$U_{1\to2} =$ area under the curve

$= -\frac{1}{2}(2.5\times10^{8}\times0.1) + \frac{1}{2}(0.05\times1.25\times10^{2})$

Class Assigned Problems #1

$m = 800\,kg$ $\quad V_c = 60\,km/h$

$V_1 = 60\,km/h$ $\quad V_2 = 0$

$\to 16.67\,m/s$

$T_1 + U_{1\to2} = \cancel{T_2} \to 0$

$T_1 = \frac{1}{2} m v_1^2$

$= \frac{1}{2}\, 800 \times (16.67)^2\, m/s$

$= 111155.56$

$0 < s < 0.75$

Area under the graph

now let $0.75m < s < 3m$

$= \int_0^{0.75} F\,ds + \int_{0.75}^{s} F\,ds$

$= 40[s]_0^{0.75} + 60[s]_{0.75}^{s}$

$= 40[0.75] + 60[s - 0.75]$

$= 40\times0.75 + 60s - 45$

$= 30 + 60s - 45$

$111.2\,kN = 30 + 60s - 45$

$111.2 + 15 = 60s$ $\qquad s = 2.10\ m$

velocity ω (omega)
Since dθ occurs during an instant of time dt, then
$\circlearrowleft+ \quad \omega = \frac{d\theta}{dt}$, magnitude

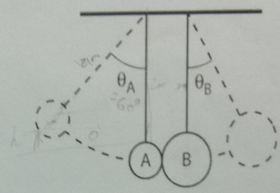
MAAE 2101 DYNAMICS | **CLASS ASSIGNED PROBLEMS** | **PROBLEM #3**

Last Name: Patel *First Name:* Rikky *PA Section* D1

Two spheres are hanging from cords as shown. The distance from the ceiling to the center of each sphere is 2m, and the coefficient of restitution is 0.75. If sphere "A" ($m_A = 2kg$) is drawn back 60° and released *from rest*, determine:

(a) The maximum angle, θ_B, that sphere "B" ($m_B = 3kg$) will swing through as a result of the impact.

(b) The angle, θ_A, that sphere "A" will rebound as a result of the impact.



Given: $m_A = 2kg$, $\theta_A = 60°$, $m_B = 3kg$, $e = 0.75$
$d = 2m$

a) Conservation of mechanical Energy:

$$T_1 + V_{g_1} = T_2 + V_{g_2}$$

$$\cancel{\tfrac{1}{2} m_A (V_A)_1^2}^{\,0} + m_A g d \cos 60 = \tfrac{1}{2} m_A (V_A)_2^2$$

$$(V_{A_2})^2 = 2 \times 9.81 \times 2 \cos 60$$

$$V_{A_2} = 4.43 \text{ m/s}$$

here $V_{A_1} = V_{A_2}$

Using $m_A V_{A_1} + \cancel{m_B V_{B_1}}^{\,0} = m_A V_{A_2} + m_B V_{B_2}$

$$(2kg)(4.43 \text{ m/s}) = 2kg\, V_{A_2} + 3kg\, V_{B_2}$$

$$V_{A_2} = \frac{2 \times 4.43}{2} - \frac{3}{2} V_{B_2}$$

$$e = \frac{V_{B2} - V_{A2}}{V_{A_1} - V_{B1}} \Rightarrow 0.75 = \frac{V_{B2} - (4.43 - 3/2\, V_{B2})}{4.43 - 0}$$

$$0.75 = \frac{V_{B2} - 4.43 + \frac{3}{2} V_{B2}}{4.43}$$

$$V_{B2} = 3.1 \text{ m/s}$$

$$T_3 + V_3 = T_4 + V_4$$

$$\tfrac{1}{2} m_B V_3^2 + 0 = 0 + mgh(1 - \cos\theta_B)$$

$$14.42 = 3 \times 9.81 \times 2 (1 - \cos\theta_B)$$

$$\theta_B = 41°$$

b) $V_{A_2} = 4.43 - \frac{3}{2} V_{B2}$

$$V_{B2} = 3.1 \text{ m/s}$$

$$V_{A2} = 0.22 \text{ m/s}$$

$$T_3 + \cancel{V_3}^{\,0} = \cancel{T_4}^{\,0} + V_4$$

$$\tfrac{1}{2} m_A V_{A_2}^2 = m_A g\, d(1 - \cos\theta_A)$$

$$\tfrac{1}{2} \times 2kg \times 0.22^2 = 2 \times 9.81 \times 2 \times (1 - \cos\theta_A)$$

$$0.0484 = 39.24(1 - \cos\theta_A)$$

$$\frac{0.0484}{39.24} = (1 - \cos\theta_A)$$

$$\theta_A = 2.846°$$

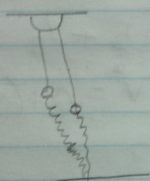
Mark 5

Tutorial

Cap #2

$$S_1 = \sqrt{(0.15)^2 + (0.25)^2} - 0.1 = 0.192 \text{ m}$$

$$S_2 = \sqrt{(0.3+0.15)^2 + (0.25)^2} - 0.3 - 0.1 = 0.115 \text{ m}$$



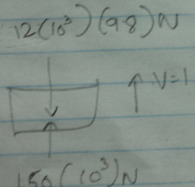
$$\beta = \text{Arc tan}\left(\frac{0.25}{0.45}\right) = 24.1^\circ$$

$$\frac{1}{2} m v_1^2 + \frac{1}{2} k s_1^2 - mg(0.3) = \frac{1}{2} m v_2^2 + \frac{1}{2} k s_2^2 - mg(0.3 \cos\beta)$$

$$\frac{1}{2}(200)(0.192)^2 - (1)(9.81)(0.3) = \frac{1}{2}(1) v_2^2 + \frac{1}{2}(200)(0.115)^2 - 1(9.81)(0.3 \cos 29.1)$$

$$v_2 = 1.99 \text{ m/s}$$

15.2



$$+\uparrow \quad m(v_y)_1 + \Sigma \int F_y\, dt = m(v_y)_2$$

$$0 + 150(10^3)(6) - 12(9.81)(6) = 12(10^3) v_2$$

$$v_2 = 16.1 \text{ m/s}$$

$$+\uparrow \quad v = v_0 + a_c t$$

$$16.1 = 0 + a_c(6)$$

$$a_c = 2.69 \text{ m/s}^2$$

$$+\uparrow \quad s = \cancel{s_0} + \cancel{v_0 t} + \frac{1}{2} a_c t^2$$

$$s = \frac{1}{2}(2.69)(6)^2 = 48.4 \text{ m}$$

$0.75 - 3.0$

$$= \int_{0.75}^{3.0} F\,ds + \int_{3.0}^{s} F\,ds$$

$$= 60[3 - 0.75] + 80[s - 3.0]$$

$$= 135 + 80s - 240$$

$$111.2\,kN = 80s - 105$$

$$s = 2.7$$

Power and Efficiency

$$P = \frac{dU}{dt} = \frac{\bar{F}\cdot d\bar{r}}{dt} = \bar{F}\cdot\frac{d\bar{r}}{dt}$$

Mechanical Efficiency η

$$\eta = \frac{\text{Power output}}{\text{Power input}}$$

WD : Work Done

Ex 1-5

40 mph , horse power = 40,000 HP

$1\,mph = \frac{88}{60}$ ft/sec

Power = F × V

Work done / against resistance $= R \times \frac{40 \times 88}{60}$

For uniform speed : work done against resistance = hP Produced

$$\therefore R \times \frac{40 \times 88}{60} =$$

$U_{1\to2}$ = area under the curve

$= -\frac{1}{2}(2.5\times10^{2}\times0.1) + \frac{1}{2}(0.05\times1.25\times10^{2})$

Class Assigned Problems #1

$m = 800\,kg$ $\quad V_c = 60\,km/h$

$V_1 = 60\,km/h \quad V_2 = 0$

$\to 16.67\,m/s$

$T_1 + U_{1\to2} = \cancel{T_2} \to 0$

$T_1 = \frac{1}{2} m v_1^2$

$= \frac{1}{2}\,800\times(16.67)^2\,m/s$

$= 111155.56$

$0 < s < 0.75$

Area under the graph

$0.5 - 3^2$

0.75

now let $0.75\,m < s < 3\,m$

$\int_0 \quad +$

$= \int_0^{0.75} F\,ds + \int_{0.75}^{s} F\,ds$

$= 40[s]_0^{0.75} + 60[s]_{0.75}^{s}$

$= 40[0.75] + 60[s - 0.75]$

$= 40\times0.75 + 60s - 45$

$= 30 + 60s - 45$

$111.2\,kN = 30 + 60s - 45$

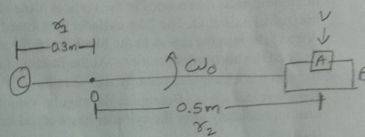
$111.2 + 15 = 60s \qquad s = 2.10\ m$

Cap #4 Solution

Cylinder speed: $T_1 + V_1 = T_2 + V_2$ $\quad$ $V = \sqrt{2 \times 9.81\,m/s \times 0.6\,m}$

$mgh = \frac{1}{2}mv^2$ $\quad$ $= 3.431$

$V = \sqrt{2gh}$



$(H_O)_1 = (H_O)_2$

$r_2 m_A V_A + m_B r_2^2 \omega_1 + m_C r_1^2 \omega_1 = (m_A + m_B) r_2^2 \omega_2 + m_C r_1^2 \omega_2$

$(0.5m)(2kg)(-3.43m/s) + (4kg)(0.5m)^2(2\,rad/s) + (6kg)(0.3m)^2(2\,rad/s) =$

$(6kg)(0.5m)^2\omega_2 + (6kg)(0.3m)^2\omega_2$

$-0.351 = 1.5\omega_2 + 0.54\omega_2$

$-0.351 = 2.04\,\omega_2$

$\omega_2 = -0.172\ rad/s$

$= 0.17\ rad/s$ CCW

MAAE 2101 DYNAMICS | CLASS ASSIGNED PROBLEMS | PROBLEM #5

Last Name: Patel *First Name:* Rikky *PA Section* D1

Water is flowing from the 150 mm diameter fire hydrant at section B with a velocity of $v_B = 15$ m/s. The static gauge pressure inside the hydrant is 50 kPa and the diameter of the base is 200 mm at section A. Given that the density of water is 1000 kg/m^3.

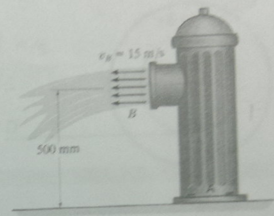
Determine:

(a) The mass flow rate of water through the hydrant.

(b) The velocity of the water v_A entering the hydrant at A

(c) The vertical and horizontal components of force at the base A

(d) The moment developed at section A



a)

$\dot{m} = \rho V_B A_B$

$\frac{dm}{dt} = \rho A_B . V_B = \rho(\pi r^2) V_B$

$\frac{dm}{dt} = 1000 \times \pi \times (0.075)^2 \times 15 = 265.07$ kg/s ✓

b) $\dot{m}_{in} = \dot{m}_{out}$

$\rho V_A A_a = \rho V_B A_B$

$V_A = \frac{\rho V_B A_B}{\rho A_a} = \frac{265.07}{1000 \times (0.100)^2 \times \pi}$

$V_A = 8.44$ m/s ✓

c) $\dot{m}(V_{Bx} - V_{Ax}) = F_x$ — ①

$\dot{m}(V_{By} - V_{Ay}) = F - F_y$ — ②

y, x

$\dot{m} \times -V_{Ay} - F = -F_y$

$F_y = \dot{m} V_A + F \Rightarrow F_y = \dot{m} V_A + P_x A_a$

$F_y = (265.07) \times (8.44) + 50 \times (\pi \times 0.100^2)$

$F_y = 3807.98$ N $\Rightarrow 3.808$ kN ✓ ↓

$F_x = \dot{m} V_{Bx} = 3.98$ kN ✓ ←

d) moment $= \dot{m}(V_B) \times d = 265.07 \times 15 \times 0.5$

$= 1.99$ KN ✓

Mark 5

CAP #6

(a) $V_B - V_b = 3$ cm/s

$V_B = CB \times \omega$

$\omega = \frac{V_B}{CB} = \frac{3}{1.5}$

$= 2$ rad/s CW

b) $V_A = CA \times \omega$

$= 6 \times 2$

$= 12$ m/s

c) $V_A - V_D = 12 - 3$

$= 9$ in/s

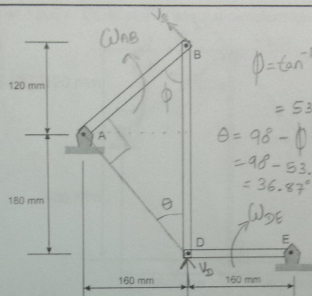
Until

Example 4-8

MAAE 2101 DYNAMICS   CLASS ASSIGNED PROBLEMS   PROBLEM #7

Last Name: Patel   *First Name:* Rikky   *Section* D1

In the position shown, bar AB of the pin-jointed frame, has a constant angular velocity of 3 rad/s counter-clockwise. Determine the angular velocity of bars BD and DE.



$\phi = \tan^{-1}\left(\frac{0.160}{0.120}\right)$

$= 53.130^\circ$

$\theta = 98 - \phi$

$= 98 - 53.130^\circ$

$= 36.87^\circ$

Bar AB - $v_B = 0.2 \times 3 = 0.6\, m/s$

Bar DE - $v_D = 0.16\, x\, \omega_{DE} = 0.16\, \omega_{DE}$

$V_{B/D} = V_B \sin\theta$

$= 0.6 \times \sin 36.87$

$= 0.36\ m/s$

$V_{B/D} = l_{BD} \times \omega_{BD}$

$= (0.120 + 0.180)\, \omega_{BD}$

$= 0.3\, \omega_{BD}$

$0.36 = 0.3\omega_{BD} \Rightarrow \omega_{BD} = 1.2$ rad/s CCW

$V_D^2 + V_{B/D}^2 = V_B^2$

$V_D = \sqrt{0.6^2 - 0.36^2} = 0.48\ m/s$

$V_D = 0.16 \times \omega_{DE}$

$\omega_{DE} = \frac{0.48\ m/s}{0.16} = 3$ rad/s

Angular Velocity of BD: 1.2 rad/s ✓

Angular Velocity of bars DE: 3 rad/s ✓

Mark	5

MAAE 2101 DYNAMICS **CLASS ASSIGNED PROBLEMS** **PROBLEM #8**

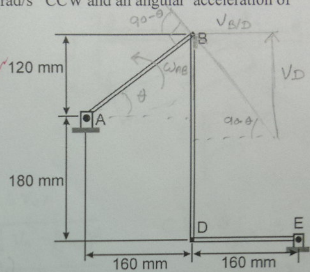
Last Name: Patel ***First Name:*** Rikky ***Section*** D1

In the position shown, bar AB has an angular velocity of 3 rad/s CCW and an angular acceleration of 6 rads/s^2 CW.

Determine:

(a) The velocity of D
(b) The angular velocity of DB
(c) The angular acceleration of BD
(d) The angular acceleration of bars BD and DE.

5/5



$\lambda^2 = 120^2 + 160^2$
$= 0.2\,m$

$\tan\theta = \frac{120}{160}$
$\theta = 36.9^\circ$

$V_B = \omega_{AB}\lambda_{AB}$
$= 3\,rad/s\,(0.2)$

$90 - \theta = 53.1^\circ$

$\sin 36.9^\circ = \frac{V_{B/D}}{V_B} \Rightarrow \boxed{V_{B/D} = 0.36\ m/s}$

a) $\cos 36.9^\circ = \frac{V_D}{V_B} \Rightarrow \boxed{V_D = 0.48\ m/s}$

$V_{B/D} = \omega_{BD}\lambda_{B/D}$

$\boxed{\omega_{BD} = \frac{V_{B/D}}{\lambda_{B/D}} = \frac{0.36}{0.3} = 1.2\ rad/s}$

$V_D = \omega_{DE} \times \lambda_{DE} \Rightarrow \omega_{DE} = \frac{V_D}{\lambda_{DE}} = 3\ rad/s$

$(a_{AB})_n = \omega^2 r_{AB}$ $(a_{AB})_t = \alpha r_{AB}$

a_n a_t θ

horizontal

$-\omega^2 r_{AB}\cos 36.9 + \alpha_{AB} r_{AB}\sin 36.9^\circ = \alpha_{BD} r_{BD} + \omega_{DE}^2 r_{DE}$

$-(3)^2 \times 0.2\cos 36.9 + 6\times 0.2\times\sin 36.9^\circ = +0.3\alpha_{BD} + (3)^2\times 0.16$

$-1.44 + 0.721 = 0.3\alpha_{BD} + 1.44$

$-2.88 + 0.721 = 0.3\alpha_{BD}$

$\boxed{\alpha_{BD} = \frac{-2.159}{0.3} = -7.2\ rad/s^2\ CCW}$

when you speify directin you remove the minus sign

$\rightarrow \alpha_{BD} = 7.2$ CCW

Vertical

$-\omega^2 r_{AB}\sin 36.9 - \alpha_{AB} r_{AB}\cos 36.9^\circ = -\omega_{BD}^2 r_{BD} + \alpha_{DE} r_{DE}$

$-(3)^2(0.2)\sin 36.9 - 6\times 0.2\times\cos 36.9^\circ = -(1.2)^2\times 0.3 + \alpha_{DE}\,0.16$

$-1.08 - 0.96 = -0.432 + 0.16\,\alpha_{DE}$

$-1.608 = 0.16\,\alpha_{DE}$ $\boxed{\alpha_{DE} = 10.05\ rad/s^2\ CCW}$

The negative sign meant it was CCW