Chapter 4

More Interest Formulas

4-1
The effective interest rate is 19.56%. If there are 12 compounding periods per year, what is the nominal interest rate?

4-2
A continuously compounded loan has what nominal interest rate if the effective interest rate is 25%? Select one of the five choices below.

(a) $e^{1.25}$
(b) $e^{0.25}$
(c) $\log_e (1.25)$
(d) $\log_e (0.25)$
(e) Other (specify) ________________________

4-3
A continuously compounded loan has what effective interest rate if the nominal interest rate is 25%? Select one of the five choices below.

(a) $e^{1.25}$
(b) $e^{0.25}$
(c) $\log_e (1.25)$
(d) $\log_e (0.25)$
(e) Other (specify) ________________________

4-4
Given: A situation where the annual interest rate is 5%. When continuous compounding is used, rather than monthly compounding, the nominal interest rate. (Select One)

(a) Increases
(b) Remains the same
(c) Decreases

4-5

A journalist for a small town newspaper has a weekly column in which he answers questions from the local populace on various financial matters. Below is one such question. Assume you are the journalist and respond to this inquiry. Be brief and specific.

Q: "I put $10,000 in a 12% six month savings certificate. When it matured, I expected to receive interest of $1,200 - 12% of $10,000. Instead, I received only $600. Why? Current six-month certificates now pay 9% interest. If I put $10,000 in a new six-month certificate, I assume (based on my previous experience) I'll get only $450 interest - one half of 9%. Wouldn't my money earn more interest if I deposit it in a savings account paying 5½ percent?"

4-6

A drug dealer will sell goods to his regular customers for $20 immediately or $22 if the payment is deferred one week. What nominal annual interest rate is the dealer receiving?

4-7

A local bank is advertising that they pay savers 6% compounded monthly, yielding an effective annual rate of 6.168%. If $2,000 is placed in savings now and no withdrawals are made, how much interest (to the penny) will be earned in one year?

4-8

A young engineer wishes to buy a house but only can afford monthly payments of $500. Thirty year loans are available at 12% interest compounded monthly. If she can make a $5,000 down payment, what is the price of the most expensive house that she can afford to purchase?

4-9

A small company borrowed $10,000 to expand the business. The entire principal of $10,000 will be repaid in 2 years but quarterly interest of $330 must be paid every three months. What nominal annual interest rate is the company paying?

4-10

A store policy is to charge 1¼% interest each month on the unpaid balance.

(a) What is the nominal interest?

(b) What is the effective interest?

4-11

Under what circumstances are the nominal and effective annual interest rates exactly equal; or is this never true?

4-12

A small company borrowed $10,000 to expand the business. The entire principal of $10,000 will be repaid at the end of two years but quarterly interest of $335 must be paid every three months. What nominal annual interest rate is the company paying?

4-13

E. Z. Marc received a loan of $50 from the S.H. Ark Loan Company that he had to repay one month later with a single payment of $60. What was the nominal annual interest rate for this loan?

4-14

A local college parking enforcement bureau issues parking tickets that must be paid within one week. The person receiving the ticket may pay either $5 immediately, or $7 if payment is deferred one week. What nominal interest rate is implied in the arrangement?

4-15

A deposit of $300 was made one year ago into an account paying monthly interest. If the account now has $320.52, what was the effective annual interest rate? Give answer to 1/100 of a percent.

4-16

Which is the better investment, a fund that pays 15% compounded annually, or one that pays 14% compounded continuously?

4-17

For a nominal interest of 16 percent, what would the effective interest be, if interest is

(a) compounded quarterly?
(b) compounded monthly?
(c) compounded continuously?

4-18

If compounding is weekly and the (one year = 12 months = 48 weeks for this problem) quarterly effective interest rate is 5%,

(a) What is the nominal annual interest rate?
(b) What is the weekly interest rate?
(c) What is the semi-annual effective interest rate?
(d) What is the effective interest rate for a two year period?

4-19

If the interest rate is 10% compounded continuously, what is the semi-annual effective interest rate?

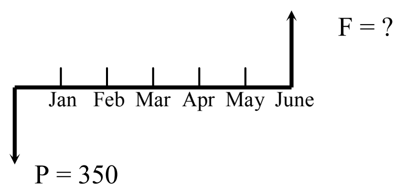
4-20

A grandfather gave his grandchild \$100 for his 10th birthday. The child's parents talked him into putting this gift into a bank account so that when he had grandchildren of his own he could give them similar gifts. The child lets this account grow for 50 years, and it has \$100,000. What was the interest rate of the account?

(a) 15.0%
(b) 15.8%
(c) 14.8%
(d) 14.0%

4-21

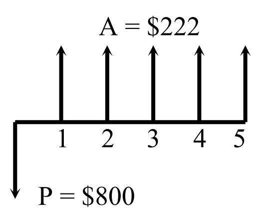
Given : Nominal interest = 9%, compounded monthly



Find: (a) F
(b) i_{eff}

4-22

Given:



Find: i %

4-23

How much should Abigail invest in a fund that will pay 9%, compounded continuously, if she wishes to have \$600,000 in the fund at the end of 10 years?

4-24

Solve for the unknown interest rate.

P = \$1,000 n = 10 years A = \$238.50 i = ?

4-25

How much will accumulate in a savings account in 15 years if \$500 is deposited in the account at the end of each quarter during that time? The account earns 8% interest, compounded quarterly. What is the effective interest rate?

4-26

Solve for the unknown value. Be sure to show your work.

$P = 1{,}000 \quad i = 12\% \quad n = 5 \quad A = ?$

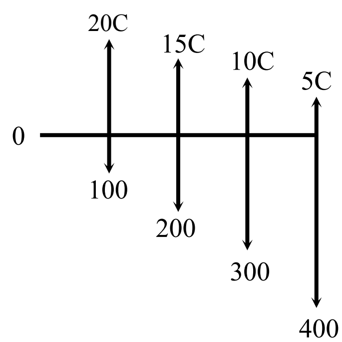
4-27

To offset the cost of buying a $75,000 house, a couple borrowed $12,500 from their parents at 6% nominal interest, compounded monthly. The loan from their parents is to be paid off in five years in equal monthly payments. The couple has saved $11,250. Their total down payment is therefore $12,500 + 11,250 = $23,750. The balance will be mortgaged at 9% nominal interest, compounded monthly for 30 years.

Find the combined monthly payment that the couple will be making for the first five years.

4-28

Find C if i = 12%



4-29

Decide whether each of the three statements below is TRUE or FALSE without referring to your book, notes, or compound interest tables.

(a) If interest is compounded quarterly, the interest period is four months.

(b) (F/A, 12%, 30) = (F/A, 1%, 360)

(c) (F/P, i %, 10) is greater than (P/F, i %, 10) for all values of i% > 0%.

4-30

A company borrowed $10,000 at 12% interest. The loan was repaid according to the following schedule. Find X, the amount that will pay off the loan at the end of year 5.

Year	Amount
1	$2,000
2	2,000
3	2,000
4	2,000
5	X

4-31

How much will Thomas accumulate in a bank account if he deposits $3,000 at the end of each year for 7 years? Use interest = 5% per annum.

4-32

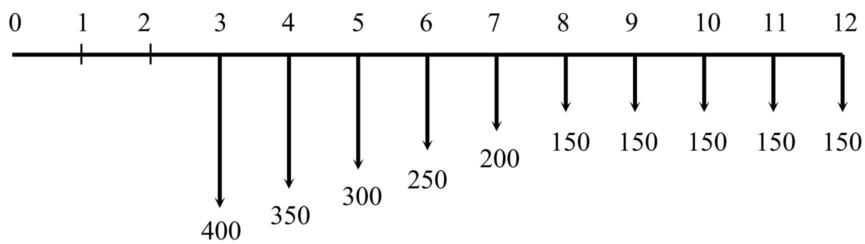
You need to borrow $10,000 and the following two alternatives are available at different banks:

(a) Pay $2,571 at the end of each year for 5 years, starting at the end of the first year. (5 payments in total.)
(b) Pay $207.58 at the end of each month, for 5 years, starting at the end of the first month. (60 payments in total.)

On the basis of the interest rate being charged in each case, which alternative should you choose?

4-33

Find the Uniform Equivalent for the following cash flow diagram if i = 18%. Use the appropriate gradient and uniform series factors.



4-34

Below is an equation to compute an equivalent annual cash flow (EACF). Determine the values of the net cash flow series that is implied by the equation.

EACF = [-8000 - 8000(P/F, 10%, 1)](A/P, 10%, 8)
+ 2000 +500(P/G, 10%, 4)(P/F, 10%, 1)(A/P, 10%, 8)
+ 750[(P/F, 10%, 6) - (P/F, 10%, 8)](A/P, 10%, 8)

Time	Net Cash Flow Series
0	
1	
2	
3	
4	
5	
6	
7	
8	

4-35

Using a credit card, Ben Spendthrift has just purchased a new stereo system for \$975 and will be making payments of \$45 per month. If the interest rate is 18% compounded monthly, how long will it take to completely pay off the stereo?

4-36

An engineer on the verge of retirement has accumulated savings of \$100,000 that are in an account paying 6% compounded quarterly. The engineer wishes to withdraw \$6,000 each quarter. For how long can she withdraw the full amount?

4-37

Charles puts \$25 per month into an account at 9% interest for two years to be used to purchase an automobile. The car he selects then costs more than the amount in the fund. He agrees to pay \$50 per month for two more years, at 12% interest, and also makes cash down payment of \$283.15. What is the cost of the automobile?

4-38

Explain in one or two sentences why (A/P, i%, infinity) = i.

4-39

A bank is offering a loan of \$20,000 with a nominal interest rate of 12%, payable in 48 months.

(a) Calculate first the monthly payments.
(b) This bank also charges a loan fee of 4% of the amount of the loan, payable at the time of the closing of the loan (that is, at the time they give the money to the borrower). What is the effective interest rate they are charging?

4-40

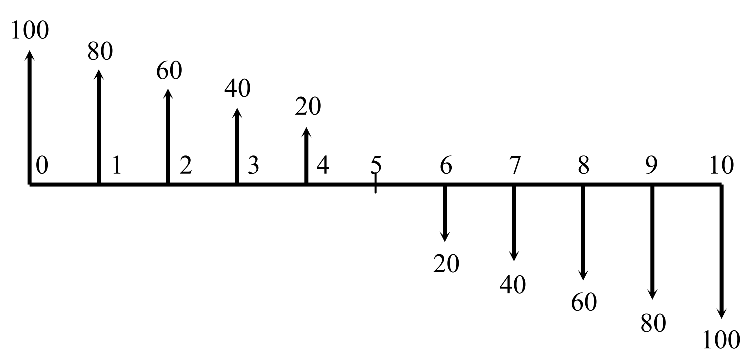
Find A if A = \$3,000(A/P, 13.5%, ∞).

4-41

Henry Fuller purchases a used automobile for \$4,500. He wishes to limit his monthly payment to \$100 for a period of two years. What down payment must he make to complete the purchase if the interest rate is 9% on the loan?

4-42

Find the present equivalent of the following cash flow diagrams if i = 18%.

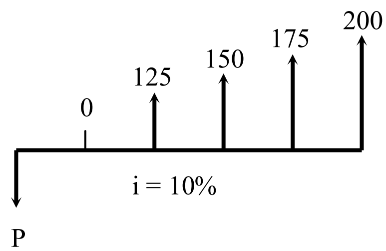


4-43

To start business, ECON ENGINEERING has just borrowed $500,000 at 6%, compounded quarterly, which will be repaid by quarterly payments of $50,000 each, with the first payment due in one year. How many quarters after the money is borrowed is the loan fully paid off?

4-44

Given:



Find: P

4-45

A tractor is bought for $125,000. What is the required payment per year to completely pay off the tractor in 20 years, assuming an interest rate of 6%?

(a) $ 1,150
(b) $ 5,550
(c) $10,900
(d) $12,750

4-1 Solution

$i_{eff} = (1 + (r/m))^m - 1 \Rightarrow r/m = (1 + i_{eff})^{1/m} - 1 = (1.1956)^{1/12} - 1 = 1.5\%$

$r = 12 \times 1.5 = 18\%$

4-2 Solution

$$e^r - 1 = 0.25$$
$$e^r = 1.25$$
$$\log_e (e^r) = \log_e (1.25)$$
$$r = \log_e (1.25)$$

Choose c.

4-3 Solution

$i_{eff} = e^{.25} - 1$

Choose e.

4-4 Solution

The answer is b: Remains the same

4-5 Solution

A: The questioner is confused about nominal vs. effective interest rates. The 9% and 12% rates are nominal annual interest rates compounded semi-annually. The effective semi-annual interest rates are 0.09/2 = 0.045 and 0.12/2 = 0.06 hence the interest earned in 6 months would be 0.045 (10,000) = \$450 and 0.06 (10,000) = \$600. The corresponding effective annual interest rates are

$$(1.045)^2 - 1 = 0.092 \text{ and } (1.06)^2 - 1 = 0.1236$$

The interest rate advertised on the savings account typically is also a nominal annual rate. Most accounts pay interest quarterly or continuously, thus the effective annual interest rate would be either

$$(1.01375)^4 - 1 = 0.05614 \quad \text{or} \quad e^{0.055} - 1 = 0.05654$$

neither of which is as good as the six month certificates.

4-6 Solution

$i = 2/20 = 10\%$

$r = 52 \times 10\% = 520\%$

4-7 Solution

Interest = Effective annual rate × principal = 0.06168 × 2,000 = \$123.36

Monthly compounding is irrelevant when the effective rate is known.

4-8 Solution

i 12%/12 = 1 %   n = (30)(12) = 360

P* = 500(P/A, 1%, 360) = 48,609
P = 48,609 + 5000
P = \$53,609

4-9 Solution

Since \$330 is interest only for one interest period, then i = 330/10,000 = 3.3% per quarter

r = 3.3 × 4 = 13.2% nominal annual

4-10 Solution

(a) r = i × m = 12(1.25) = 15%

(b) $i_{eff} = (1 + i)^n - 1 = (1.0125)^{12} - 1 = 16.075\%$

4-11 Solution

The nominal interest rate equals the effective interest rate when there is yearly (annual) compounding (i.e., m = 1).

4-12 Solution

i = 335/10,000 = 3.35%

r = i × m = 4 × 3.35 = 13.40%

4-13 Solution

Interest = \$10 in one month
i = 10/50 = 20%
r = i × m = 20 × 12 = 240%

4-14 Solution

$i = (7 - 5)/5 = 40\%$ per week

$r = i \times m = 52(40) = 2080\%$

4-15 Solution

$i_{eff} = 20.52/300 = 6.84\%$

4-16 Solution

$i = 15\%$; $n = 1$ $F = P(1 + 0.15)^1 = 1.1500 P$

$r = 14\%$ cont; $n = 1$ $F = P e^{.14} = 1.1503 P$

14% compounded continuously is slightly better

4-17 Solution

(a) $i_{eff} = [(1.04)^4 - 1](100) = 16.986\%$

(b) $i_{eff} = [(1.01333)^{12} - 1](100) = 17.222\%$

(c) $i_{eff} = e^{rn} - 1 = e^{.16(1)} - 1 = 17.35\%$

4-18 Solution

(a) $i_P = (1 + r/m)^P - 1 \rightarrow r = m\{(1 + i_P)^{1/P} - 1\}$

$i_{gr} = i_{12} = 0.05 = (1 + r/48)^{12} - 1$

$r = 48\{1.05^{1/12} - 1\} = 19.555\%$ per year compounded weekly.

(b) $i_{WK} = r/m = 19.555 / 48 = 0.4074\%$ per week

(c) $i_{SA} = i_{24} = i_P = (1 + i)^P - 1 = (1.004074)^{24} - 1 = 10.25\%$ per 1/2 year.

(d) $i_{2\ YR.} = i_{96} = i_P = (1 + i)^P - 1 = (1.004074)^{96} - 1 = 47.75\%$ per 2 years.

4-19Solution

$i_{t =} e^{rt} - 1$

$i_{1/2} = e^{(0.1)(1/2)} - 1 = e^{.05} - 1 = 1.0512711 - 1 = 5.127\%$ per 1/2 year

4-20 Solution

$\$100,000 = \$100(1 + i)^{50}$
$i = 14.8\%$

The answer is c.

4-21 Solution

(a) $F = 350(F/P, 0.75\%, 6) = 366.10$
(b) $i_{eff} = (1 + i)^m - 1 = (1.0075)^{12} - 1 = 9.38\%$

4-22 Solution

$P = A(P/A, i\%, 5)$
$800 = 222(P/A, i\%, 5)$
$(P/A, i\%, 5) = 800/222$
$= 3.6$

From tables $i = 12\%$

4-23 Solution

$r = 0.09$
$n = 10$

$P = F(P/F, 9\%, \infty) = Fe^{-rn} = 600,000(0.40657) = \$243,941.80$

4-24 Solution

$A = P(A/P, i\%, 10)$
$238.50 = 1,000(A/P, i\%, n)$
$(A/P, i\%, n) = 238.50/1,000$
$= 0.2385$

From tables, $i = 20\%$

4-25 Solution

$i = 8/4 = 2\%$ $n = (4)(15) = 60$

$F = 500 (F/A, 3\%, 60) = \$57,025.50$

Effective interest rate $= (1 + 0.02)^4 - 1 = 8.24\%$

4-26 Solution

A = 1,000(A/P, 12%, 5) = $277.40

4-27 Solution

Payment to parents:

12,500(A/P, ½%, 60) = $241.25

Borrowed from bank: 75,000 - 23,750 = $51,250

Payment to bank

51,250(A/P, ¾%, 360) = $412.56

Therefore, monthly payments are $241.25 + 412.56 = $653.81

4-28 Solution

$$
\begin{aligned}
20C(P/A, 12\%, 4) - 5C(P/G, 12\%, 4) &= 100(P/A, 12\%, 4) + 100(P/G, 12\%, 4) \\
40.105C &= 716.4 \\
C &= 17.86
\end{aligned}
$$

4-29 Solution

(a) FALSE. If interest is compounded quarterly, each interest period is 3 months long.

(b) FALSE. If we assume, for example, we are talking about 30 years or 360 months, the (F/A, 12%, 30) does not provide monthly compounding of interest. The (F/A, 1%, 360) does. (This is a common error among beginning students.)

(c) TRUE. Since i % > 0 %, (1 + i) > 0. Thus (F/P, i %, 10) = $(1+i)^{10} > 1$ and (P/F, i %, 10) = $(1 + i)^{-10} < 1$ for all values of i % > 0%.

4-30 Solution

$$
\begin{aligned}
10{,}000 &= 2{,}000\,(P/A, 12\%, 4) + X(P/F, 12\%, 5) \\
3{,}926 &= X(0.5674) \\
X &= 3{,}926/0.5674 \\
&= \$6{,}919.28
\end{aligned}
$$

4-31 Solution

F = 3,000(F/A, 5%, 7) = $25,962.

4-32 Solution

Alternative a:

$10{,}000 = 2{,}571(P/A, i, 5)$
$(P/A, i, 5) = 10{,}000/2{,}571$
$= 3.890$

From tables; i $\approx$ 9% (nominal = effective rate since compounded annually)

Alternative b:

$n = 5 \times 12 = 60$

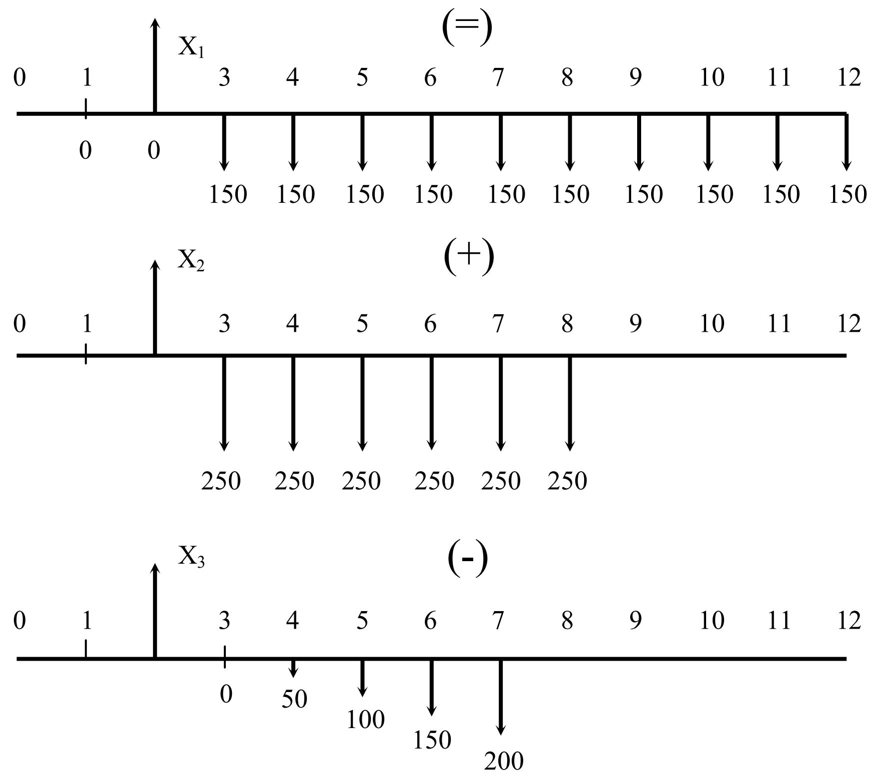
$10{,}000 = 207.58(P/A, i, 60)$
$(P/A, i, 60) = 10{,}000/ 207.58$
$= 48.174$

From tables i =0.75%

The nominal annual interest rate is: $12 \times 0.75 = 9\%$
but the effective interest rate is: $(1 + (0.09/12))^{12} - 1 = 9.38\%$

Therefore, choose the first alternative.

4-33 Solution



$X_1 = 150\ (P/A, 18\%, 10) = 674.10$
$X_2 = 250\ (P/A, 18\%, 5) = 781.75$

$X_3 = 50\ (P/G,\ 18\%,\ 5) \quad = 261.55$

$X = X_1 + X_2 - X_3 = \$1{,}194.30$

$P = 1{,}194.30\ (P/F,\ 18\%,\ 2) = 857.75$
$A = 857.75\ (A/P,\ 18\%,\ 12) = \$178.93$

4-34 Solution

Time	Net Cash Flow Series				
0	- 8,000 =	- 8,000			
1	-6,000 =	- 8,000	+ 2,000		
2	2,000 =		+ 2,000		
3	2,500 =		+ 2,000	+ 500	
4	3,000 =		+ 2,000	+ 1,000	
5	3,500 =		+ 2,000	+ 1,500	
6	2,750 =		+ 2,000		+750
7	2,000 =		+ 2,000		
8	1,250 =		+ 2,000		- 750

4-35 Solution

$i = 18\%/12 = 1\frac{1}{2}\%$

$975 = 45(P/A,\ 1\frac{1}{2}\%,\ n)$
$(P/A,\ 1\frac{1}{2}\%,\ n) = 975/45$
$= 21.667$

From tables n is between 26 and 27 months. The loan will not be completely paid off after 26 months. Therefore the payment in the 27th month will be smaller.

4-36 Solution

$i = 6\%/4 = 1\frac{1}{2}\%$

$6{,}000 = 100{,}000(A/P,\ 1\frac{1}{2}\%,\ n)$
$(A/P,\ 1\frac{1}{2}\%,\ n) = 0.0600$

From tables n = 19 quarters or 4¾years

Note: This leaves some money in the account but not enough for a full $6,000 withdrawal

4-37 Solution

P = 283.15 + 25(F/A, ¾%, 24) + 50(P/A, 1%, 24)
= 283.15 + 654.70 + 1,062.15
= \$2,000 ← cost of auto

4-38 Solution

In order to have an infinitely long A series, the principal must never be reduced. For this to happen only the interest earned each period may be removed. Removing more than the interest would deplete the principal so that even less interest is available the next period.

4-39 Solution

(a) The monthly payments:

n = 48 ; i = 12%/12 = 1% per period (month)

20,000(A/P, 1%, 48) = \$526.00

(b) Actual money received = P = 20,000 – 0.04(20,000) = \$19,200

But A = 526.00; n = 48

Recalling that A = P(A/P, i, n)
526 = 19,200(A/P, i, 48)
(A/P, i, 48) = 526/19,200
= 0.02739

for i = 1¼% the A/P, factor @ n = 48 = 0.0278

for i = 1% the A/P factor @ n = 48 = 0.0263

by interpolation i ≈ 1 + ¼((0.0263 - 0.02739)/(0.0263 - 0.0278))

i ≈ 1.1817%

the effective interest rate = $(1 + 0.011817)^{12}$ - 1 = 0.1514 = 15.14%

4-40 Solution

A=P × *i* when n = ∞

A = 3,000 × 0.135 = \$405

4-41 Solution

$$P = P' + A(P/A, \tfrac{3}{4}\%, 24)$$
$$4{,}500 = P' + 100(21.889)$$
$$P' = 4{,}500 - 2{,}188.90$$
$$= \$2{,}437.60 \leftarrow \text{down payment}$$

4-42 Solution

$$P = 100 + 80(P/A, 18\%, 10) - 20(P/G, 18\%, 10) = \$172.48$$

4-43 Solution

$$i = 6/4 = 1\tfrac{1}{2}\%$$

$$500{,}000 = 50{,}000(P/A, 1\tfrac{1}{2}\%, n)(P/F, 1\tfrac{1}{2}\%3)$$
$$(P/A, 1\tfrac{1}{2}\%, n) = 500{,}000/[50{,}000(0.9563)]$$
$$= 10.46$$

From tables n = 12 payments plus 3 quarters without payments equal 15 quarters before loan is fully paid off.

4-44 Solution

$$P = [125(P/A, 10\%, 4) + 25(P/G, 10\%, 4)](P/F, 10\%, 1) = 459.73$$

4-45 Solution

$$A = 125{,}000(A/P, 6\%, 20)$$
$$= \$10{,}900$$

Answer is c.