Midterm Test 2 - Answer Key
Econ 2129, March 2013 - Al Slivinski

Section I

1. cross-price demand elasticity
2. no units
3. decrease
4. the market price (half credit for 'marginal cost', which is true *only* at the profit-maximizing quantity, while 'market price' is always true.)
5. differentiated *or* imperfect substitutes.

Section II

6. Ingrid has gathered some 60 data points, each of which is of the form:

$$(p_m^j, p_d^j, p_c^j, m^j, c^j)$$

where p_m^j is the price of milk recorded on the jth day, m^j is the litres of milk sold on the jth day, etc., for $j = 1, 2, ..., 60$. The question asked was how Ingrid could use *this data* to provide the store owner with useful info *about the demand for 2% milk at his store.*

Some said she could calculate various price elasticities for milk, such as the own-price elasticity. How does one do this using this data? It isn't obvious. One could pick two days, say 1 and 7, on which the price of milk differed, and then calculate the elasticity as we learned. That is, it would be:

$$\varepsilon(1,7) = \frac{\frac{m^1 - m^7}{m^1 + m^7}}{\frac{p_m^1 - p_m^7}{p_m^1 + p_m^7}}$$

and this calculation would spit out a (hopefully negative) number. There are two problems with this. The small problem is that there is no reason to expect one would get the same answer if one used the data from days 6 and 13, or 57 and 20 or any other two days.

A bigger problem is the fact that all elasticities are 'ceteris paribus' calculations, meaning all other variables are supposed to be held constant, and the above calculation doesn't pay any attention to that - it ignores the values of all the other variables on those two days.

What one needs is an estimate of the actual demand for milk at this store, since *that* is what we used to calculate elasticities. We also learned how one can use this kind of data to estimate a demand function.

Ingrid could use the data gathered on just p_m and m to use simple linear regression to estimate a linear demand function for milk of the form:

$$m = \alpha + \beta p_m$$

but this doesn't help her with the ceteris paribus problem, as it ignores all the other data she gathered. The better thing she can do is estimate a multiple linear regression. How do we do that? We estimate a demand function that includes (almost) all of the variables Ingrid has data on, which will look like:

$$m = \alpha + \beta p_m + \gamma p_c + \delta p_d$$

There is plenty of reason to think demand for milk is influenced by the prices of soft drinks and cookies, and doing this will give us an estimate of those influences. How do we do this? The same way we estimate a simple linear regression - find the values of $\alpha, \beta, \gamma, \delta$ that minimize this expression:

$$MSE = \frac{1}{60} \sum_{j=1}^{60} \left[m^j - \left(\alpha + \beta p_m^j + \gamma p_c^j + \delta p_d^j \right) \right]^2$$

which is the average 'error' the estimated function makes in predicting each m^j value based on the corresponding values of $\left(p_m^j, p_c^j, p_d^j\right)$ in all 60 data points.

(Note that Ingrid doesn't use the data on c, which I threw in as a red herring. It would be useful if one wanted also to estimate a demand function for cookies, but you weren't asked about that, and it's actually a lot harder to estimate a *system* of related demand functions.)

Now, with this estimated demand function,

$$m = \alpha^0 + \beta^0 p_m + \gamma^0 p_c + \delta^0 p_d$$

with α^0 indicating the estimated value of the parameter α that minimizing the MSE spits out, and so on for the other three parameters (any reasonably good calculator, let alone a PC, can do this), one can actually calculate elasticities in the correct 'ceteris paribus' manner. And, although it will still be true that one can expect to get different elasticity numbers depending on which price range one calculates them over, there are two things Ingrid can now do.

First, she can tell the owner what the own-price elasticity for milk is when the *other* prices are held constant at specific chosen values, and second, she can ask the owner what price range (for milk) he wants to know the elasticities for.

As an example, Ingrid can say 'If we hold the prices of soft drinks and cookies at what you are currently charging, which is \$1.50/litre and \$2/oz, respectively, and if we consider a price range that includes what you are currently charging for milk, which is \$4/litre, then the own price elasticity for milk at your store is ε^*, where ε^* is calculated as:

$$\varepsilon^* = \frac{\frac{m'-m''}{m'+m''}}{\frac{3.5-4.5}{3.5+4.5}}$$

and

$$\begin{aligned} m' &= \alpha^0 + \beta^0 3.5 + \gamma^0 2 + \delta^0 1.5 \\ m'' &= \alpha^0 + \beta^0 4.5 + \gamma^0 2 + \delta^0 1.5 \end{aligned}$$

This would be useful to the owner, as the value of ε^* would give him an estimate of what would happen to his total revenue from milk sales as he varied his price around the price he is currently charging, while holding his other prices constant. He could estimate a whole host of other such effects, also, like - what would be the effect on his sales of milk if he changed his cookie or soft drink prices?

There are a million reasons for the owner to be cautious about using these estimates. Of course they are only estimates, but 60 data points is quite a few. The most important caution is not that no data on income was used, since people's incomes don't change much over two months, so an income variable would likely not vary even if you included it in the data. More of a worry is other missing data; we know retail selling of just about anything is a monopolistically competitive industry, and that means that the prices for milk, cookies and soft drinks charged at other convenience stores (particularly those close by) can be expected to have an influence on this store's sales, since those are the closest imperfect substitutes for what this store sells, and those prices probably *did* change over the two months. Without data on those prices, there is no way to know how much of the variance in m in Ingrid's data is due to that rather than the variables she does have data on.

A secondary worry is that although 60 data points is quite a few, they were gathered over 60 consecutive days, and so if there is anything seasonal in the demand for milk at the store, the demand function might look different if the data had been gathered over a different two months. This suggests that using the estimated demand during other months of the year might be a mistake.

7. The diagram is attached as Fig. 1. The horizontal axis measures q_j, the amount of its own differentiated product sold by firm j, while the vertical axis is

in units of \$/unit of product. $P_j(q_j, P_{-j}, X_j)$ is the inverse demand for this firm's product, which depends on the amount it wants to sell, the prices of other firms in the industry and of any other substitutes or compliments for this firm's product, all indicated as part of P_{-j}, and finally, X_j, which is anything else that might affect demand for its product. From this is derived the firm's marginal revenue, mr_j, which therefore depends on the same variables, and the firm's total cost function is $c_j(q_j)$, whereas its marginal cost function is $mc_j(q_j)$ and its average cost is $c_j(q_j)/q_j$.

Such a firm maximizes profit by producing the output q_j^* at which $mc_j(q_j^*) = mr_j(q_j^*, P_{-j}, X_j)$, as indicated on the diagram, and the price it charges is $p_j^* = P_j(q_j^*, P_{-j}, X)$, as indicated on the diagram, since this is the price at which it can sell q_j^* units. The profit the firm earns is given by the rectangle in the diagram which a horizontal size of q_j^* and vertical size of $\left[p_j^* - \frac{c_j(q_j^*)}{q_j^*}\right]$,and so has area equal to:

$$q_j^* \left[p_j^* - \frac{c_j(q_j^*)}{q_j^*}\right] = \pi^*$$

If the firm's landlord raises the rent on its premises, this increases the firm's costs, but *not* its marginal costs, as rent is a fixed cost. One doesn't pay more rent when one produces more, so the only impact on the diagram is to change the firm's average costs to $\bar{c}_j(q_j)/q_j$, which is the red curve on the diagram. Thus, the firm's pricing and output decisions are unaffected by this change, but its profits are reduced, as shown.

As to entry or exit, in my diagram the firm is still making a profit after the rent increase, so it will not exit, and since only this one firm's rent has increased, there will be no other firm entry or exit induced by it.

It is possible that the rent increase is large enough to drive this firm's profit below zero, (by raising $\bar{c}_j(q_j)/q_j$ above the inverse demand curve) in which case this firm would eventually exit. That *might* be enough to allow some other entrepreneur to enter this industry, but it would certainly not cause any other firms to exit.

8. We discussed (at least) *five* kinds of advertising in such an industry. I'll pick two and answer for those.

Comparative advertising: This occurs when a firm's advertisements include a direct comparison of its product with the product(s) of one or more of its competitors. The basis of the comparison could be prices or attributes of the products themselves.

Niche advertising: This occurs when a firm's ads convey the message that its product is ideally suited for a particular target group of potential buyers. For example, an ad that stresses that 'breakfast cereal bars' are perfect for busy people who don't want to take a lot of time for breakfast in the morning.

Advertising is always intended to *increase* the demand for the product being advertised. In the case of comparative advertising, the idea is that this increase in demand will arise by attracting buyers who would otherwise purchase the products being compared, and in that sense, would *decrease* the demand for those products. In the attached Fig 2, the effect of such advertising on the *demand of the firm doing the advertising* is shown. The curves are the same as those defined in Fig. 1, since this is the same type of (monopolistically competitive) firm, **except that -**

the inverse demand curve is originally $P_j(q_j, P_{-j}, A'_j, X_j)$, and moves outward to $P_j(q_j, P_{-j}, A''_j, X_j)$, because of the increase in advertising from A'_j to A''_j. This in turn means the associated marginal revenue curve changes from $mr_j(q_j, P_{-j}, A'_j, X_j)$ to $mr_j(q_j, P_{-j}, A''_j, X_j)$.

The impact on equililbrium for this firm is as follows. With the lower advertising expenditure of A'_j, the equilibrium price and quantity for firm j is (p'_j, q'_j). With the higher level of advertising A''_j the new equilibrium involves higher price and output at (p''_j, q''_j).However, while the increase in output *must happen*, because the new $mc = mr$ intersection occurs at a higher quantity, it does not have to be true that the price the firm charges rises. The price changes because of two things: first, the firm produces more, which would imply a lower price if nothing else happened. Second, however, the demand function also increases, which would imply a higher price if nothing else happened. The net impact on price therefore depends on which of these effects is larger.

Note finally that advertising is not free: this firm's advertising costs would rise from $\alpha_j(A'_j)$ to $\alpha_j(A''_j)$. This would have no impact on marginal costs, so the diagram is completely accurate. However, it would increase total costs, and therefore average costs. To depict this, one needs to add (two) average cost functions, which would be denoted as:

$$\frac{c_j(q_j) + \alpha_j(A'_j)}{q_j} \text{ and } \frac{c_j(q_j) + \alpha_j(A''_j)}{q_j}$$

with the second lying above the first. One could then show the impact on firm j's profits of all this, but I didn't really expect people to go this far because i) the diagram would be very messy and ii) one could not come to a definite conclusion as to whether profit increased by looking at such a diagram (although one would hope firm j's CEO didn't spend money on advertising that decreases profit.)

For *comparative advertising* the intent of firm j is to attract those who would otherwise buy from a rival, call it firm k. If that works, then there is a direct impact on firm k which is the opposite of that above. That is because included in the 'other stuff' variable X_k in k's inverse demand function $P_k()$ is A_j, firm $j's$ comparative advertising. So, a second diagram in this case could be drawn for firm k in which its inverse demand (and therefore the associated mr_k) *shifts in*, with the result that q_k goes down, and p_k again *might* go down.

For the case of *niche advertising* by firm j, the diagram in Fig 1 works to explain what happens again (you need not draw another one). It is less clear here whether this has an impact on any of the other firms. One could take the reasonable position that since monopolistically competitive firms are all small and there are many of them that one firm's additional advertising has a negligible effect on any other single firm unless the advertising is specifically targeted at one of them, as is true with comparison advertising.

8. The *definition:* Davi-Coal's Cournot reaction function relates the profit-maximizing output of coal by D-C to each level of output that D-C might expect All-Coal to produce. Thus, it is written in the form:

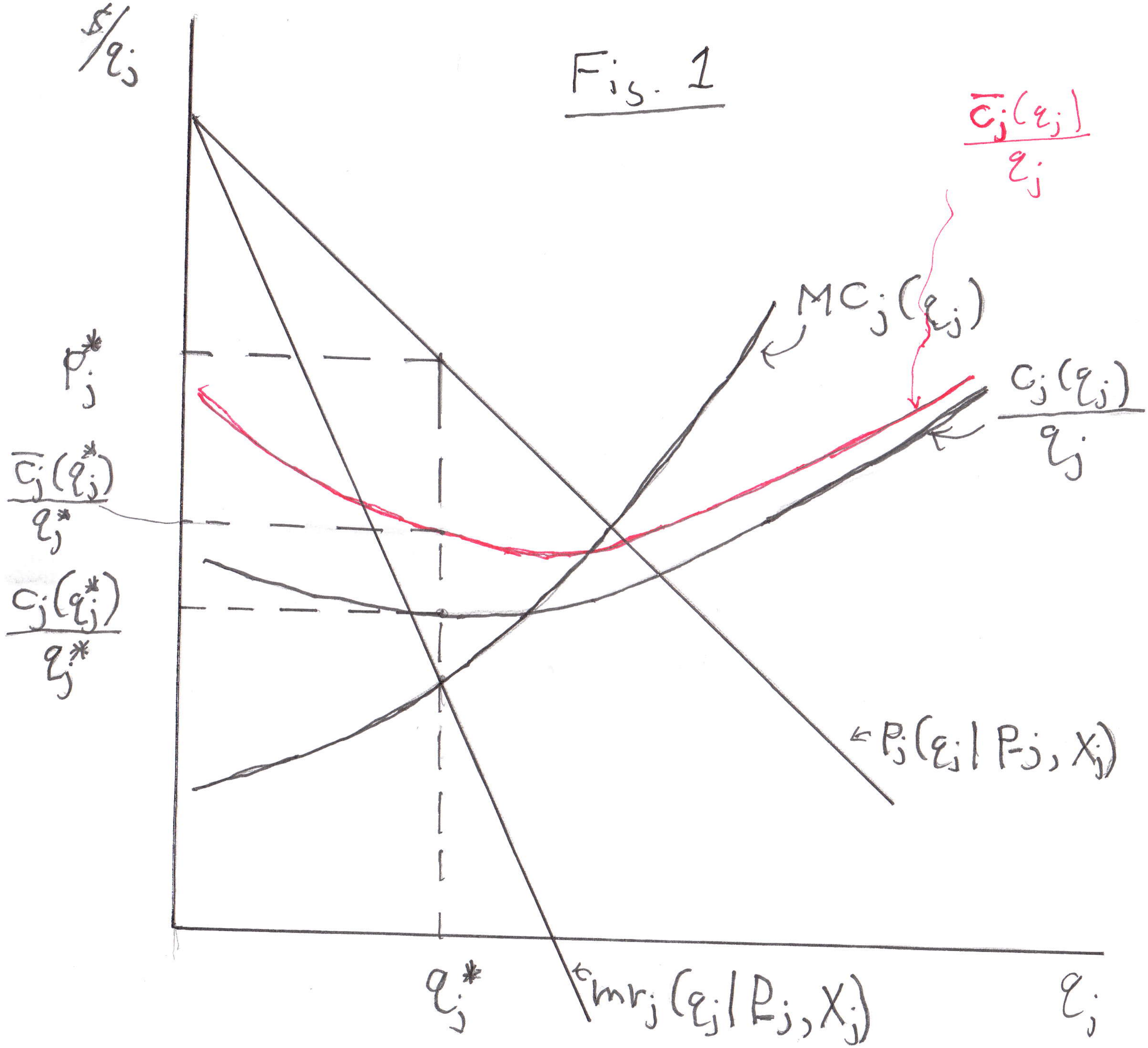
$$r_d(q_a) = q_d$$

where q_a is A-C's coal production and q_d is D-C's coal production.

Fig. 3 shows how this would be derived. In the diagram $P(Q)$ is the *market* demand for coal in Slivinia by all electricity generators, $mc_d(q_d)$ is D-C's marginal cost of producing coal, and $P_d(q_d|q_a')$ is the 'residual demand' that remains to D-C when D-C believes that A-C is going to produce q_a' units of coal for sale. Note that $P_d(q_d|q_a')$ is shifted in horizontally from $P(Q)$ by exactly the q_a' units that A-C produces. This generates a profit-maximizing output of q_d' for D-C, where its $mc_d(q_d') = mr_d(q_d'|q_a')$. So, we have one point on D-C's reaction function, namely $q_d' = r_d(q_a')$. Note also that one could use the residual demand to determine what the market price would be if $q_d' + q_a'$ were actually produced, but this is not necessary to answer the question.

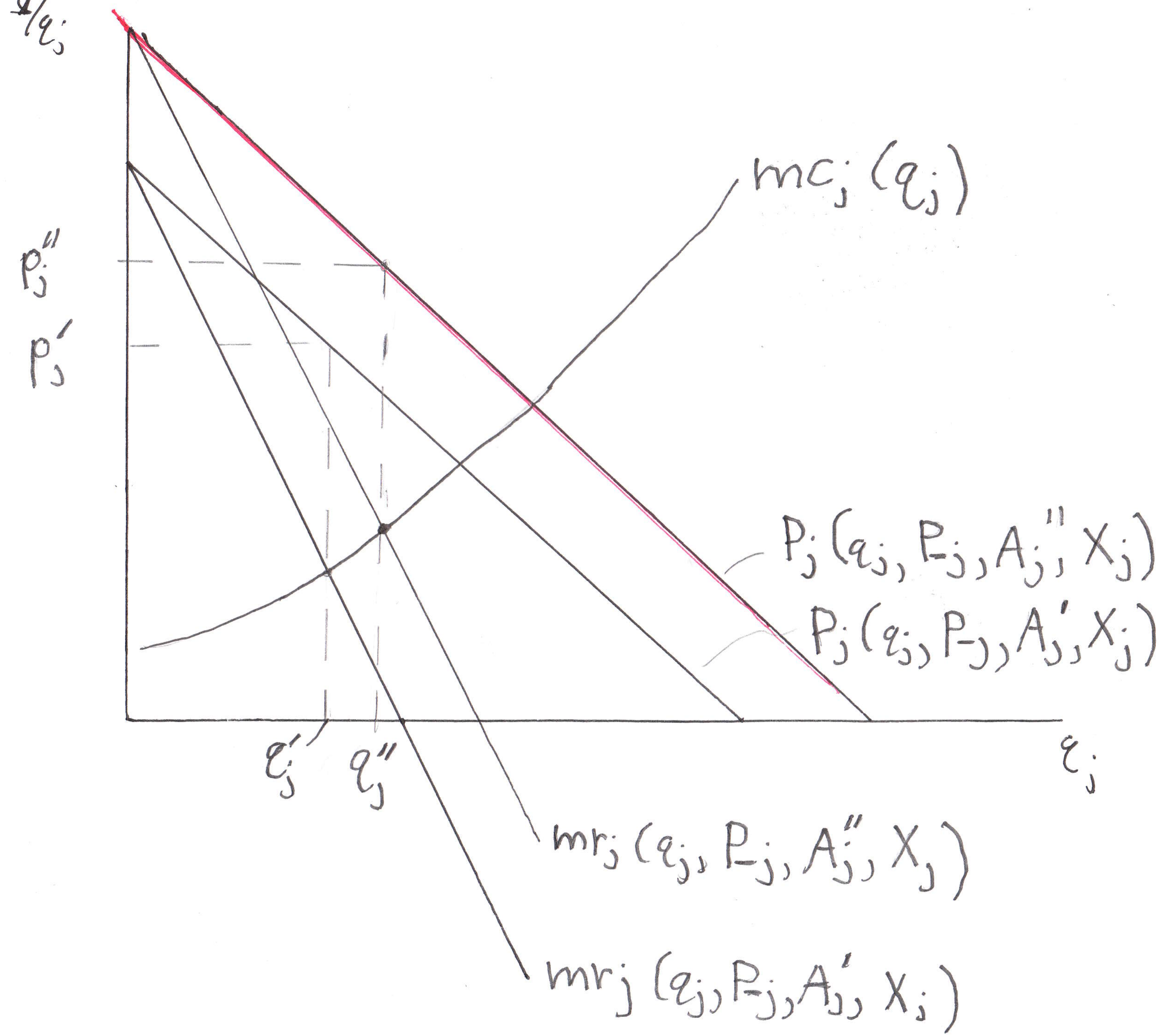
Now, if instead D-C believes that A-C will produce $q_a'' > q_a'$, then D-C (believes) it is faced with the residual demand $P_d(q_d|q_a'')$, and the associated $mr_d(q_d|q_a'')$, and this changes the profit maximizing output for D-C to q_d''. Since the increase in (expected) production by A-C from q_a' to q_a'' resulted in a decrease in D-C's production from q_d' to q_d'', we can conclude that the reaction function $r_d(q_a)$ is a decreasing function of q_a.

Fig. 1



$/q_j
$\overline{c}_j(q_j)/q_j$
$MC_j(q_j)$
p_j^*
$c_j(q_j)/q_j$
$\overline{c}_j(q_j^*)/q_j^*$
$c_j(q_j^*)/q_j^*$
$P_j(q_j | P_{-j}, X_j)$
q_j^*
$mr_j(q_j | P_{-j}, X_j)$
q_j

Fig. 2



$/q_j
p_j''
p_j'
mc_j (q_j)
P_j (q_j, P_-j, A_j'', X_j)
P_j (q_j, P_-j, A_j', X_j)
q_j'
q_j''
q_j
mr_j (q_j, P_-j, A_j'', X_j)
mr_j (q_j, P_-j, A_j', X_j)

Fig. 3