 (c) What is the rank of C?

2 (d) What is dim(null(C))?

a) $C \sim \begin{bmatrix} -1 & 2 & 6 & 5 & 4 \\ 0 & 2 & 6 & -10 & 0 \\ 0 & 0 & -5 & 38 & 16 \\ 0 & 0 & 0 & -1 & 0 \end{bmatrix}$ $\therefore$ basis of Col(C) is $\left\{ \begin{bmatrix} -1 \\ -1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 6 \\ 12 \\ 1 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ -5 \\ -2 \\ -11 \end{bmatrix} \right\}$

pivot columns

b) pivots in rows 1-4, so these rows form a basis of Row C

also $\begin{bmatrix} -1 \\ 2 \\ 6 \\ 5 \\ 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ 6 \\ -10 \\ 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -5 \\ 38 \\ 16 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$

c) Rank C = dim Col C = 4

d) rk C + dim Null C = 5 $\therefore$ dim Null C = 1.

[8] 2. Let $A = \begin{bmatrix} -1 & -2 & -1 & 3 \\ 2 & 0 & 2 & 2 \\ 6 & -8 & -2 & 14 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 4 & 4 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

(a) Give a basis of null(A).

2 (b) Verify the Rank Theorem. 6, 4 for row reducing, 2 for basis

$A \sim \begin{bmatrix} 1 & -2 & -1 & 3 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

b) Rk(A) = 2

dim (Null (A)) = 2

2+2 = # cols of A = 4.

$\therefore$ basis of Null(A) = $\left\{ \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

[5] 3. Indicate if the following statements are TRUE or FALSE.

T / F [a] The set $\{v_1, v_2, \ldots, v_6\}$ is linearly independent if and only if $0v_1 + 0v_2 + \cdots + 0v_6 = 0$.

T / F [b] If $x = cy$ for some scalar c, then $\dim(\text{Span}(x, y)) = 1$.

T / F [c] Two linearly independent vectors in $\mathbb{R}^3$ span $\mathbb{R}^2$.

T / ~~F~~ [d] Let $A = \begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}$, and $u = \begin{bmatrix} +2 \\ +1 \end{bmatrix}$. Then u is in null(A). Note the change.

T / F [e] Elements of Col(A) can be taken from any echelon form of the matrix A.

[6] 4. Determine **by inspection** if the following sets are linearly independent or linearly dependent and **explain your reasoning.** (2 each)

(a) $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, [0\ \ 0\ \ 0\ \ 0]^T, [0\ \ 2\ \ 0\ \ 1]^T,$ (b) $\begin{bmatrix} 2 \\ 1 \\ -3 \\ 2/3 \end{bmatrix}, \begin{bmatrix} 1 \\ 1/2 \\ 3/2 \\ 1/3 \end{bmatrix}$. (c) $\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 12 \\ -13 \end{bmatrix}$.

(a) l.d

b/c zero vector

(b) l.i

$-3 \neq 2(3/2)$

so not scalar multiples

(c) l.d

TOO many vectors

[12] 5. Let $w_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \\ 4 \end{bmatrix}, w_2 = \begin{bmatrix} 2 \\ 4 \\ 2 \\ 4 \end{bmatrix}, w_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \\ -2 \end{bmatrix}$ and let $W = \text{Span}(w_1, w_2, w_3)$. Let $b = \begin{bmatrix} 2 \\ 4 \\ 4 \\ 4 \end{bmatrix}$.

1 (a) How many vectors are in W? ∞'ly many

(b) Let A be the matrix with w_1, w_2, w_3 as columns. Is $b \in \text{Col}(A)$? Is $b \in W$?

6

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 2 & 4 & 3 & 4 \\ -1 & 2 & 1 & 4 \\ 4 & 4 & 2 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & 4 & 2 & 6 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

Inconsistent, so

$b \notin \text{Col}(A) = W$.

(c) Find the dimension of Row(A). (Hint: Can you re-use your computations from (c)?))

2

dim Row (A) = # pivots in $A = 3$

(consider the coefficient part of c))

(d) Give two sets of vectors that span $\mathbb{R}^4$.

$\{w_1, w_2, w_3, b\}$, $\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}\right\}$

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