PRACTICE PHYSICS 101 EXAM

Problem 1

I. A student, holding a 500 Hz tuning fork, is walking at 1.50 m/s on a direct line between two parallel smooth reflecting walls. What frequency of sound are heard by the student?

Answer

At the wall the student approaches, $f' = f\left(\frac{v}{v - v_s}\right)$ because the wall is a stationary "observer" watching a moving source.
For the sound reflected by the wall, the wall is a stationary "source" that reflects the same frequency it "hears", and the student is a moving observer, so if the student hears frequency f'', where:

$$f'' = f'\left(\frac{v + v_o}{v}\right) = f\left(\frac{v + v_o}{v - v_s}\right) = 500Hz\left(\frac{341m/s + 1.5m/s}{341m/s - 1.5m/s}\right) = 504.4Hz$$

Similarly, for receding wall, (the student is going away from the wall, the velocities have inverse signs):

$$f'' = f\left(\frac{v + v_o}{v - v_s}\right) = 500Hz\left(\frac{341m/s - 1.5m/s}{341m/s + 1.5m/s}\right) = 495.6Hz$$

II. A column of air in a tube is found to have standing waves at frequencies of 380, 532 and 684 Hz. There are no standing wave frequencies between the above frequencies.

a) What is the fundamental frequency of the tube?

Answer

Resonant frequency for sound waves in a tube are uniformly spaced. Notice the frequency spacing here is 152 Hz because 684Hz-532Hz=532Hz-380Hz=152Hz. Keep subtracting multiples of 152Hz until you reach a frequency <152Hz and you'll reach the fundamental frequency $f_{fundamental}$=76Hz.

b) Is the tube open at both ends or open at one end and closed at the other end?

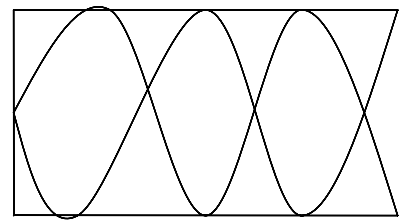
Answer

For a tube open at both ends, the fundamental frequency is the same as the frequency of spacing. For a tube closed at one end, the fundamental frequency is on half of the frequency of spacing. The latter is true, so the tube is closed at one end.

c) Draw a displacement curve for the 532Hz standing wave.

Answer

The fundamental frequency has one node on the close end of the tube and no nodes between the ends of the tube. The open end of the tube has an antinode. For each multiple of 152Hz we add to the fundamental frequency to get to 532Hz wave, we add a node.



532Hz=76Hz+152Hz×3, so we add 3 nodes between the ends of the tube.

d) the air in the tube is now replaced with carbon dioxide which has a speed of sound of 280m/s. What are the new frequencies corresponding to the given standing wave frequencies?

Answer

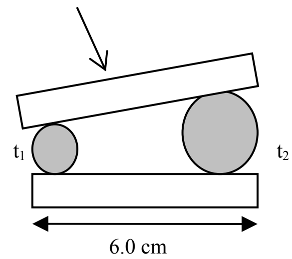
Since geometry of the tube is unchanged, the resonant wavelength are unchanged. Since wavelength is unchanged, $\lambda = \frac{v_{air}}{f_{air}} = \frac{v_{CO_2}}{f_{CO_2}}$.

→ $f_{CO_2} = f_{air} \times \frac{v_{CO_2}}{v_{air}} = f_{air} \times \frac{280m/s}{341m/s} = 0.8211 f_{air}$

so, the new frequencies are 312Hz, 436.8Hz and 561.6Hz.

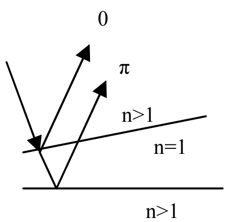
Problem 2

Two flat glass plates, each of length 6.0cm, are separated at each end by two support of thickness t_1=0.00125 mm and t_2=0.00575 mm, as shown in the figure. The air between the glass plates forms a thin film. Green laser light of wavelength 500 nm is incident from the top. When viewed from above, an interference pattern consisting of alternating bright and dark fringes is observed.



a) What is the separation of the two dark fringes?

Answer



For dark fringes, we want the phase difference between the two reflected waves to be π+2mπ for integer values of m.

$$\Delta\phi = \pi + 2m\pi = \pi + 2\times\frac{2nt}{\lambda}\times\pi$$

→ 2nt=2t=mλ (n=1 in air)

If the pieces of glass extended until they touched, this formula would give the number of gaps between fringes, m, to the left of wherever you choose to measure the thickness t. thus, you can count the number of fringes along the 6 cm stretch by the following formula:

$$m_2 - m_1 = \frac{2t_2 - 2t_1}{\lambda} = \frac{2}{500\times10^{-9}}(5.75\times10^{-6} - 1.25\times10^{-6})m = 18$$

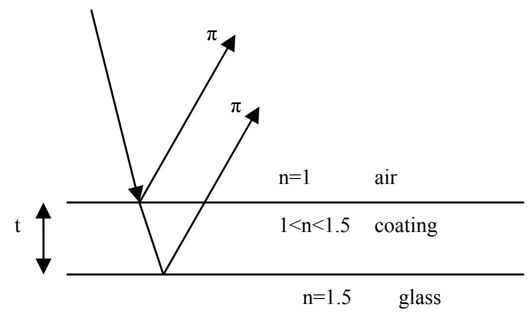
The far left end of the plate has a dark fringe since $m_1=2t_1\lambda=5$ is an integer.

The far right edge of the plate has a dark fringe since $m_2=2t_2\lambda=23$ is n integer. If there are 18 gaps between fringes, then the spacing between fringes is 6cm/18=0.33cm.

b) What must be the index of refraction of a 150 nm thick coating (with index of refraction less than 1.5) on a glass surface (n=1.5) if it is to eliminate reflection of 650 nm light incident on the coating from air?

Answer

We want destructive interference. Light rays reflected off the coating and light rays reflected off the glass both receive π phase shifts. Therefore, we only need to consider phase shift due to the path difference. We want $2nt=\lambda/2$ for the thinnest coating that blocks λ=650nm light.

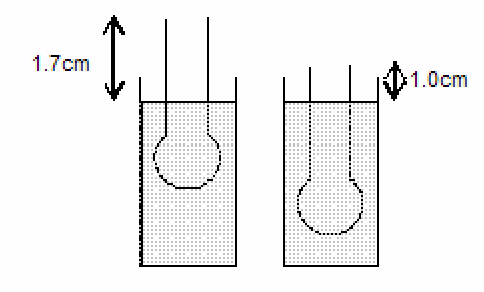


Thus:

$$n = \frac{\lambda}{4t} = \frac{650nm}{4 \times 150nm} = 1.08$$

Problem 3

I. A hydrometer consists of a spherical bulb with a cylindrical stem. The cross-sectional area of the stem is 4.0 cm^2. The total volume of bulb and stem is 16.8 cm^2. When immersed in water, the hydrometer floats with 1.7 cm of the stem above the water surface. In alcohol, 1.0 cm of the stem is above the water surface. Find the density of alcohol.

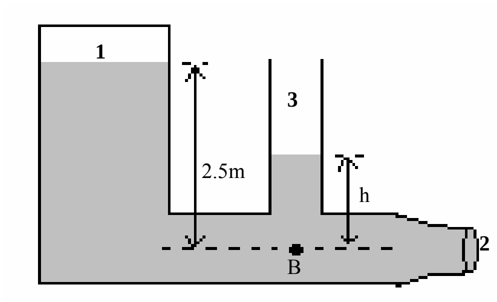


Answer

Volume under water=V_{water}=$V_{hydrometer}$-$V_{above\ water}$
=16.8cm^3-(1.7 cm×4cm^2)=10 cm^3
Mass of hydrometer=m=$\rho_{water}V_{water}$=1g/cm^3×10cm^3=10g
Volume under alcohol=$V_{alcohol}$=$V_{hydrometer}$-$V_{above\ alcohol}$
=16.8cm^3-(4cm^2×1cm)=12.8cm^3

→ $$\rho_{alcohol} = \frac{m}{V_{alcohol}} = \frac{10g}{12.8cm^3} = 0.78125g/cm^3$$

II. Water stands to a height of 2.50 m in a large tank which contains compressed air maintained at an absolute pressure of 1.100×10^5 N/m^2. The horizontal outlet of the pipe has cross-sectional areas of 0.0300 m^2 and 0.0100 m^2 at the larger and smaller sections.



a) What is the velocity of the fluid as it leaves the outlet into the atmosphere?

Answer

Use Bernoulli's equation at point 1 and 2 as seen in the diagram.
For a large tank, v_1≈0. Define h_2=0.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$v_2 = \sqrt{\frac{2}{\rho}(P_1 - P_2 + \rho g h_1)}$$

$$= \sqrt{\frac{2m^3}{10^3 kg}(1.1\times10^5 N/m^2 - 1.013\times10^5 N/m^2 + 10^3 kg/m^3 \times 9.81N/kg \times 2.5m)}$$

$= \sqrt{66.45 m^2 / s^2} = 8.15 m/s$

b) What is the velocity of the fluid at the point B, as shown in the figure?

Answer

$$A_2 v_2 = A_B v_B \quad \rightarrow \quad v_B = \frac{v_1 A_1}{A_B} = \frac{8.15 m/s \times 0.01 m^2}{0.03 m^2} = 2.7 m/s$$

c) To what height h does water stand in the open ended pipe?

Answer

Use Bernoulli's equation between points 1 and B. Again, use $v_1 \approx 0$ for a large tank and define $h_B=0$.

$$P_1 + \rho g h_1 = P_B + \frac{1}{2}\rho v_2^{\ 2} \quad \rightarrow \quad P_B = P_1 + \rho g h_1 - \frac{1}{2}\rho v_2^{\ 2}$$

$$P_B = 1.1 \times 10^5 N/m^2 + 10^3 kg/m^3 \times 9.81 N/kg \times 2.5 m - \frac{1}{2} \times 10^3 kg/m^3 \times (2.7 m/s)^2$$

P_B=130880 N/m^2

The height h is only affected by the static pressure at point B, so define P_3=101.3kPa (atmospheric pressure) and since

$$P_B + \rho g h_B = P_3 + \rho g h \quad \rightarrow \quad h = \frac{P_B - P_3}{\rho g} = \frac{(130880 - 101300) N/m^2}{1000 kg/m^3 \times 9.81 N/kg} = 3.015 m$$

Problem 4

A simple harmonic oscillator, consisting of a mass, m, on a frictionless horizontal surface connected to a spring, has a period of 0.8s and an amplitude of 8.0cm.

a) Write an equation for displacement, x, as a cosine function of time if at time t=0s the mass is at x=-0.30cm from the equilibrium position.

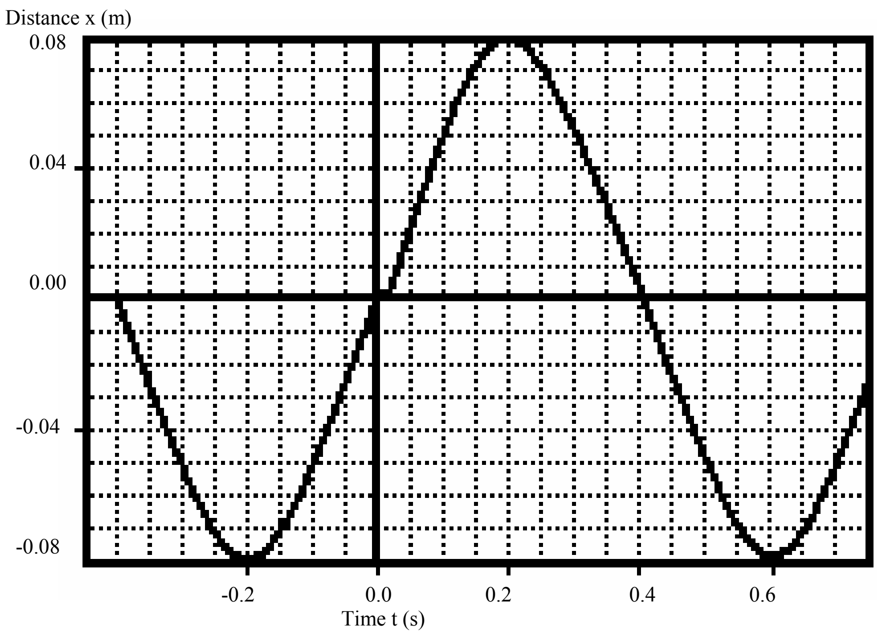
Answer

We seek an equation of the form $x(t)=A\cos(\omega t+\varphi)$.
Since $f=1/T=1.25\ s^{-1}$, then $\omega=2\pi f=2.5\pi\ s^{-1}$.
We can write an equation $x(t)=0.08\ m \times \cos(2.5\pi\ s^{-1} \times t + \varphi)$.
We are given x(0)=-0.03m so:
$x(0)=-0.03=0.08\times\cos\varphi$ → $\cos\varphi=-0.267$ → $\varphi=105.5°$
Therefore, the answer is: $(t)=0.08\ m \times \cos(2.5\pi\ s^{-1} \times t + 105.5°)$.

b) Draw a diagram of displacement as a function of time, starting at time t=0 with a positive velocity.

Answer

In an x vs. t graph, there is positive velocity if v=dx/dt>0. Hence, we want the slope of such a graph to be positive at t=0s. Also, the graph should have amplitude A=8cm and period T=0.8s.



c) It is found that a force of 5.0 N stretches the spring 3.0 cm further than does a force of 3.0 N. Find the value of the mass m.

Answer

Use the equation F=kx. Let x_1 and x_2 be the distances that the mass has been stretched. From the information given, we know $3N=kx_1$, $5N=kx_2$ and $x_2-x_1=3cm=0.03m$. So:

$5N-3N=kx_2-kx_1$ → $2N=k(x_2-x_1)=k(3\ cm)$

→ $k=\dfrac{2N}{0.03m}=666.7N/m$

→ $m=\dfrac{k}{\omega^2}=\dfrac{666.7N/m}{(2.5\pi\, s^{-1})^2}=10.81kg$

d) Find an expression for the acceleration, a, in terms of the angular velocity, ω, when the displacement is 4.0 cm.

Answer

$$a=\frac{d^2x}{dt^2}=\frac{d^2}{dt^2}\left[A\cos(\omega t+\phi)\right]=-A\omega^2\cos(\omega t+\phi)=-\omega^2 x$$

When the displacemement is x=4 cm=0.04m, the acceleration is: A=-0.04ω^2.

e) At what displacement is the potential energy equal to the kinetic energy?

Answer

We must find x such that $E_k = \frac{1}{2}mv^2 = U = \frac{1}{2}kx^2$. Usinf the general form of the harmonic oscillator equation x(t)=Acos(ωt+φ), one can find:

V=dx/dt=-ωAsin(ωt+φ) → $E_k = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi)$

A judicious substitutions of $\sin^2\theta$=1-$\cos^2\theta$ and m=k/ω^2 gives us:

$$E_k = \frac{1}{2}\left(\frac{k}{\omega^2}\right)\omega^2 A^2 \left[1 - \cos^2(\omega t + \phi)\right] = \frac{1}{2}k\left[A^2 - x^2\right]$$

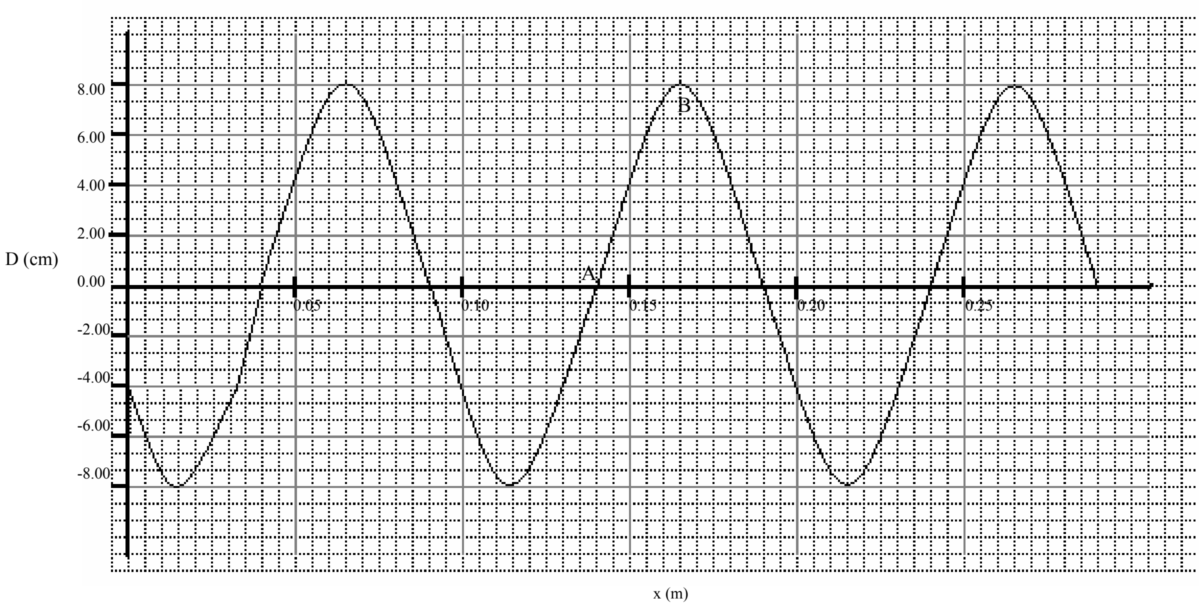
Substituting into the equation E_k=U gives:

$$E_k = \frac{1}{2}k\left[A^2 - x^2\right] = U = \frac{1}{2}kx^2 \quad \rightarrow \quad \frac{1}{2}kA^2 = kx^2$$

$$\rightarrow x = \sqrt{\frac{A^2}{2}} = \sqrt{\frac{(0.08m)^2}{2}} = 0.056568m$$

Problem 5

I. The graph shows a plot at t=0.50s of the displacement of a 3.0 Hz wave traveling in the –x direction along a string.



a) What is the speed of the wave?

Answer

We are given the frequency f=3Hz. To get the wavelength λ, we can measure the distance between two peaks on the graph. We get λ=0.165m-0.065m=0.1m. Thus, v=λf=0.1m$\times$3Hz=0.3m/s.

b) Write an equation describing the wave with all the constants evaluated.

Answer

We can fit the equation to a function of the form:
$D(x,t)=A\sin(kx+\omega t+\varphi)$.
We use this form instead of $D(x,t)=A\sin(kx-\omega t+\varphi)$ because the wave is moving in the –x direction. A cosine function can be done too.
The amplitude A can be read from the graph: A=0.08m.
We also have:

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{0.1m}=20\pi m^{-1};\qquad \omega=2\pi f=2\pi\times 3Hz=6\pi\ Hz$$

$$D(x,t)=0.08m\times\sin(20\pi\times x+6\pi\times t+\phi)\ .$$

We know that D(x,t)=A at x=0.065m and t=0.1s, so $\sin(kx+\omega t+\varphi)=1$
→ $kx+\omega t+\varphi=\pi/2+n\pi$, where n is an integer. You can use any number n. Let's arbitrarily pick n=2 to get a nice small number for φ.
Thus:

$$\phi=\frac{\pi}{2}+2\pi-kx-\omega t=\frac{5\pi}{2}-20\pi\times 0.065-6\pi\times 0.1=-1.8\pi=-5.65\,rad\,(=-324°)$$

Therefore $D(x,t)=0.08m\times\sin(20\pi\,x+10\pi t-1.8\pi)$

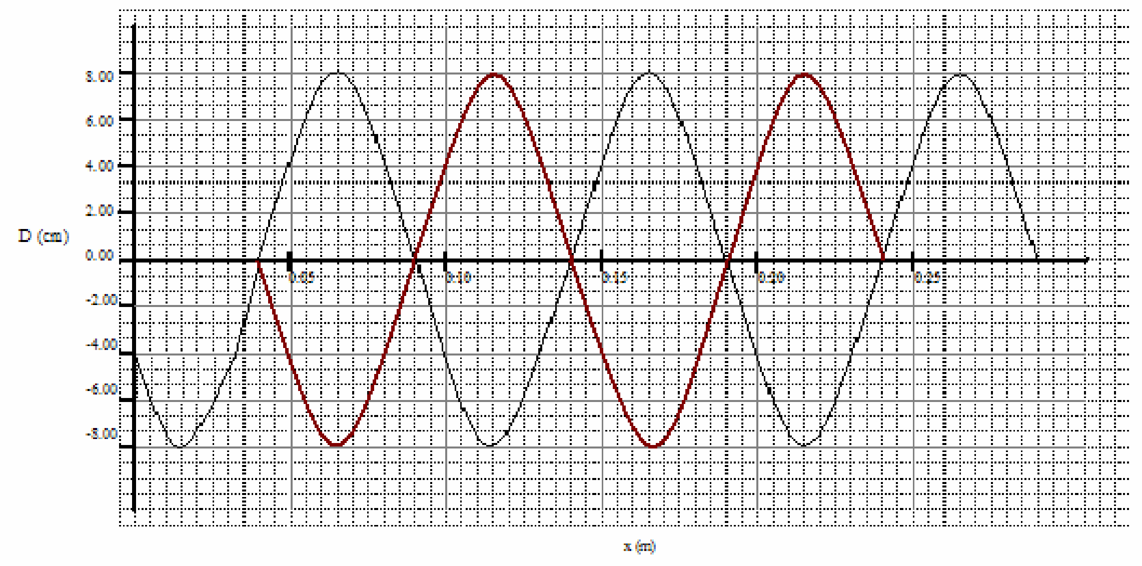
c) What is the transverse speed of the string a t=0.10s for points A and B?

Answer

Since B is at maximum of the displacement, $v_B=0$.
Since A is at equilibrium position, $v_a=v_{max}=A\omega=0.08m\times 6\pi\ s^{-1}$
$v_a=1.5m/s$.

d) On the graph, sketch the displacement at t=0.

Answer
Substitutiong t=0 in our equation from b) is like shifting the original graph by a phase of −3π radians.
To see this, note that at t=0.1s, ωt=2π×3Hz×0.1s=3π. With t=0, we must remove this phase contribution due to time by subtracting π from the phase. This has the effect of shifting the graph half a wavelength to the right.



e) Write the equation of the wave that, when added, will produce a standing wave.

Answer
$D(x,t) = 0.08m \times \sin(-20\pi\, x + 6\pi t - 1.8\pi)$ will work.
So will $D(x,t) = 0.08m \times \sin(20\pi\, x - 6\pi t - 1.8\pi)$.
Basically, any sinusoidal wave of form
$D(x,t) = 0.08m \times \sin(\pm 20\pi\, x \pm 10\pi t - 1.8\pi)$ will work.

II. A jet plane emits 8.0×10^5 J of sound energy per second.

a) What is the sound level in decibels 40.0m away?

Answer

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{8\times10^5 W}{4\pi\times(40.0m)^2} = 39.79W/m^2$$

$$\rightarrow \beta(dB) = 10\log_{10}\frac{39.79W/m^2}{10^{-12}}dB = 135.99dB$$

b) Air absorbs sound at a rate of 7.0 dB/km. What will the sound level 1.5 km away from the plane be, taking into account air absorption?

Answer

Let position A be at 10m away from the plane. Let position B be at 1000 m away from the plane. Without considering the air absorption, $\beta_1 - \beta_2 = 20\log_{10}\frac{r_2}{r_1}dB = 31.48dB$. Now, also taking absorption into account,

$$\beta_2 = \beta_1 - 30.46dB - \beta_{absorbed} = 135.99 - 31.48 - 7\times1.5 = 94dB$$

Problem 6

a) A steel tape measure gives the length of a brass rod to be 300.00 cm when both the tape measure and the rod are at 25°C. What would the tape measure read when both are at 60°C?
[α_{brass}=19×10^{-6} K^{-1} ; α_{steel}=11×10^{-6} K^{-1}]

Answer

The temperature difference is ΔT=60°C-25°C=35°C=35 K (since it's a difference of temperature, ΔT is the same in °C or in K).

The steel tape measure will expend by:

$$\Delta L_{steel} = L_{0\_steel}\alpha_{steel}\Delta T = 300cm \times 11 \times 10^{-6} K^{-1} \times 35 = 0.1155cm$$

The brass rod will expend by:

$$\Delta L_{steel} = L_{0\_steel}\alpha_{steel}\Delta T = 300cm \times 19 \times 10^{-6} K^{-1} \times 35 = 0.1995cm$$

The tape is therefore shorter than the rob by 0.084cm. The tape will therefore measure that the rod is 300.084 cm at 40°C.

b) The average rate at which heat flows through Mars' surface is 70 W/m^2 and the average thermal conductivity of Mars' crust is k=4.1 W/m/K.
What is the temperature at the base of the crust if its thickness is 20km and the surface temperature is 210°C?

Answer

The heat flow for conduction is given by: $H = kA\frac{\Delta T}{L}$

Therefore

$$\Delta T = T_{base} - T_{surface} = \frac{LH}{kA} = \frac{L}{k} \times \text{heat flow} = \frac{20\text{km}}{4.1\,\text{W/m/K}} \times 70W/m^2 = 341.5K \; (= 341.5°C)$$

where $\frac{H}{A} = \frac{1}{A}\frac{dQ}{dt} = \text{heat flow through a surface}$

(since it's a difference of temperature, ΔT is the same in °C or in K)
Since $T_{surface}$=210°C, we have:
$T_{base}=T_{surface}+\Delta T$=210°C+341.5°C=551.5°C

c) How long would it take in space to turn a 1.4 kg sphere of water with surface area 0.06 m^2 into ice in an environment with temperature T_{ext}=5K? (assume ε=1.0, L_{sol_w}=3.33 and that the sphere is "floating" in space)

Answer

The change of temperature of the ice will come from radiation. The heat current between the ice (T_{ice}=0°C) and the outside is therefore:

$$H = \frac{dQ}{dt} = \varepsilon\sigma A T^4 = 1 \times 5.67 \times 10^{-8} W/m^2/K^4 \times 0.06 m^2 \times 273^4 = 18.9W$$

To become ice, the water needs to "loose":

$$Q = mL = 1.4kg \times 3.33 \times 10^5 J/kg = 466{,}200 J$$

It will therefore take:

$$t = \frac{Q}{dQ/dt} = \frac{466{,}200\,J}{18.9\,J/s} = 24{,}666.67s = 6\text{h } 51 \text{ min } 6.7s$$