Question 1 (30%)

A car weighing 50 kg is supported by a steel cable through a wheel on an inclined track, as shown in Fig. 1. The friction coefficient between the car and the track is 0.4 and the friction coefficient between the cable and the wheel is 0.2. The cable is fixed on a rigid 2 m-long rod, see Fig.1. Calculate the value of the force *P*, applied at the free end of the rod, required to raise the car.

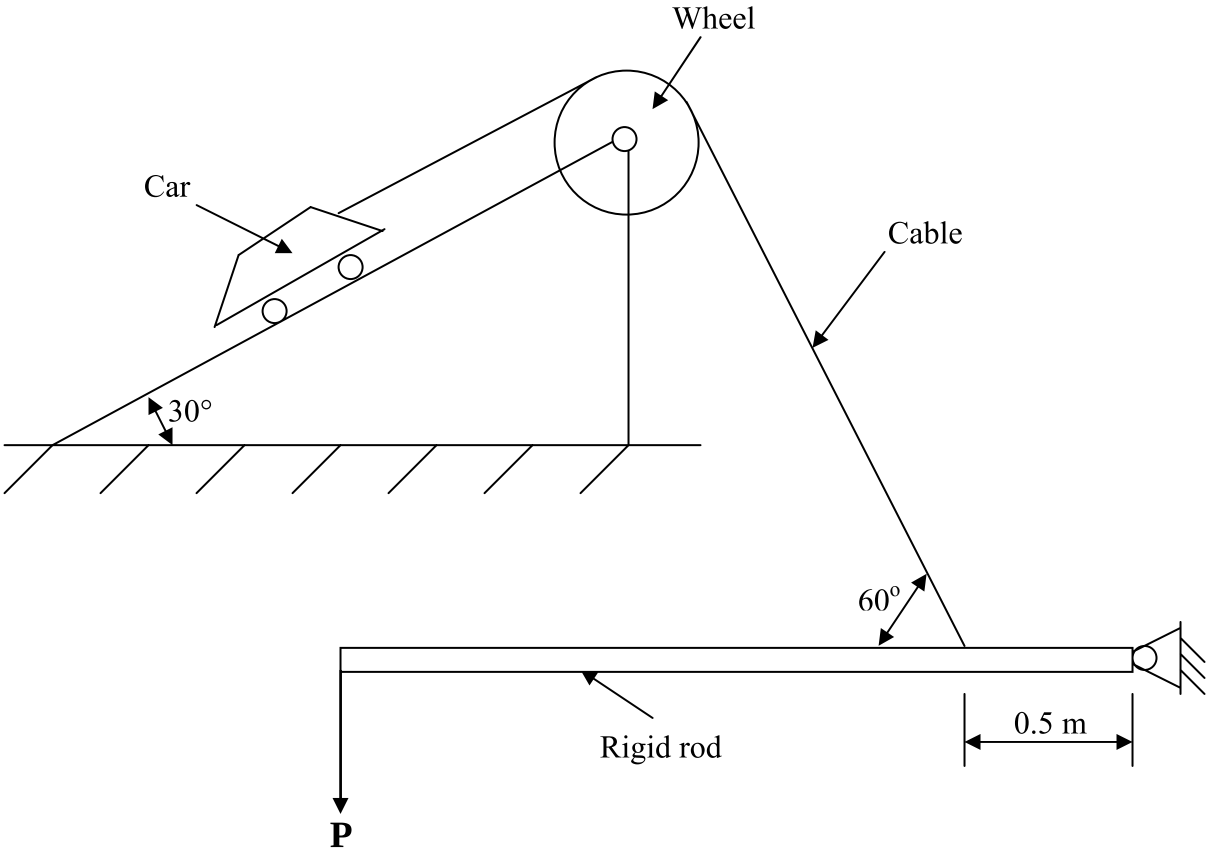
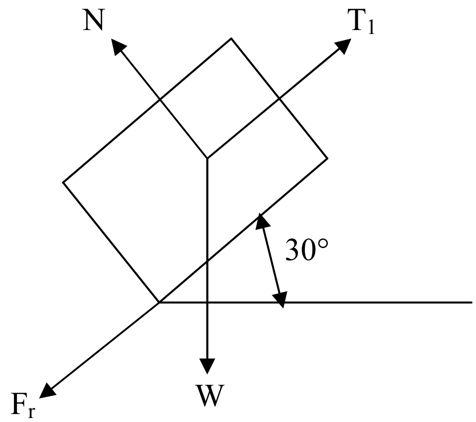


Fig. 1

Solution:

Consider force equilibrium of the car,



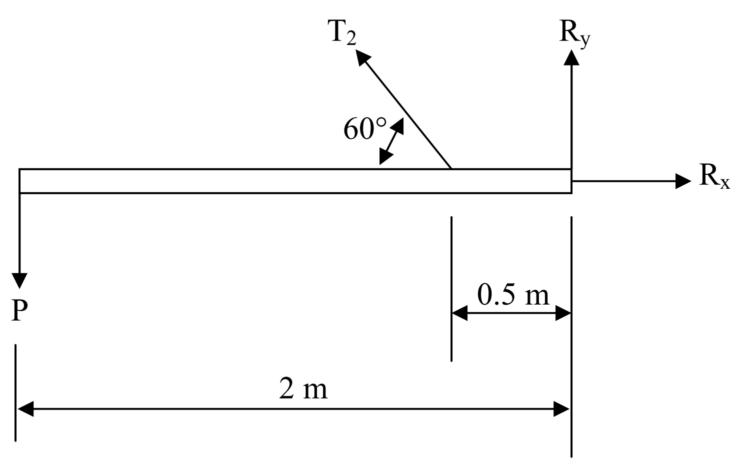
$$T_1 - F_r - W\sin 30° = 0$$
$$N - W\cos 30° = 0$$
$$F_r = \mu N = 0.4N$$

Solving the above equations yields $T_1 = 414.74$ N.

Consider the wheel,

$\frac{T_2}{T_1} = e^{\mu\beta}$, $\beta = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$, then $T_2 = T_1 e^{\mu\beta} = 414.74 e^{0.2\times\frac{\pi}{2}} = 567.73$ N.

Consider force equilibrium of the rod,



$$P \cdot 2 = (T_2 \sin 60°) \cdot 0.5 = (567.73 \sin 60°) \cdot 0.5 \text{, then}$$
$$P = 122.92 \text{ N}$$

Question 2 (40%)

In the system of Fig. 2, wires *a* and *b* are all made of aluminium alloy with $E = 70$ GPa and have a same diameter of 5 mm. They are jointed at *A* in a roof and the other ends are fixed on a rigid 3 m-long rod, as shown in Fig. 2. The rod is supported by a spring with the stiffness $k = 2$ kN/mm at *B* (Line *AB* is perpendicular to the rod). Calculate the stresses and deformations of wires *a* and *b*, respectively, when a force of 50 kN is applied at the free end of the rod, see Fig. 2.

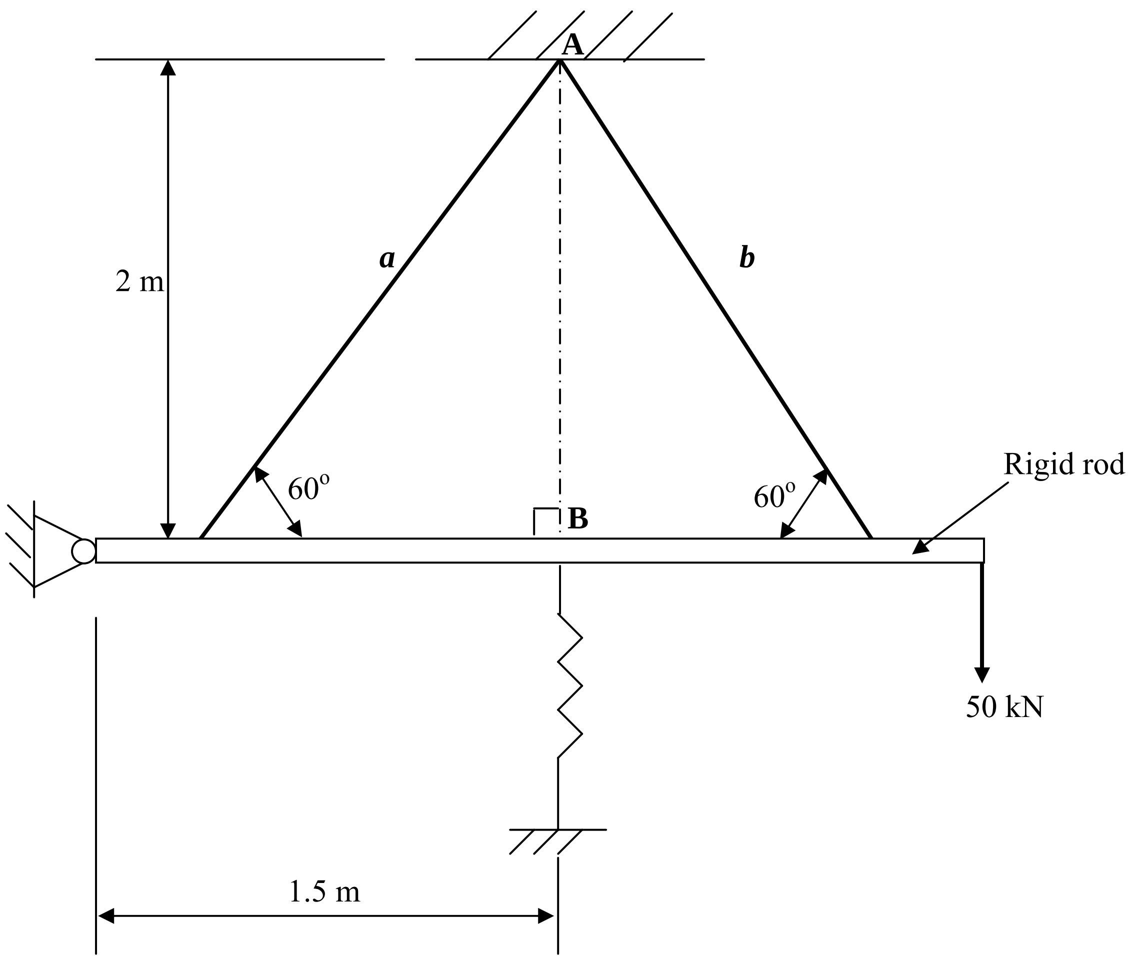
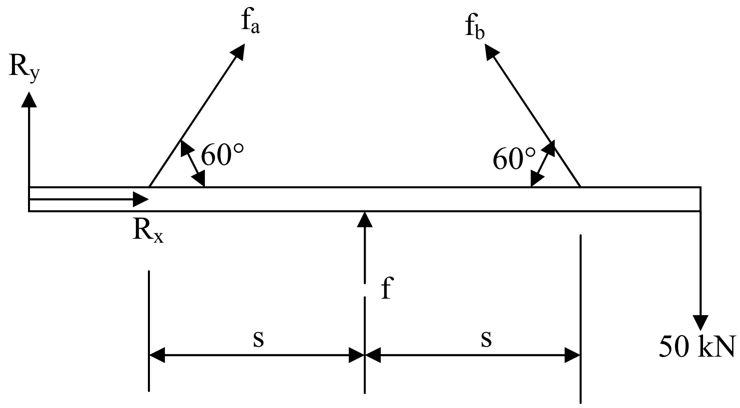


Fig. 2

Solution:

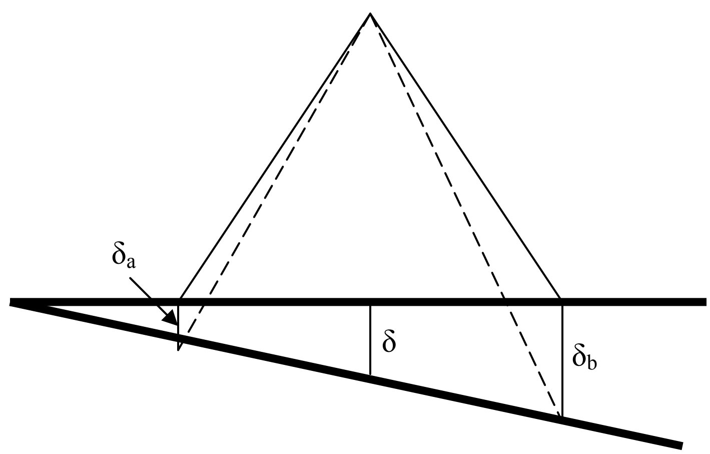
Consider force equilibrium of the rod,



$$s = \frac{2}{\tan 60°} = 1.155 \text{ m}$$

$$50 \times 3 - f \cdot 1.5 - (f_b \sin 60°)(1.5 + 1.155) - (f_a \sin 60°)(1.5 - 1.155) = 0$$
$$1.5 f + 2.3 f_b + 0.3 f_a = 150$$

Consider deformation of the wires,



$$\frac{\delta_a}{\delta} = \frac{1.5 - 1.155}{1.5} = \frac{0.345}{1.5},$$

$$\frac{\delta_b}{\delta} = \frac{1.5 + 1.155}{1.5} = \frac{2.655}{1.5}$$, then $\delta_a = 0.23\delta$ and $\delta_b = 1.77\delta$.

$$f_a = \sigma_a A = \varepsilon_a EA = \frac{\Delta l_a}{l} EA,$$

$$f_b = \sigma_b A = \varepsilon_b EA = \frac{\Delta l_b}{l} EA.$$

Note: $\delta_a \neq \Delta l_a$ and $\delta_b \neq \Delta l_b$, $\Delta l_a = \delta_a \sin 60°$ and $\Delta l_b = \delta_b \sin 60°$.

$$l = \frac{2}{\sin 60°} = 2.3 \text{ m.}$$

Therefore $f_a = \frac{\Delta l_a}{l} EA = \frac{EA}{l}\delta_a \sin 60° = \frac{EA}{l}\cdot 0.1992\delta$,

$$f_b = \frac{\Delta l_b}{l} EA = \frac{EA}{l}\delta_b \sin 60° = \frac{EA}{l}\cdot 1.5328\delta.$$

$$1.5k\delta + 2.3\times 1.5328\delta\frac{EA}{l} + 0.3\times 0.1992\delta\frac{EA}{l} = 150$$

$$1.5\times 2\times 10^3\delta + 3.5852\times\frac{70\times 10^6}{2.3}\pi\left(\frac{5}{2}\right)^2\times 10^{-6}\delta = 150$$

$$\delta = 0.0292 \text{ m} = 29.2 \text{ mm}$$

$$\Delta l_a = 0.23\times 29.2\sin 60° = 5.82 \text{ mm}$$

$$\sigma_a = \frac{\Delta l_a}{l}E = \frac{5.82}{2.3\times 10^3}\times 70\times 10^3 = 177.13 \text{ MPa}$$

$$\Delta l_b = 1.77\times 29.2\sin 60° = 44.76 \text{ mm}$$

$$\sigma_b = \frac{\Delta l_b}{l}E = \frac{44.76}{2.3\times 10^3}\times 70\times 10^3 = 1362.26 \text{ MPa}$$

Question 3 (30%)

A shaft consists of a thin-walled cylinder and a rod, with one end fixed in the wall, a torque T_1 applied at the free end and a torque T_2 applied at the joint (T_1 and T_2 are in opposite directions), as shown in Fig. 3. The cylinder and the rod are made of a same material with the shear modulus G and have the same radius R and length l. The thickness of the cylinder is t. Determine the ratio T_1/T_2 which causes an equal amount of maximum shear stress in the cylinder and the rod. Also determine the angle of twist at the joint and at the free end, respectively, when the cylinder and the rod have a same amount of maximum shear stress.

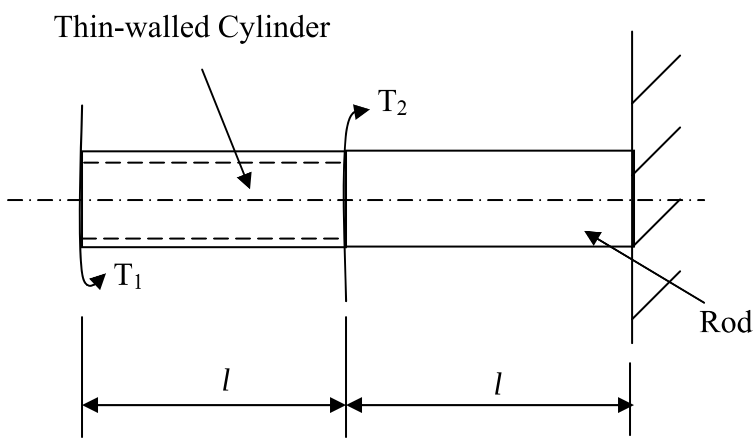
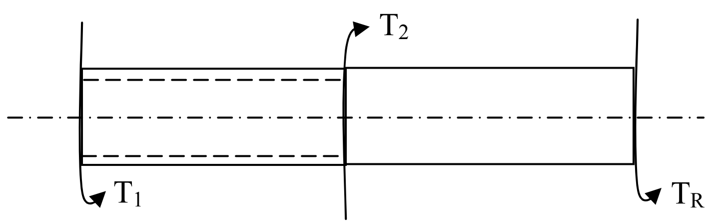


Fig. 3

Solution:

Consider force equilibrium of the whole system,



$$T_1 + T_R = T_2$$

For the cylinder, $T_c = T_1$, $\tau_{c\max} = \dfrac{T_c}{2\pi R^2 t} = \dfrac{T_1}{2\pi R^2 t}$.

For the rod, $T_r = T_R = T_2 - T_1$, $\tau_{r\max} = \dfrac{T_r R}{J} = \dfrac{(T_2 - T_1)R}{\dfrac{\pi R^4}{2}} = \dfrac{2(T_2 - T_1)}{\pi R^3}$.

Since $\tau_{c\max} = \tau_{r\max}$, $\dfrac{T_1}{2\pi R^2 t} = \dfrac{2(T_2 - T_1)}{\pi R^3}$, therefore $\dfrac{T_1}{T_2} = \dfrac{4t}{R + 4t}$.

Consider the deformations, for the rod, $\dfrac{G\theta_r}{l} = \dfrac{T_r}{J} = \dfrac{T_2 - T_1}{\dfrac{\pi}{2}R^4}$, then $\theta_r = \dfrac{l}{G}\left(\dfrac{T_2 - T_1}{\dfrac{\pi}{2}R^4}\right) = \dfrac{2l}{\pi R^4 G}(T_2 - T_1)$;

For the cylinder, $\theta_c = \theta_r - \theta_1$, $\dfrac{G\theta_1}{l} = \dfrac{T_c}{2\pi R^3 t} = \dfrac{T_1}{2\pi R^3 t}$,

$$\theta_1 = \frac{l}{G}\left(\frac{T_1}{2\pi R^3 t}\right),$$

$$\theta_c = \frac{2l(T_2 - T_1)}{\pi R^4 G} - \frac{l}{G}\left(\frac{T_1}{2\pi R^3 t}\right)$$

$$= \frac{l}{G\pi R^3}\left(\frac{2}{R}(T_2 - T_1) - \frac{T_1}{2t}\right)$$

$$=0$$