

Solution

University of Concordia
Department of Mechanical and Industrial Engineering
MECH 344/X-Machine Element Design
Quiz #3- Thursday Dec. 3, 2009

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This examination paper has 5 pages.

TIME ALLOWED: 1.15 minutes

Special notes and instructions

1. OPEN BOOK EXAM (ONLY TEXT BOOK)
2. ENCS standard non-programmable calculator is allowed.
3. Write directly on this exam and bank pages provided. If necessary, write on the back of the facing page.
4. Write your Concordia ID on the space provided on the top of each page.
5. Show work in sufficient detail.
6. DO NOT UNSTAPLE PAGES.

I have read the above instructions

Signature: _____

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Student ID: _____

Problem 1: (60 Marks)

Figure below shows the input shaft of the crane gear box, supported in the gear box by bearings *A* and *B* and driven by electric motor. The applied forces and geometry and dimensions of the hollow shaft and square shaft key are:

$$F_t = 206 \text{ N} \quad ; \quad F_r = 75 \text{ N} \quad ; \quad T = 2.06 \text{ N.m}$$

$$a = 66 \text{ mm} \quad ; \quad b = 84 \text{ mm} \quad ; \quad L = 150 \text{ mm}$$

$$d = 6 \text{ mm} \quad ; \quad d_p = 20 \text{ mm} \quad ; \quad D = 12 \text{ mm}$$

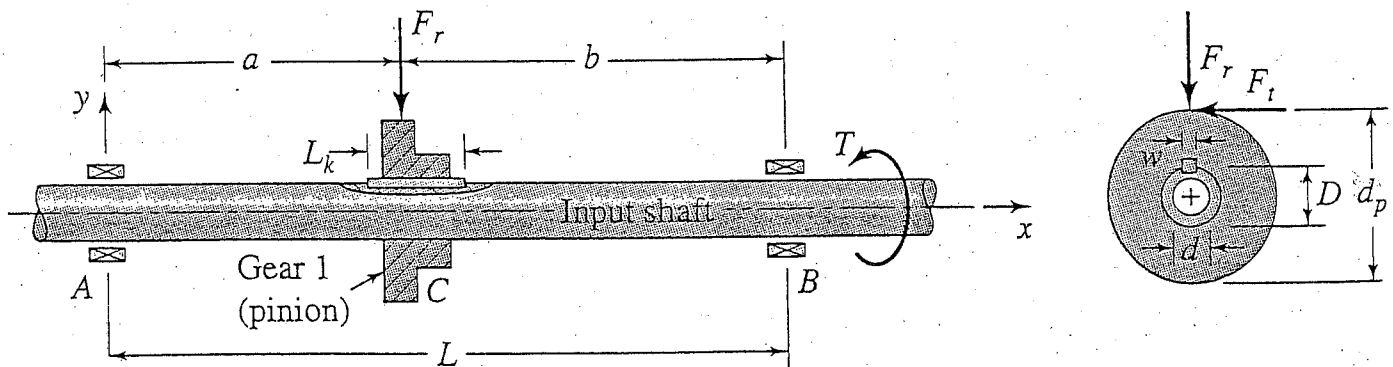
$$w = 2.4 \text{ mm} \quad ; \quad L_k = 25 \text{ mm}$$

$$I = \frac{\pi}{64}(D^4 - d^4) \quad , \quad J = 2 I$$

Assume bearings acts as simple supports. Shaft and shaft key are made of 1030 cold roll steel ($S_{ut} = 520 \text{ MPa}$, $S_y = 440 \text{ MPa}$, $E = 210 \text{ GPA}$) with machined surface. The fatigue stress concentration factor at keyway is assumed to be $k_f = 2$. The reliability of 99.9% and room temperature are considered.

- Draw moment and torsion diagram and identify the critical location on the shaft
- Find fatigue safety factor for the shaft
- Find safety factor for the key against shear and bearing.

Note: Moment is fluctuating and torque is steady. Use general method which is based on von Mises stress and modified-Good man diagram.

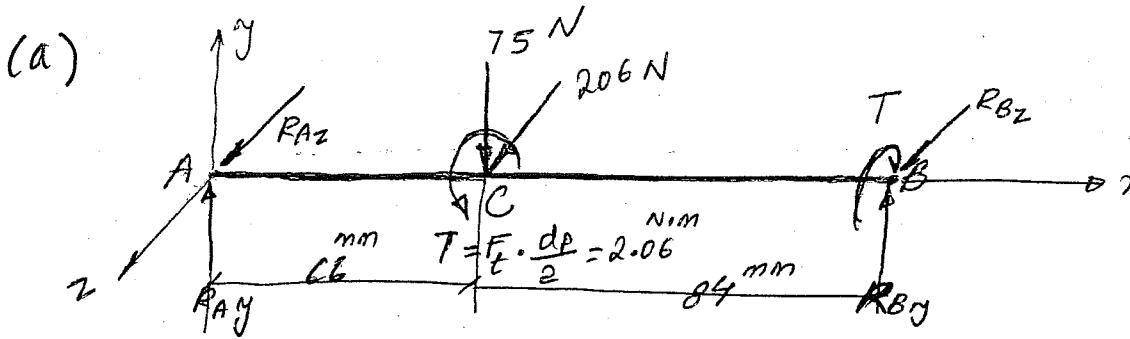


Problem 2: (40 Marks)

A 7/16-in dia UNC, Grade 7 bolt with cold-rolled threads is preloaded to 70% of its proof strength when clamping a 2.75-in-thick sandwich of solid steels. Find safety factor against fatigue failure and joint separation when a load 5000 lb (peak) fluctuating external load is applied. Assume bolt length of 3 in, reliability of 99% and room temperature.

-END-

Solution Q. # 1 :



Equilibrium loading diagram

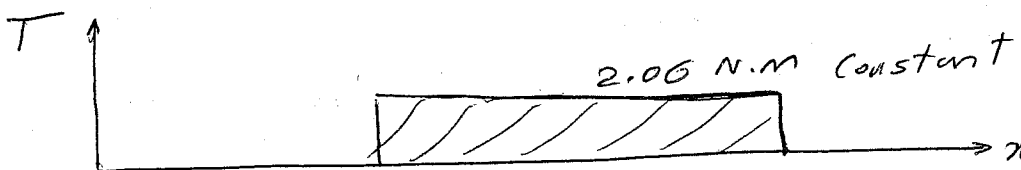
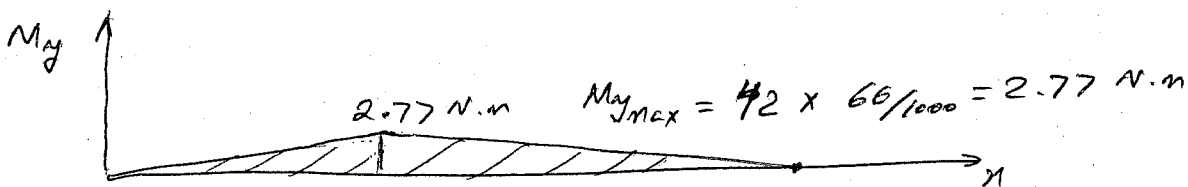
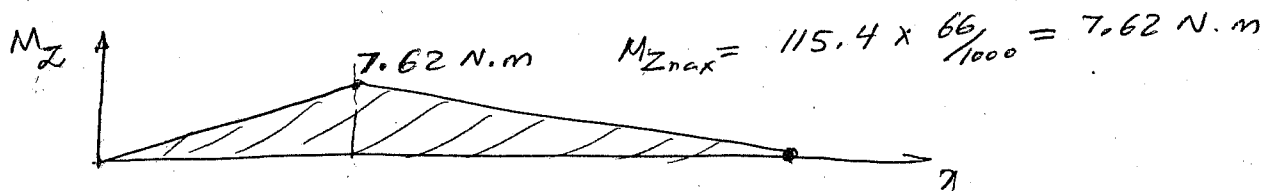
$$\begin{cases} R_{Ay} + R_{Bz} = 75 \text{ N} \\ R_{Bz} \times 150 - 75 \times 66 = 0 \end{cases} \quad \begin{cases} R_{Ax} + R_{Bx} = -206 \text{ N} \\ R_{Bx} \times 150 + 206 \times 66 = 0 \end{cases}$$

Reactions

$$\Rightarrow R_{Bz} = 33 \text{ N} \uparrow \quad R_{Bx} = -90.6 \text{ N} \nearrow$$

$$R_{Ay} = 75 - 33 = 42 \text{ N} \uparrow \quad R_{Ax} = 90.6 - 206 = -115.4 \nearrow$$

Maximum Moment on planes $x-y$ and $x-z$ occurs at point C.



The mean and alternating moments and torques are:

substit

$$M_c = \sqrt{(2.77)^2 + (7.62)^2} = 8.11 \text{ N.m} \quad \begin{cases} M_m = 0 \\ M_a = 8.11 \text{ N.m} \end{cases}$$

$$T_m = 2.06 \text{ N.m} \quad T_a = 0$$

(b)

$$S_e = C_{load} C_{size} C_{surf} C_{Temp} C_{reliab} S_e'$$

$$S_e' = 0.5 \times 520 = 260 \text{ MPa}$$

$$C_{load} = 1, \quad C_{size} = 1.189 (12)^{-0.097} = 0.934$$

$$C_{surf} = A (S_{ut})^b = 4.51 (520)^{-0.265} \approx 0.860$$

$$C_{Temp} = 1, \quad C_{reliab} = 0.753$$

$$\Rightarrow S_e = 1 \times 0.934 \times 0.860 \times 1 \times 0.753 \times 260 \approx 157.3 \text{ MPa}$$

$$\sigma_a = K_f \frac{M_a C}{I} = 2 \times \frac{8.11 \times 10^3 \times 12/2}{\frac{\pi}{64} (12^4 - 6^4)} \approx 102 \text{ MPa}$$

$$\tau_a = 0$$

$$\sigma_m = 0, \quad \tau_m = K_{fsm} \frac{T C}{J} = 1 \times \frac{2.06 \times 10^3 \times 12/2}{\frac{\pi}{32} (12^4 - 6^4)} \approx 6.5 \text{ MPa}$$

$$\Rightarrow \sigma_a' = \sqrt{\sigma_a^2 + 3\tau_a^2} = 102 \text{ MPa}$$

$$\sigma_m' = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{3 \times 6.5^2} = 11.26 \text{ MPa}$$

$$\Rightarrow \frac{1}{N_f} = \frac{\sigma_a'}{S_e} + \frac{\sigma_m'}{S_{ut}} = \frac{102}{157.3} + \frac{11.26}{520}$$

$$\Rightarrow N_f \approx 1.5$$

(c)

$$\text{fat key: } \tau = \frac{F_t}{A_{shear}} = \frac{F_t}{wL_k} = \frac{206}{2.4 \times 25} = 3.43 \text{ MPa}$$

$$\text{shear yield strength } S_{ys} = 0.577 S_y = 0.577 \times 440 \approx 253.9 \text{ MPa}$$

according to von-Mises

due to shear

$$N_{bearing} = \frac{S_y}{\sigma_b} \approx 64$$

\Rightarrow safe

$$N_{key} = \frac{S_{ys}}{\tau} = \frac{253.9}{3.43} \approx 74$$

$$\sigma_{bearing} = \frac{F_t}{A_{bearing}} = \frac{F_t}{w/2 \times L_k} = \frac{206}{2.4/2 \times 25} = 6.87 \text{ MPa}$$

bearing should not occur.

shear should not occur at shaft key

PROBLEM # 2

Solution Quiz #3

Statement:

A 7/16-in dia UNC, Grade 7 bolt with rolled threads is preloaded to 70% of its proof strength when clamping a 2.75-in-thick sandwich of solid steel. Find the safety factors against fatigue failure, yielding, and joint separation when a static 5000-lb (peak) fluctuating external load is applied. Use 99% reliability.

Units:

$$\frac{\text{ksi}}{\text{mm}^2} := 10^3 \cdot \text{psi}$$

Given:

Bolt diameter $d := 0.4375 \cdot \text{in}$

Preload fraction $f_p := 0.70$

Clamp length $l_m := 2.75 \cdot \text{in}$

Number of bolts $N_{bolts} := 1$

Applied load $P_{tot} := 5000 \cdot \text{lbf}$

Material properties for Grade 7:

Proof strength $S_p := 105 \cdot \text{ksi}$

Yield strength $S_y := 115 \cdot \text{ksi}$

Ultimate strength $S_{ut} := 133 \cdot \text{ksi}$

Young's modulus $E := 30 \cdot 10^6 \cdot \text{psi}$

Solution:

1. Determine the load per bolt.

$$P_{max} := \frac{P_{tot}}{N_{bolts}}$$

$$P_{max} = 5000 \text{ lbf}$$

$$P_{min} := 0 \cdot \text{lbf}$$

2. Get the tensile stress area from Table 14-1.

$$A_t := 0.1063 \cdot \text{in}^2$$

3. Calculate the preload.

$$F_i := f_p \cdot S_p \cdot A_t$$

$$F_i = 7813 \text{ lbf}$$

4. Define the bolt length and calculate the shank area.

Bolt length

$$l := 3.00 \cdot \text{in}$$

Shank area

$$A_b := \frac{\pi \cdot d^2}{4}$$

$$A_b = 0.15 \text{ in}^2$$

5. Find the lengths of thread l_{thd} and shank l_s of the bolt as shown in Figure 14-21:

$$l_{thd} := 2 \cdot d + 0.25 \cdot \text{in}$$

$$l_{thd} = 1.125 \text{ in}$$

$$l_s := l - l_{thd}$$

$$l_s = 1.875 \text{ in}$$

from which we can find the length of thread l_t that is in the clamp zone:

$$l_t := l_m - l_s$$

$$l_t = 0.875 \text{ in}$$

6. Find the stiffness of the bolt from equation 14.11a.

$$k_b := \frac{1}{\frac{l_t}{A_t \cdot E} + \frac{l_s}{A_b \cdot E}}$$

$$k_b = \frac{\text{lbf}}{\text{in}}$$

7. Calculate the material stiffness from equation 14.17, using the constants from Table 14-9a.

Steel

$$A_s := 0.78715$$

$$b := 0.62873$$

$$k_m := d \cdot E \cdot A \cdot e \cdot \left(\frac{d}{l_m} \right)$$

$$k_m = 1.142 \times 10^7 \frac{\text{lbf}}{\text{in}}$$

8. The joint stiffness factor from equation 14.13c is

$$C := \frac{k_b}{k_m + k_b}$$

$$C = 0.113$$

9. The portions of the applied load P felt by the bolt and the material can now be found from equations 14.13.

$$P_b := C \cdot P_{max} \quad P_b = 563.1 \text{ lbf}$$

$$P_m := (1 - C) \cdot P_{max} \quad P_m = 4436.9 \text{ lbf}$$

10. Find the resulting loads in bolt and material after the load P is applied.

$$F_b := F_i + P_b \quad F_b = 8376 \text{ lbf}$$

$$F_m := F_i - P_m \quad F_m = 3376 \text{ lbf}$$

11. Calculate the alternating and mean components of the fluctuating bolt load.

$$F_{alt} := \frac{F_b - F_i}{2} \quad F_{alt} = 281.5 \text{ lbf}$$

$$F_{mean} := \frac{F_b + F_i}{2} \quad F_{mean} = 8095 \text{ lbf}$$

12. The fatigue stress-concentration factor $K_f := 3.0$ for rolled threads is taken from Table 14-8. The mean stress-concentration factor is taken as $K_{fm} := 1$ in this case. The mean and alternating stresses in the bolt are:

$$\sigma_{alt} := K_f \cdot \frac{F_{alt}}{A_t} \quad \sigma_{alt} = 7.95 \text{ ksi}$$

$$\sigma_{mean} := K_{fm} \cdot \frac{F_{mean}}{A_t} \quad \sigma_{mean} = 76.15 \text{ ksi}$$

13. The stress at the initial preload is

$$\sigma_{init} := K_{fm} \cdot \frac{F_i}{A_t} \quad \sigma_{init} = 73.50 \text{ ksi}$$

14. An endurance strength must be found for this material. Using the methods of Section 6.6 we find for $S_{ut} = 133 \text{ ksi}$

$$S'_e := 0.5 \cdot S_{ut} \quad S'_e = 66.5 \text{ ksi}$$

15. From the tables and formulas in Section 6.6 we have:

Load $C_{load} := 0.70$

Size $C_{size} := 1$

Surface $A_{\text{surf}} := 2.70 \quad b_{\text{surf}} := -0.265$

$$C_{surf} := A_{\text{surf}} \cdot \left(\frac{S_{ut}}{\text{ksi}} \right)^{b_{\text{surf}}} \quad C_{surf} = 0.739$$

Temperature $C_{temp} := 1$

Reliability $C_{reliab} := 0.814$

and the endurance limit is

$$S_e := C_{load} \cdot C_{size} \cdot C_{surf} \cdot C_{temp} \cdot C_{reliab} \cdot S'_e \quad S_e = 28.00 \text{ ksi}$$

16. The corrected endurance strength and the ultimate tensile strength are used in equation 14.16 to find the safety factor from the Goodman line.

$$N_f := \frac{S_e(S_{ut} - \sigma_{init})}{S_e(\sigma_{mean} - \sigma_{init}) + S_{ut}\sigma_{alt}} \quad N_f = 1.5$$

17. The load required to separate the joint and the safety factor against joint separation are found from equations 14.14c and 14.14d.

$$N_{sep} := \frac{F_i}{P_{max}(1 - C)} \quad N_{sep} = 1.8$$