Answer Key - Midterm 3, Econ 2129

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Section I

1. Cournot
2. price......price(s)
3. dominant
4. Nash equilibrium
5. mixed *or* random

Section II

6. In a Cournot market, the firms are producing homogeneous products, which means that the market price is determined by the total quantity produced by all the firms through the (inverse) demand function. That is, $p = P(Q)$, where Q is total production by all the firms. Further, the strategic decision variable for each firm is its own output, q_i, for each firm $i = 1, 2, ..., n$.

When the firms compete in this way, then the outputs and price are determined each period as a Cournot-Nash equilibrium (CNE), which is determined by the 'no-regrets' condition that for each firm i, $q_i^c = r_i(Q_{-i}^c)$, where $r_i(Q_{-i}^c)$ is firm i's reaction function, which determines the profit-maximizing output for firm i given any total output by all the other firms, which is Q_{-i}. So, each CNE output q_i^c is determined as the solution to this problem

$$\text{Choose the value of } q_i \text{ to maximize } \pi_i\left(q_i|Q_{-i}^c\right) = q_i P(q_i + Q_{-i}^c) - c_i(q_i)$$

This means that - to take one example - any change in any one firm's marginal costs will alter the value of q_i^c a firm wants to choose, and therefore alter Q^c, the total CNE output. Since the CNE price is determined in the market as $p^c = P(Q^c)$, this must in turn cause the CNE price to vary.

One the other hand, if these same firms collude in such a market, they do so by agreeing on production quotas q_i^0 for each firm i, with the collusive price then being determined by the market again as $p^0 = P(Q^0)$, where Q^0 is just the sum of all the quotas. Now, if any firm i's marginal costs change, they may again *want* to alter their output, but we know also that such a collusive agreement must be held together over time by something that eliminates the inevitable incentive to cheat on one's quota. If any firm unilaterally alters its production away from the agreed-on quota they run the risk of triggering a response from the other firms, because they view it as cheating on the collusive agreement. So, the same changes in firm costs that cause output fluctuations (and therefore price fluctuations) in a CNE will not induce output fluctuations in a collusive arrangement, and no output fluctuations means no price fluctuations.

7. As noted in the answer above, the key properties here are an oligopoly (few firms) producing a homogeneous product whose price is determined in a market, with the strategic decisions made by firms being the quantity they produce. These decisions are based on what they expect the other firms in the industry to produce. The best examples of these types of markets are resource-extraction industries like potash, oil, nickel, etc., in which there are a few major firms. There is typically a world market in which the going price (as well as futures prices in many cases) are determined, and firms know that the more they (or their rivals) produce, the lower will be that market price. So, the market for oil (not gasoline) is a good example, but note also that OPEC is *not* a good example, as it is a particular organization of oil-producing countries which includes a large fraction of total production. However, the oil *industry* or *market* includes many non-OPEC member producers these days, so the question is whether this entire industry is a good example of a Cournot market. To the extent that the OPEC members are able to manipulate the price by changing their collective output they are behaving as a *cartel* and are not in fact behaving as Cournot asserted firms in this industry would.

In a Bertrand market, a few firms produce differentiated products, meaning they each face their own negatively-sloped demand function which includes as important factors the prices charged by the other firms, and the key strategic decision variable is their own price. So, gasoline stations are a good example because location (as well as differences in the other goods and services they sell) provide product differentiation, and it is clear that the key decision they make each day is what price to post for gasoline. Most any retail sales market can be characterized this way, **if** it includes only a few sellers.

As analyzed in class, a market with homogeneous products can *not* be reasonably represented as a Bertrand oligopoly, since that model predicts that the BNE price will be equal to (constant) marginal cost, which in turn implies that any firm with any fixed costs (and it seems obvious all firms have some fixed costs) will earn $\pi < 0$ and necessarily go out of business. So, no market that has firms actually operating in it can be a homogeneous-product Bertrand market.

8.a) As noted in class, the best way to analyze any one-shot game like this is to write out the best-responses for each player. With $D = 3$, these are:

$r_a(X) = U \quad r_b(T) = Y$
$r_a(Y) = U \quad r_b(U) = Z$
$r_a(Z) = U \quad r_b(V) = Y$

i) The above, which states which choice gives a player the highest payoff for a given choice by the other player, immediately tells us that A has a dominant strategy of U, as that is her best response to *anything* B chooses, whereas B has no such strategy.

ii) It also tells us that there is exactly one Nash equilibrium as only the choices of U, Z by A and B respectively are best-responses to one another.

b) Now, with $D = 0$, only one of the above best-responses changes, that being that now $r_a(Y) = T$ rather than U.

i) This then implies that neither player has a dominant strategy, and that there are now two strategy pairs that are Nash equilibria, these being (U, Z) and (T, Y)

9. a) The extensive form game tree is attached on the last page, and the notation is explained as follows:

The two players are E and I, the potential entrant and the incumbent monopolist. E has the first move, choosing to enter (*in*) or not (*out*). If E chooses *out* the game ends, but if E chooses to enter, then I has a choice to prey (p) or compete (c).

The payoffs from each set of possible choices are depicted in parentheses at each terminal node, with the first entry being the payoff (present discounted value of a profit stream) for E and the second entry being the same for I.

The payoffs are explained as follows:

0 is the future stream of profits E earns if *out* is chosen. The same choice by E gives I a payoff of

$$V_m = \sum_{t=0}^{\infty} \delta^t \pi_I^m$$

where δ is the discount rate used by firm I and π_I^m is the profit earned by I in each period that it operates as a monopoly.

If E chooses *in* and I responds by choosing p, then this is interpreted as a decision by I to engage in predatory pricing, and so the payoffs are,

for E,

$$V_E^- = \sum_{t=0}^{T} \delta^t \pi_E^-$$

where $\pi_E^- < 0$ is the loss that will be incurred by E each period it operates while I is predatory pricing, and $T + 1$ is the period in which it runs out of financial resources and ceases to operate.

The payoff to I in this case is

$$V_I^- = \sum_{t=0}^{T} \delta^t \pi_I^- + \sum_{t=T+1}^{\infty} \delta^t \pi_I^m$$

where $\pi_I^- < 0$ are the losses I incurs while it is predatory pricing, and $T = 1$ is again the period in which it can begin to earn monopoly profits again, with E having ceased to operate.

If on the other hand I chooses c after E chooses in then the payoffs are:

for E,

$$V_E^c = \sum_{t=0}^{\infty} \delta^t \pi_E^b$$

where now $\pi_E^b > 0$ is the profit E earns in each period of a Bertrand (I'm assuming - it could just as well be Cournot) Nash equilibrium competing with I, and for I,

$$V_I^c = \sum_{t=0}^{\infty} \delta^t \pi_I^b$$

where $\pi_I^b > 0$ is I's BNE profit each period.

Simple arithmetic implies that $V_I^- < V_m$ and $V_E^- < 0 < V_E^c$.

b) The second implication of arithmetic tells us that if E expects I to choose p if he chooses in, then E will in fact choose out, since $V_E^- < 0$. This is entry-deterrence. However, *credible* entry deterrence requires that it be in I's own best interest to choose p rather than c in that event, and that is true only if $V_I^- > V_I^c$, and the arithmetic doesn't tell us if this is true. It will be true if

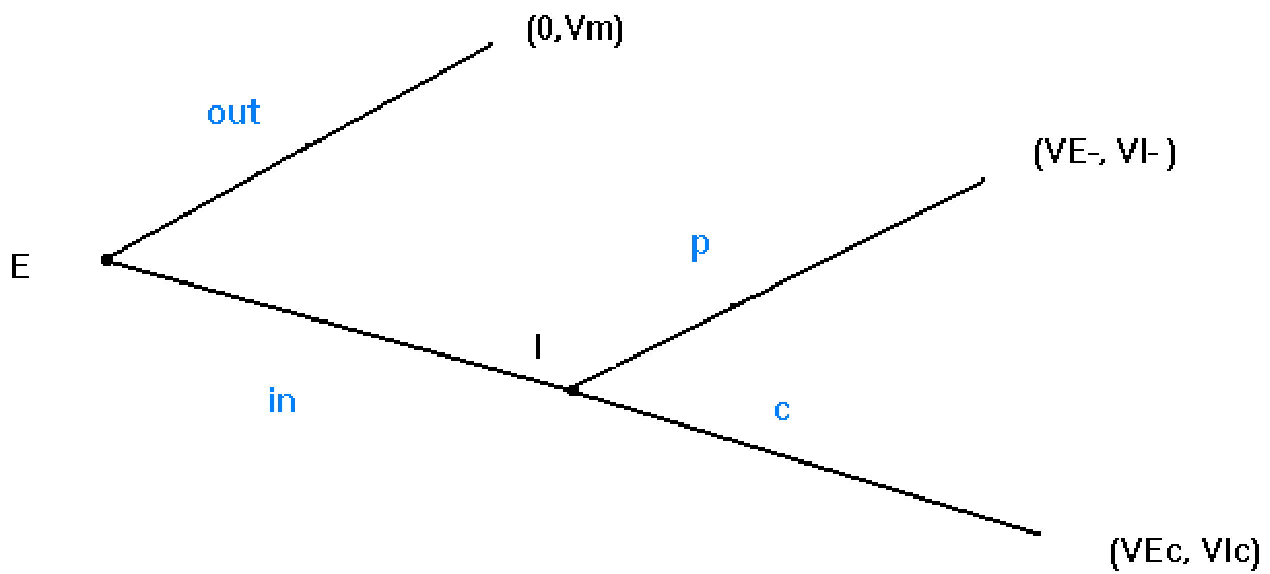
$$\sum_{t=0}^{T} \delta^t \pi_I^- + \sum_{t=T+1}^{\infty} \delta^t \pi_I^m > \sum_{t=0}^{\infty} \delta^t \pi_I^b$$

The first T terms on the left side are clearly less than the corresponding terms on the right side, since $\pi_I^- < 0 < \pi_I^b$ (if $\pi_I^- > 0$ then it isn't predatory pricing by I, just good competition). However, the remaining terms on the left side are greater than those on the right ($\pi_I^m > \pi_I^b$), since I will earn more profit as a monopolist than as part of a duopoly. So, whether this condition for credibility holds or not turns to a great extent on how big T is; that is, how many periods of losses by E

(and by I) are required before E drops out and leaves I to earnπ_I^m. (The other things that matter, of course are the differences between π_I^b, π_I^m, and π_I^b.)

c) It is certainly true that Walmart Canada - being player I in the game above - *could* engage in predatory pricing once Target - player E- enters the Canadian market. There is nothing to stop Walmart from lowering its prices to the point where Target would have to lower its prices below its own cost to sell anything; this would imply that Walmart's prices would be so low that it would lose money on its sales, also. As noted in b) above, though, this is only in Walmart's interest if it eventually induces Target to exit from Canada, and so generates a bigger stream of profits than simply competing with Target as in a Bertrand market. Since Target is already a multi-national company with many stores in the US and other places, it would certainly take a long period of losses before Target would decide that entry into Canada had been a mistake, inducing it to exit and leave Walmart alone. In other words, T must be very large, which suggests that it will *not* be true for Walmart that

$$\sum_{t=0}^{T} \delta^t \pi_I^- + \sum_{t=T+1}^{\infty} \delta^t \pi_I^m > \sum_{t=0}^{\infty} \delta^t \pi_I^b$$



Entry-deterrence extensive form game tree