CONCORDIA UNIVERSITY

Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	209	ALL Except EC
Examination	**Date**	**Pages**
Final	December 2012	3
Instructors		**Course Examiner**
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Special Instructions

- ▷ Ruled booklets to be used.
- ▷ Only approved calculators are allowed.

MARKS

[9] **1.** (a) Let $f(x) = \frac{3-x^2}{2x^3-x^2+9}$. Find the limits:

(i) $\lim_{x\to 1} f(x)$ (ii) $\lim_{x\to\infty} f(x)$

(b) Given that $\lim_{x\to -3} g(x) = 4$ and $\lim_{x\to -3} h(x) = -5$, find the limit

$$\lim_{x\to -3} \sqrt{g(x) - h(x)}.$$

(c) True or False; if $\lim_{x\to 4} k(x) = 5$, then $\lim_{x\to 3} k(x) = 4$. Explain your answer.

[13] **2.** (a) If $g(x) = -3x^4 + 2x^2 - \pi$, find $g'(x)$.

(b) If $f(x) = (\ln(x) + x)(2x^2 - 5)$, find $f'(x)$.

(c) If $y = \frac{(e^x - x)}{(x^2 - 2x)}$, find y'.

(d) If If $y = \sqrt[3]{x^5 - 7}$, find y' =?

(e) Find y' if $e^y = y^3 - 2x$.

[11] 3. Given the price-demand equation

$$.03x + 4p = 30$$

4 (A) Express the demand x as a function of price p.

4 (B) Express the revenue R as a function of the price p.

3 (C) Find the elasticity of demand, $E(p)$.

[11] 4. A small machine shop manufactures drill bits used in the petroleum industry. The shop manager estimates that the total daily cost (in dollars) of producing x bits is

$$C(x) = 1,000 + 25x - 0.1x^2$$

4 (A) Find $\bar{C}(x)$ and $\bar{C}'(x)$.

4 (B) Find $\bar{C}(10)$ and $\bar{C}'(10)$, and interpret these quantities.

3 (C) Use the results in part (B) to estimate the average cost per bit at a production level of 11 bits per day.

[10] 5. Find dy for $y = f(x) = \sqrt{x} + 3$. Evaluate dy for

5 (A) $x = 4$ and $dx = 0.1$. $= \frac{1}{40} = .025$

5 (B) $x = 9$ and $dx = 0.12$. $= \frac{1}{50} = .02$

[12] 6. Compute the following:

2 each

(a) $\int e^{-3x}\, dx$

(b) $\int (4x^3 - 7x^6)\, dx$

(c) $\int (x+9)^{-8}\, dx$

(d) $\int (ex^5 - x^2)\, dx$

(e) $\int \frac{x^2}{7 - x^3}\, dx$

(f) $\int_3^4 \frac{1}{x-2}\, dx$

[6] / **7.** Find the absolute maximum and absolute minimum value of $f(x) = x^3 - 12x$ on the interval $[-3, 3]$.

[6] / **8.** Is there a function f from the reals to the reals which is not continuous, but has a continuous square? Justify your answer.

[11] / **9.** Use the graphing strategy to analyze the function

$$h(x) = \tfrac{2x-1}{x^2}.$$

State all pertinent information and sketch a graph of h.

[11] / **10.** The Lorenz curve for a small country is $f(x) = x^{2.3}$. Graph the curve and find the Gini index for this country.

1. A. i. $\lim\limits_{x\to 1} \dfrac{3-x^2}{2x^3-x^2+9} = \dfrac{2}{2-1+9} = \dfrac{2}{10} = \dfrac{1}{5}$.

ii. $\lim\limits_{x\to \infty} \dfrac{3-x^2}{2x^3-x^2+9} = \dfrac{-\infty}{\infty}$ I.F.

$$\Rightarrow \lim_{x\to\infty} \frac{x^2\left(\frac{3}{x^2}-1\right)}{x^3\left(2-\frac{1}{x}+\frac{9}{x^3}\right)}$$

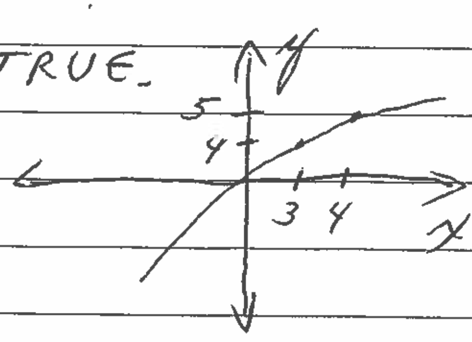
$$= \lim_{x\to\infty} \frac{\left(\frac{3}{x^2}-1\right)}{x\left(2-\frac{1}{x}+\frac{9}{x^3}\right)} = \frac{0-1}{\infty(2-0+0)} = 0$$

B. $\lim\limits_{x\to -3} \sqrt{g(x)-h(x)} = \sqrt{\lim\limits_{x\to -3} g(x) - \lim\limits_{x\to -3} h(x)}$

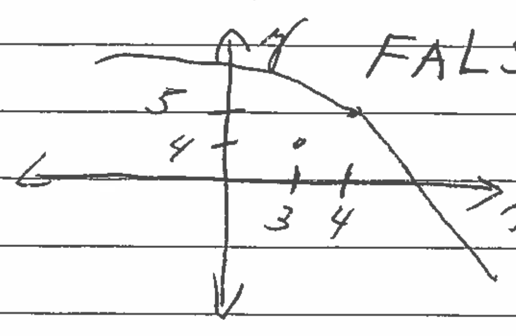
$= \sqrt{4-(-5)} = \sqrt{9} = 3$.

C. True or False, depends on function.

TRUE.



FALSE



2. A. $y = -3x^4 + 2x^2 - \pi$

$y' = -12x^3 + 4x.$

B. $y = (\ln(x) + x)(2x^2 - 5)$

$y' = (\ln x + x)(4x) + (2x^2 - 5)\left(\frac{1}{x} + 1\right).$

C. $y = \frac{e^x - x}{(x^2 - 2x)}$

$$y' = \frac{(x^2 - 2x)(e^x - 1) - (e^x - x)(2x - 2)}{(x^2 - 2x)^2}.$$

D. $y = \sqrt[3]{x^5 - 7} = (x^5 - 7)^{1/3}$

$y' = \frac{1}{3}(x^5 - 7)^{-2/3}(5x^4)$

E. $e^y = y^3 - 2x$

$e^y y' = 3y^2 y' - 2$

$e^y y' - 3y^2 y' = -2$

$y'(e^y - 3y^2) = -2 \Rightarrow y' = \frac{-2}{e^y - 3y^2}.$

3. $.03x + 4p = 30$

A. $.03x = 30 - 4p$

$$x = \frac{30 - 4p}{.03} = 1000 - \frac{400}{3}p.$$

B. $R = xp = \left(1000 - \frac{400}{3}p\right)p$

$$R = 1000p - \frac{400}{3}p^2.$$

C. $E(p) = -\dfrac{p\,f'(p)}{f(p)}$

$$= \frac{-p\left(-\frac{400}{3}\right)}{\left(1000 - \frac{400}{3}p\right)} = \frac{400p}{3000 - 400p}.$$

4. $C(x) = 1{,}000 + 25x - 0.1x^2$

A. $\bar{C}(x) = \dfrac{C(x)}{x} = \dfrac{1000 + 25x - 0.1x^2}{x}$

$\bar{C}(x) = \dfrac{1000}{x} + 25 - .1x.$

$\bar{C}'(x) = -\dfrac{1000}{x^2} - 0.1.$

B. $\bar{C}(10) = \dfrac{1000}{10} + 25 - 0.1(10)$

$\bar{C}(10) = 100 + 25 - 1 = \$124.$

average cost ~~after~~ of producing 10 units is \$124.

$\bar{C}'(10) = -\dfrac{1000}{(10)^2} - 0.1 = -10 - 0.1$

$\bar{C}'(10) = -10.1.$

approximate cost of producing 1 more unit after 10 units produced is decreasing by \$10.1.

OR

Rate of change of cost is $-\$10.1$/unit after 10 units produced!

c. $\overline{C}(x+1) \approx \overline{C}(x) + \overline{C}'(x)$

$\overline{C}(11) \approx \overline{C}(10) + \overline{C}'(10)$

$\overline{C}(11) \approx 124 + (-10.1)$

$\overline{C}(11) \approx 113.9.$

Exact answer: $\overline{C}(11) = 114.8.$

5. $y = f(x) = \sqrt{x} + 3.$

$dy = f'(x)\,dx;\; f'(x) = \frac{1}{2} x^{-1/2}$

A. $dy = \left(\frac{1}{2}(4)^{-1/2}\right) .1 = \left(\frac{1}{4}\right)(.1) = \frac{1}{40}.$

B. $dy = \left(\frac{1}{2}(9)^{-1/2}\right)(.12) = \left(\frac{1}{6}\right)(.12) = \frac{1}{50}.$

6. A. $\int e^{-3x}\,dx = \dfrac{e^{-3x}}{-3} + C$

B. $\int (4x^3 - 7x^6)\,dx = x^4 - x^7 + C.$

C. $\int (x+9)^{-8}\,dx = \dfrac{(x+9)^{-7}}{-7} + C.$

D. $\int (ex^5 - x^2)\,dx = \dfrac{ex^6}{6} - \dfrac{x^3}{3} + C.$

E. $\int \dfrac{x^2}{7-x^3}\,dx = -\dfrac{1}{3}\ln|7-x^3| + C.$

F. $\int_3^4 \dfrac{1}{x-2}\,dx = \ln|x-2|\Big|_3^4$

$= \ln|4-2| - \ln|3-2|$

$= \ln(2) - \ln(1) = \ln(2).$

$= .69.$

7. $f(x) = x^3 - 12x \quad [-3, 3]$

$f'(x) = 3x^2 - 12 = 0$

$x^2 = 4$

$x = \pm 2 \Rightarrow$

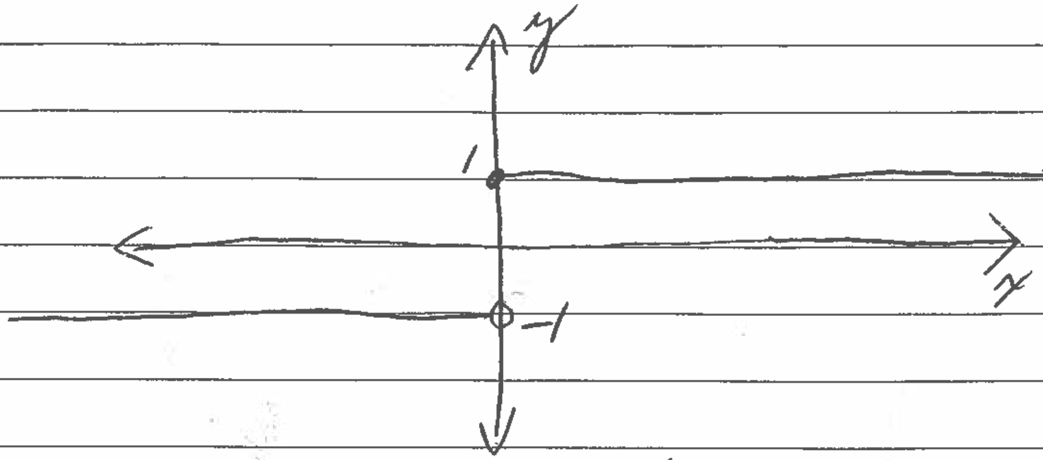
$f(-3) = +9$

$f(-2) = 16$ A. Max.

$f(2) = -16$ A. min.

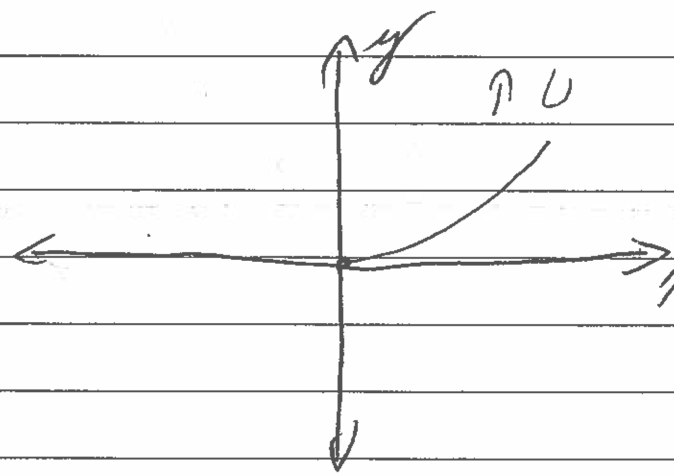
$f(3) = -9$

8.



$$f(x) = \begin{cases} 1 \text{ if } x \geq 0 \\ -1 \text{ if } x < 0 \end{cases}$$

10. $f(x) = x^{2.3}$ D.: $x \geq 0$ R.: $y \geq 0$



$$G.I. = 2\int_0^1 [x - f(x)]\,dx$$

$$= 2\int_0^1 [x - x^{2.3}]\,dx$$

$$= 2\left(\frac{x^2}{2} - \frac{x^{3.3}}{3.3}\right)\Big|_0^1$$

$$G.I. = 2\left(\frac{1}{2} - \frac{1}{3.3}\right) - 0 = \frac{2(1.3)}{6.6} = \frac{1.3}{3.3}.$$

9. $f(x) = \frac{2x-1}{x^2}$.

D.: $\mathbb{R}/0$ R.: $y \le 1$

x-inter.: $1/2$

y-inter.: $\emptyset$

V.A.: $x=0$

H.A.: $y=0$

C.P.: $(1,1)$

P.I.: $(3/2,$

Inc.: $(0,1)$

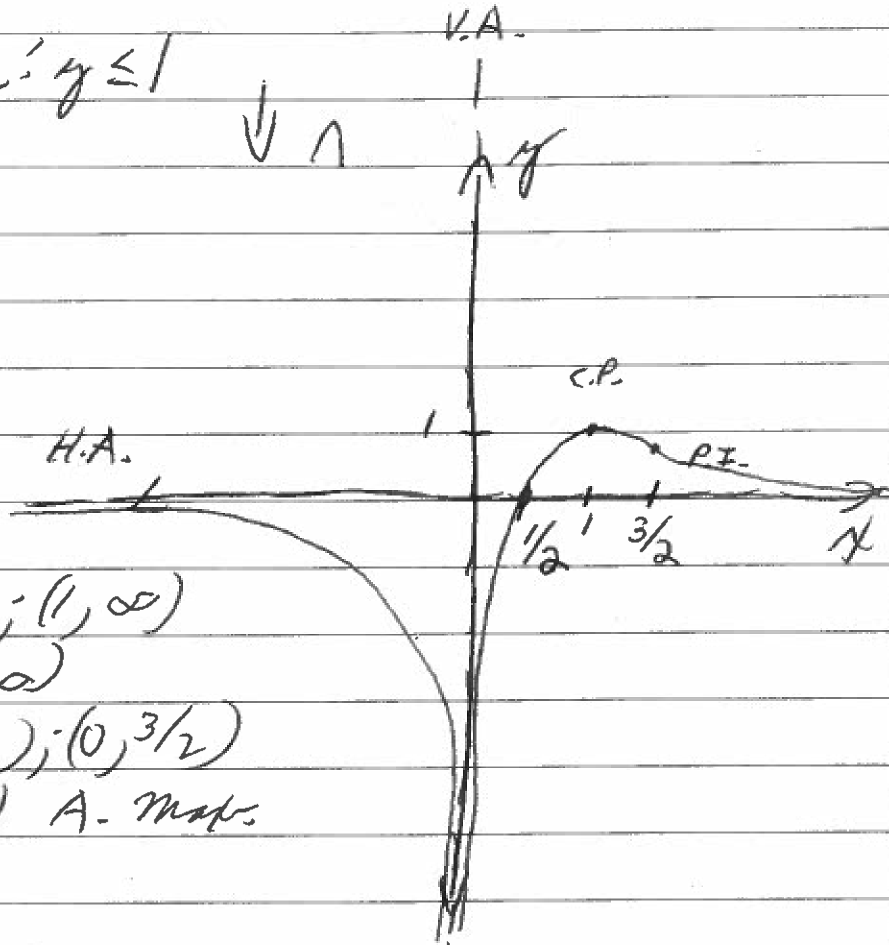
Dec.: $(-\infty,0); (1,\infty)$

C.U.: $(3/2,\infty)$

C.D.: $(-\infty,0); (0,3/2)$

MAX.: $(1,1)$ A. Max.

MIN.: $\emptyset$



$$f(x) = \frac{2x-1}{x^2} = 2x^{-1} - x^{-2}; \cancel{f'(x) = 2}$$

$$f'(x) = \frac{x^2(2) - (2x-1)(2x)}{x^4}$$

$$f'(x) = \frac{-2x^2+2x}{x^4} = 0$$

$$\frac{2-2x}{x^3} = 0 \Rightarrow \text{C.V.: } x=1$$

$$f'(x) = 2x^{-3} - 2x^{-2}$$

$$f''(x) = -6x^{-4} + 4x^{-3} = 0 \Rightarrow 2x^{-4}(3-2x) = 0$$

$$x = 3/2$$

S.O.T.

$f''(1) < 0 \Rightarrow (1,1)$ is a L. max. ∩

C.T.

$f''(-1) < 0$ ∩

F.D.T.

$f'(-1) < 0$ ↓