1.3, 1.5

Polynomial (p 27)

$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$

n is non-negative integer.
("th degree")

$a_n, a_{n-1}, \dots, a_0$ are constants
("the coefficient")

eg. $P(x) = -3x + 2$

first degree: -3'
coefficients: -3, 2.

Power Functions (p.28)

$f(x) = x^a$

a is a constant

Rational Functions (p.30)

$P(x)$ + $Q(x)$ are polynomial functions then

$\frac{P(x)}{Q(x)}$ is a rational function

Exponential F^n (p.32)

$f(x) = a^x$

Where a is a <u>positive</u> constant.

Laws of exponents.

$a^{x+y} = a^x a^y$

1.6 9/12.

prev. missed: Log Laws.

Trigonometric Functions
- angles
- triangles
- "wave-like appearance of a graph,

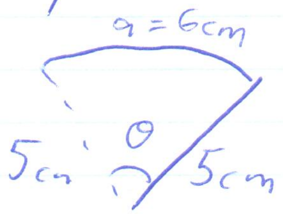
Angles.
- measured in radians.

radian:

$\theta = 1$ rad

ex I / 17 $r = 5$cm. $a = 6$cm. $\theta = ?$

$\theta = 6/5$ radians



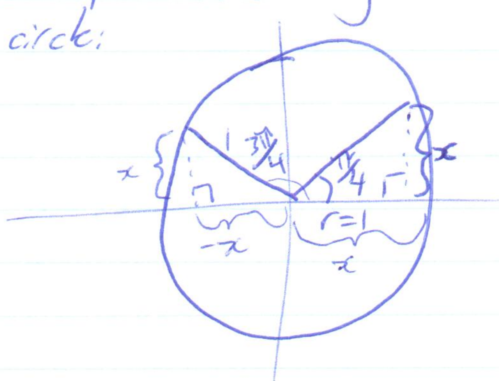
Triangles

$\cos\theta$	$\sec\theta$
$\sin\theta$	$\csc\theta$
$\tan\theta$	$\cot\theta$

$\rightarrow$ obv.

We might combine our ideas about circles + triangles to come up $\bar{w}$ a way of remembering values taken by trig Fns.

Unit circle:



$\tan \pi/4 = 1$ ~~[illegible]~~ $= \frac{x}{x}$

$\cos \pi/4 = \frac{x}{1} = x$

$\sin \pi/4 = \frac{x}{1} = x$

$x^2 + x^2 = 1^2$

$x = \pm \frac{1}{\sqrt{2}}$

Now consider $\frac{3\pi}{4}$

Special Triangles.

$$\sin\frac{3\pi}{4} = \frac{opp}{hyp}$$
$$= x$$
$$\cos\frac{3\pi}{4} = -x$$
$$\tan\frac{3\pi}{4} = -1$$

etc.

$$x = \frac{1}{\sqrt{2}}$$

$$\cos 0 = 1$$
$$\sin 0 = 0$$
$\tan 0 =$ ~~undefined~~, 0

$$\cos\frac{\pi}{2} = 0$$
$$\sin\pi/2 = 1$$
$\tan\pi/2$ = undefined.

– Wave-like graphs.

etc.

9/14

* Assignment #1 Posted on Webct Due Sept. 21 - end of Class

There are a number of formulae involving trig fns. Memorize (know) from appendix D!

$$\sin^2 x + \cos^2 x = 1$$

$$\left(\frac{\sin x}{\cos x}\right)^2 + 1 = \left(\frac{1}{\cos x}\right)^2$$

$$\tan^2 x + 1 = \sec^2 x$$

$$1 + \left(\frac{1}{\tan^2 x}\right) = \csc^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

_ Two Principal Identities

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\cos(x+y) = \cos x \cos y - \sin x \sin y$$

Take these two and replace y w̄ $-y$

$$\sin(x-y) = \sin x \cos(-y) + \cos x \sin(-y)$$

$$= \sin x \cos y - \cos x \sin y$$

$$\cos(x-y) = \cos x \cos y + \sin x \sin y$$

_ Double Angle Formulae

$$\sin(2x)$$

$$= \sin(x+x) = \sin x \cos x + \cos x \sin x$$

$$= 2\sin x \cos x$$

$$\cos(2x)$$

$$= \cos^2 x - \sin^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\therefore \cos^2 x - (1 - \cos^2 x) = \cos^2 x$$

$$2\cos^2 x - 1 = \cos(2x)$$

etc. $1 - 2\cos^2 x = \sin(2x)$

$$\cos(2x) = 1 - 2\sin^2 x$$

_ Half Angle Formulas

$$2\cos^2 x - 1 = \cos(2x).$$

$$\cos^2 x = \frac{(\cos(2x) + 1)}{2.}$$

9/14.2

$$\sin^2 x = \frac{(1-\cos(2x))}{2}$$

~~tan(x+y).~~

$\tan(x+y)$

$$= \frac{\sin(x+y)}{\cos(x+y)} = \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y - \sin x \sin y}$$

$$= \frac{\sin x \cancel{\cos y}}{\cos x \cancel{\cos y}} + \frac{\cancel{\cos x} \sin y}{\cancel{\cos x} \cos y} \div \left(1 - \frac{\sin x \sin y}{\cos x \cos y}\right)$$

$$= \frac{\tan x \tan y}{1 - \tan x \tan y}$$

$\tan(x-y)$

$$= \frac{\sin(x-y)}{\cos(x-y)}$$

$$= \frac{\sin x \cos y - \cos x \sin y}{\cos x \cos y + \sin x \sin y}$$

complete.

$\sin x \sin y = ?$

$$\sin(x+y) = \sin x \cos y + \sin y \cos x$$
$$+ \quad \sin(x-y) = \sin x \cos y - \sin y \cos x$$
$$\sin(x+y) + \sin(x-y) = 2 \sin x \cos y$$
$$\therefore \sin x \cos y = \tfrac{1}{2}(\sin(x+y) + \sin(x-y))$$

$\sin x \sin y \ldots$

$\cos x \cos y \ldots$

9/16

$$\cos(\tan^{-1}(x)) = \pm \frac{1}{\sqrt{1+x}} x$$

$\theta = \tan^{-1} x$

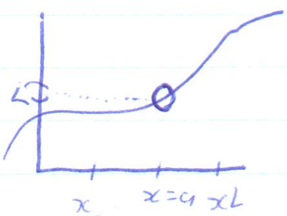
— Chapter 2 s.2.

Defn

$$\lim_{x \to a} f(x) = L$$

For some fn $f(x)$ defined on some open interval $x_0 < x < x_1$ that includes a.

* $f(a)$ may be undefined.



Blindfolded Cartographer.

ex. $f(x) = \frac{(x-1)}{(x^2-1)}$

$$\lim_{x \to 1} \frac{x-1}{x^2-1} \doteq 0.5$$

x	$\frac{x-1}{x^2-1}$
0.9	0.5263.
0.99	0.5025
0.999	0.5003
0.9999	0.5000...
1.001	0.4998
1.01	0.4975
1.1	0.4762

9/16.2.

$$\lim_{t \to 0} \frac{\sqrt{t^2+9}-3}{t^2}$$

t	$f(t)$
± 1.0	0.16228
± 0.5	0.16553
± 0.1	0.16662
± 0.05	0.16666
± 0.01	0.166

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

x	$\frac{\sin x}{x}$
± 0.5	0.959
± 0.1	0.9983
± 0.01	0.999983
± 0.001	. . .
0	

2.3

9/21

missed: p. 99-on - limit laws.

Review example 1 on p. 99

ex II (101)/ $\lim_{x \to 5} (2x^2 - 3x + 4)$

$$f_x + g(x) + h(x)$$

$$= \lim (2x^2) + \lim (-3x) + \lim (4)$$

$$= 2\lim(x^2) - 3\lim(x) + 4$$

$$= 39$$

Direct Substitution Property

- statement ~ limits of polynomial functions, $P(x)$

$$\lim_{x \to a} P(x) = P(a)$$

We can extend this property to rational functions

$$R(x) = \frac{P(x)}{Q(x)}$$

To cover our behinds we stipulate that "a" in $\lim_{x \to a} R(x)$ is in domain of $R(x)$

- Another Useful Idea

If $f(x) = g(x)$ for all x's near a, but maybe $f(x) \neq g(x)$ at $x = a$

then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$

provided limits exist.

2.3 con't. 9/22

ex V/ $\lim_{h\to 0} \frac{(3+h)^2-9}{h}$

↓

not defined by $h=0$

- find another f_n $(g(h))$ for $h \neq 0$, to see if we can compute $\lim g(h)$
- we look for $g(h)$ by simplifying original f_n

$$\frac{(3+h)^2-9}{h} = \frac{9+6h+h^2-9}{h}$$

$g(h) = 6+h$ ← b/c it is defined at $h=0$

$$\therefore \lim_{h\to 0} \frac{(3+h)^2-9}{h} = \lim_{h\to 0}(6+h)$$

direct substitution property: $= 6$

ex VI $\lim_{t\to 0}\left(\frac{\sqrt{t^2+9}-3}{t^2}\right)$

$f(t)$ is undefined at $t=0$

Try to find $g(t)$, $t \neq 0$.

*multiply by conjugate ↘

$$\frac{\sqrt{t^2+9}-3}{t^2} = \frac{\sqrt{t^2+9}-3}{t} \cdot \frac{\sqrt{t^2+9}+3}{\sqrt{t^2+9}+3}$$

$$= \frac{t^2+9-3^2}{t^2\left(\sqrt{t^2+9}+3\right)}$$

$$= \frac{1}{\sqrt{t^2+9}+3}$$

$\to \lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2} = \lim_{t\to 0} \frac{1}{\sqrt{t^2+9}+3}$

$\therefore$ limit of $\sum$ of functions in denominator exists and is not 0...

*conjugate?

9/22.2

One-sided limits can help.

$$\lim_{x\to 0} |x|$$

$$\downarrow$$

$$|x| = \begin{cases} -x & \text{if } x<0 \\ 0 & \text{if } x=0 \\ x & \text{if } x>0 \end{cases}$$

$x<0$: approaching $x=0$ from left (below).

∴ $f(x)$ is continuous... $\lim_{x\to 0^-} (-x) \overset{\text{dir sub.}}{=} 0$

$x>0$: approaching $x=0$ from right (above)

" " $\lim_{x\to 0^+} (x) \overset{\text{direct. sub.}}{=} 0$

The left hand limit exists and is 0.
The right hand limit exists and is 0.
∴ $\lim_{x\to 0} |x|$ exists and is 0

$$\lim_{x\to 0} \frac{|x|}{x}$$

$x<0$

$$\frac{|x|}{x} = \frac{-x}{x} = -1$$

$x>0$

$$\frac{|x|}{x} = \frac{x}{x} = 1$$

$$\lim_{x\to 0^-} \frac{|x|}{x} = \lim_{x\to 0^-} (-1) = -1$$

$$\lim_{x\to 0^+} \frac{|x|}{x} = \lim (1) = 1$$

∴ $\lim_{x\to 0^-} \neq \lim_{x\to 0^+}$, $\lim_{x\to 0}$ does not exist.

9/23

ex I/from 2.5

$$f(x) = \begin{cases} \sqrt{x-4} & x > 4 \\ 8-2x & x < 4 \end{cases}$$

$\lim_{x\to 4} f(x) = ?$

$\lim_{x\to 4^-} f(x) = \lim_{x\to 4^-} (8-2x) = 8-2(4) = 0.$

direct substitution

$\lim_{x\to 4^+} f(x) = \lim \sqrt{x-4} = \sqrt{\lim x-4} = \sqrt{0} = 0.$

stays positive b/c $x=4$ is approached from above.

law ~ roots of $f(x) \geq 0$

direct substitution.

$\therefore$ left side limit = right side limits, the limit exists and is equal to 0.

Theorem of 2.5

- limits preserve order.

if $f(x) = g(x)$

then $\lim_{x\to a} f(x) = \lim_{x\to a} g(x)$

- Squeeze theorem

$f(x) \leq g(x) \leq h(x)$ for all x's near a (except perhaps at $x=a$)

<u>If</u> $\lim_{x\to a} f(x) = L = \lim_{x\to a} h(x)$

<u>Then</u> $\lim_{x\to a} g(x) = L$

9/23.2.

$$\lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right)$$

What not to do: $= \lim_{x \to 0} x^2 \cdot \lim_{x \to 0} \sin \frac{1}{x}$

$= 0 \cdot \text{DNE}$

$= \text{DNE}$

Why? Cannot use those limit laws. unless it can be assumed all limits exist.

What to do: $1 \leq \sin(\frac{1}{x}) \leq 1$

$-x^2 \leq x^2 \sin(\frac{1}{x}) \leq x^2$

$f(x) \leq g(x) \leq h(x)$.

Squeeeeeeze.

$\lim_{x \to 0} f(x) = \lim_{x \to 0} (-x^2) = 0.$

$\uparrow$

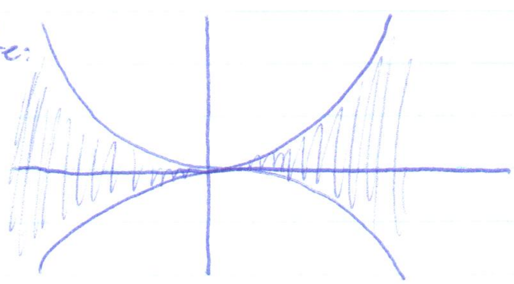
$\cancel{0} \rightarrow$ direct substitution

$\lim h(x) = \lim (x^2) = 0$

$\therefore \lim f(x) = \lim g(x) = L$

$\therefore \lim_{x \to 0} \left(x^2 \sin\left(\frac{1}{x}\right)\right) = L = 0.$

Note:



$g(x)$ must be somewhere in shaded area.

* 2.6 for Monday *

2.5 Continuity

9/26

A function $f(x)$ is continuous at $x=a$, provided: ① $\lim_{x \to a} f(x) = L$ exists

② $L = f(a)$

Types of discontinuity: removable (hole)
infinite (vertical asymptote)
jump.

Just like we had one-sided limits, we can also have one-sided continuous.

$f(x)$ is right continuous at $x=a$ if $\lim_{x \to a^+} f(x) = L = f(a)$.

9/28

Last time:

Show $f(x) = 1 - \sqrt{1-x^2}$ is continuous $[-1,1]$

Background

- consider some $a \in (-1,1)$ (so $-1 < a < 1$)
- check "foundation" for limit laws
  - consider just the "$\sqrt{1-x^2}$" part.

$\lim_{x \to a} \sqrt{1-x^2}$ → b/c we know that for x's near a $1-x^2 > 0$

$\therefore$ $1-x^2 > 0$, we are justified in applying limit law on $\sqrt{\ }$

$= \sqrt{\lim_{x \to a} (1-x^2)}$

direct. sub.

$= \sqrt{1-a^2}$

$\therefore$ limit exists.

- consider only the "1" part

$\lim_{x \to a} 1 = 1$

$\therefore$ limit exists.

$\therefore$ limits of each "component" fns exists

$\therefore$ $\lim_{x \to a} (1 - \sqrt{1-a^2}) = \lim_{x \to a} 1 - \lim_{x \to a} \sqrt{1-x^2}$

$= 1 - \sqrt{1-a^2} = f(a)$

Shown that for $a \in (-1,1)$

1. The limit of $f(x)$ as $x \to a$ exists.
2. $\lim_{x \to a} f(x) = f(a)$

$\therefore$ $f(x)$ is continuous for all $a \in (-1,1)$

Now we need to deal $\bar{w}$ end points. $a = \pm 1$

consider $a = 1$

→ use left limit

$$\lim_{x \to 1^-} 1 = 1$$

$$\lim_{x \to 1^-} \sqrt{1 - x^2}$$

$$\lim (1 - x^2) = 0$$

$$\therefore \lim \sqrt{1-x^2} = \sqrt{\lim (1 - x^2)}$$

$$\text{dir sub} \quad = \sqrt{0}$$

$$= 0$$

Both "pieces" continuous from left at $x = 1$

$$\therefore \lim_{x \to a^-} f(x) = \lim_{x \to 1^-} (1 - \sqrt{1 - x^2})$$

$$= \lim 1 - \lim \sqrt{1 - x^2}$$

$$= 1 - 0$$

$$= 1 = f(1)$$

$\therefore f(x)$ is continuous from left at $a = +1$

(cont for from right at -1)

thus: f is continuous on $(-1, 1)$

" " at $+1$

" " " -1

$\therefore$ " " on $[-1, 1]$

So... we are pretty bored at this point. Computing limits is, for lack of a better word, dumb.

Luckily ... ~~mmmm~~ Theorem many of my favourite f_{ns} are continuous on their domains.

Polynomials, rational, root, Trig, Inverse trig f_{ns}, log, exponential

9/28.3

If f is continuous at $x=a$
and g " "
then $f+g$ " "
$f-g$ " "
cf " "
$f \cdot g$ " "
$\frac{f}{g}$ " " (if $g(a) \neq \emptyset$)

ex I/ $f(x) = \dfrac{\ln x + \tan^{-1} x}{x^2 - 1}$

Where is x continuous?

Numerator $\quad \ln x + \tan^{-1} x$

$\underbrace{\text{cts on } (0,\infty) \quad \text{cts on } (-\infty,\infty)}_{\text{both are cts on } (0,\infty)}$

Denominator $\quad x^2-1$

$\underbrace{\text{cts on } (-\infty,\infty) \quad \text{non-zero for } (-\infty,-1)(-1,1)(1,\infty)}_{\text{Both cts and } \neq 0 \text{ for } (-\infty,-1)(-1,1)(1,\infty)}$

Both cts + non-zero for xs in $(0,1)\,(1,\infty)$

Summary of Strategy
1. Find largest interval(s) on which numerator is cts.
2. " " denominator " " + $\neq 0$.
3. Find intersection.

9/29

$$\lim_{x \to \pi} \frac{\sin x}{2+\cos x}$$

Numerator

- $\sin x$ cts on own domain
- $\pi \in$ domain of $\sin f(x)$

$\therefore \sin x$ is cts at π

Denominator

- sum of 2 f_{ns} (each is cts at π)

$\therefore 2+\cos x$ is cts at π

$\because -1 \leq \cos x \leq +1$

$\therefore 2+\cos x \geq 2-1$

$\therefore 2+\cos x \geq 1 > 0$

$\therefore$ the quotient $\frac{\sin x}{2+\cos x}$ must also be continuous at π

$\therefore$ we can use direct substitution.

— Another useful theorem

If (1) f is cts at b

and (2) g is a f_n such that $\lim_{x \to a} g(x) = b$

Then $\lim_{x \to a} f(g(x)) = f(b)$

Why do we care?

$$\lim_{x \to 1} \left[\arcsin\left(\frac{1-\sqrt{x}}{1-x}\right)\right]$$

$f(y) = \arcsin(y)$

$g(x) = \frac{1-\sqrt{x}}{1-x}$

What is b?

$$\lim_{x \to 1} \left(\frac{1-\sqrt{x}}{1-x}\right)$$

$a = 1$

What is b?

$\lim_{x\to 1}\left(\frac{1-\sqrt{x}}{1-x}\right) \longrightarrow$ if $x\neq 1$, then $\frac{1-\sqrt{x}}{1-x} = \frac{\cancel{1-\sqrt{x}}}{\cancel{(1-\sqrt{x})}(1+\sqrt{x})}$

$= \lim_{x\to 1}\left(\frac{1}{1+\sqrt{x}}\right)$ $\qquad = \frac{1}{1+\sqrt{x}}$

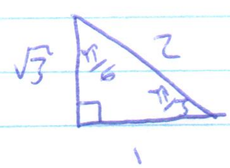
$= \frac{1}{1+\sqrt{1}} = \frac{1}{2} = b$

Now we can assert that

① $f(y) = \arcsin(y)$ is cts on its domain and $b = \frac{1}{2} \in$ to that domain.

② $\lim_{x\to a} g(x) = \lim_{x\to 1} \frac{1-\sqrt{x}}{1-x} = \frac{1}{2} = b$

$\therefore$ we can invoke our theorem to say... $\lim_{x\to 1} \arcsin\left(\frac{1-\sqrt{x}}{1-x}\right) = f(b)$



$= \arcsin\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

If ① g is cts at a

and ② f is cts at $g(a)$

Then $f(g(x))$ is cts at $x = a$

$f \cdot g(x)$

Why do we care?

ex/ Determine where $F(x) = \ln(1+\cos x)$ is cts.

$f(y) = \ln y$

$g(x) = 1 + \cos x$

$F = f \cdot g$

We can say f is cts at any $y = b$ in its domain $(0, \infty)$

$\cos x$ cts on $\mathbb{R}$

1 " " "

(①) checks out for any x in $\mathbb{R}$

② checks out if $g(x)$ is in domain of f

→ $1 + \cos x > 0$?

$1 + \cos x > 0$ if $x \neq \pm k\pi$, for k being ~~and~~ any odd integer.

Provided $a \neq \pm\pi, \pm 3\pi, \pm 5\pi \ldots$
we can say that
g is cts at a ① ✓
$g(a) \in$ domain of f (positive g(a))
∴ f is cts at g(a) ② ✓
∴ as long as $a \neq \pm\pi, \pm 3\pi \ldots$

$f(g(x)) = \ln(1 + \cos x)$
is cts at $x = a$.

– Intermediate Value Theorem

f cts on $[a, b]$ ①

② $f(a) \neq f(b)$ → $f(a) < f(b)$
→ $f(a) > f(b)$

Number N is b/w
$f(a)$ + $f(b)$ ③

If ① + ② + ③

Then There must be at least one number c, b/w a + b for which $f(c) = N$

9/30

(Con't from 9/29).

Why might we care? $\underbrace{Ax^2 + Bx + C = 0}$

We already know how to find the roots of relatively simple fns.

eg. $4x^3 - 6x^2 + 3x - 2 = 0$. We can show that there must be at least one soln on the interval $[1, 2]$ using the Intermediate Value Theorem (IVT)

How? 1. The roots of the given eqn is a polynomial fn ∴ cts on $[1,2]$.
2. $f(1) = -1 = f(a)$
$f(2) = 12 = f(b) \neq f(a)$
3. Say $N = 0$

$f(a) = -1 < N = 0 < f(b) = 12$
∴ by IVT there must be value c b/w 1 and 2 | $f(c) = N = 0$.
$4c^2 - 6c^2 + 3c - 2 = 0$

Not course material. Look it, obv.

course material again — 2.6 Limits at Infinity

$f(x) = \dfrac{x^2 - 1}{x^2 + 1}$ What happens to $f(x)$ when $|x|$ is large?
$f(x) \to 1$
as $|x|$ becomes large.

eg. $\lim_{x \to \infty} \dfrac{x^2 - 1}{x^2 + 1} = 1$

$\lim_{x \to \infty} f(x) = L$

etc.

Hilroy

10/3

* Assignment #2 Due Oct 12^{th}

2.6 Last class: limits at infinity $\lim_{x \to \infty} f(x) = L$

$$\lim_{x \to -\infty} f(x) = L.$$

- **Theorem** if $r > 0$ is a rational number then

$$\lim_{x \to \infty} \frac{1}{x^r} = 0$$

"Also same for $\lim_{x \to -\infty}$ (as long as x is defined for all x's."

Limits at infinity tell us something ~ horizontal asymptotes.

If $\lim_{x \to \infty} f(x) = L$ then $y = L$ is a horizontal asymtote to graph of $f(x)$

"

eg. Find horizontal asymtotes to graph of $\arctan x$.

$$\lim_{x \to \infty} \arctan(x) = \frac{\pi}{2}$$

$$\lim_{x \to -\infty} \arctan(x) = \frac{-\pi}{2}$$

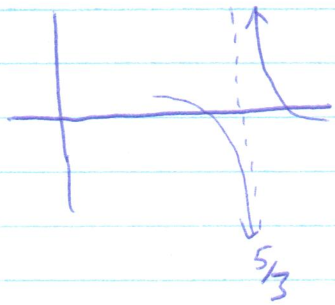
$\therefore$ <etc>

ex I / Find all asymptotes to $f(x) = \dfrac{\sqrt{2x^2+1}}{3x-5}$

denominator: look at $\lim_{x \to 5/3} f(x)$ $\sqrt{2x^2+1} \to$ finite + positive.

$$\lim_{x \to 5/3^-} f(x) = -\infty \quad (\text{denominator} \to 0^-)$$

$$\lim_{x \to 5/3^+} f(x) = \infty \quad \text{<dbv>}$$



<u>Horizontal</u> $\lim_{x\to\infty}$ ~~ $= \frac{\sqrt{2}x}{3x}$ → guessing

$\lim_{x\to-\infty}$ ~~ $= -\frac{\sqrt{2}}{3}$ → guessing

$$\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{3x-5}$$

$$= {''} \quad \frac{(\frac{1}{x})\sqrt{2x^2+1}}{(\frac{1}{x})3x-5}$$

$$= {''} \quad \sqrt{\frac{2x^2+1}{x^2}} \div (3-5x)$$

$$= {''} \quad \frac{\sqrt{2+\frac{1}{x^2}}}{3-5x}$$

$$= \frac{\sqrt{2}}{3}$$

$$\lim_{x\to-\infty} \frac{\sqrt{2x^2+1}}{3x-5}$$

$$= {''} \quad \frac{-\frac{1}{\sqrt{x^2}}\sqrt{2x^2+1}}{\frac{3x-5}{x}}$$

*Must be $-\frac{1}{\sqrt{x^2}}$ to avoid changing sign of x.

= <etc>

10/5.1

Last time: 2.6 limits at $\pm\infty$ + horizontal asymptotes

ex $\lim_{x\to\infty}\left(\sqrt{x^2+1}-x\right)$ guess: $\emptyset$

$$\lim_{x\to\infty}\left\{\left(\sqrt{x^2+1}-x\right)\frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}+x}\right\}$$

$$= \text{"} \quad \frac{x^2+1-x^2}{\sqrt{x^2+1}+x}$$

$$= \text{"} \quad \frac{1}{\sqrt{x^2+1}+x} \quad \begin{matrix}\longrightarrow 1 \\ \text{as } x\to\infty\end{matrix}$$

denominator $\to\infty$ as $x\to\infty$

$=\emptyset$

$\lim_{x\to 2^+} \arctan\left(\frac{1}{(x-2)}\right) \to$ guess $\pi/2 \to [\arctan\infty]$

$y = g(x) = \frac{1}{x-2}$
then

$\lim_{x\to 2^+} g(x) = \infty$

so " $\arctan[g(x)] = \lim_{y\to\infty} \arctan y = \frac{\pi}{2}$

b/c arctan, being an inverse trig f^n, is cts.

Recall: we had for $r>\emptyset$ rational $\lim_{x\to\infty} \frac{1}{x^r} = 0$ as long as x^r is defined for all x's.

We can also use: $\lim_{x\to\infty} f(x) = \lim_{t\to 0^+} f\left(\frac{1}{t}\right)$

We can also have $\lim_{x\to\infty} f(x) = \infty$
or
$= -\infty$
<etc>

10/7

Last class: 2.7
- slope of graph at pt. $P=(a, f(a))$
- approximated w/ secants
  - in a limit secant becomes tangent

$$\underbrace{\lim_{x \to a} \underbrace{\left(\frac{f(x)-f(a)}{x-a}\right)}_{\text{slope of secant}}}_{\text{slope of tangent of } f(a) \text{ at } x=a}$$

ex from 2.7 / Suppose a ball is dropped from a height of 450m.
a) Ball's velocity after 5 s?
b) Ball's speed when hits ground?

$$s(t) = -4.9t^2$$

Last time: - derivative of a f^n

- differentiability of a f^n
- read 2.7

Thoerem

If f is diff'able at $x=a \rightarrow f$ is cts at $x=a$.

Read 3.1 + 3.2.

3.3

$\frac{d}{dx}(\sin x) \overset{?}{=}$ <obv>

$=$

Chapter 3.4 10/14

The Chain Rule

If ① g is diff'able at x.
② f " " $g(x)$
Then $f \circ g(x) = f(g(x))$
is diff'able at x.

Moreover,
$$f \circ g'(x) = f'(g(x)) \cdot g'(x)$$
Sometimes written:
$y = f(u) \qquad u = g(x) \qquad \frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

p.200 ex/ $F(x) = \sqrt{x^2+1}$
$$\frac{dy}{dx} = ?$$

$$F(x) = f(g(x))$$
$$g(x) = x^2+1 \qquad f(u) = \sqrt{u}$$
$$g'(x) = 2x$$

$$f(u) = \frac{d}{du} u^{1/2}$$
$$= \frac{1}{2} u^{-1/2}$$
$$= \frac{1}{2\sqrt{u}}$$

$$F'(x) = f'(g(x)) \cdot g'(x)$$
$$= f'(x^2+1) \cdot 2x$$
$$= \frac{x}{\sqrt{x^2+1}}$$

10/14.2

~~On the domain $x \neq 1.5$: $9-4x^2$~~

p.201 ex / $y=\sin(x^2)$

$f'(u)=\cos u$

$g'(x)=2x$

$\frac{dy}{dx}=f'(x^2)\cdot g'(x)$

$=2x\cos(x^2)$

ex II / $y=\sin^2(x)$

$=(\sin x)^2$

$\frac{dy}{dx}=2(\sin x)^{2-1}\cdot\frac{d}{dx}(\sin x)$

$=2\sin x\cos x$

$y=(x^3-1)^{100}$

$\frac{dy}{dx}=100\cdot(x^3-1)^{99}\cdot 3x^2$

$=300x^2\cdot(x^3-1)^{99}$

$f(x)=\frac{1}{\sqrt[3]{x^2+x+1}}=(x^2+x+1)^{-\frac{1}{3}}$

$\frac{dy}{dx}=\left(\frac{-1}{3}\right)(x^2+x+1)^{-\frac{4}{3}}\cdot(2x+1)$

$g(t)=\left(\frac{t-2}{2t+1}\right)^9$ *

$=9\left(\frac{t-2}{2t+1}\right)^8\ \frac{1(2t+1)-2(t-2)}{(2t+1)^2}$

$=\ \frac{5}{(2t+1)^2}$

10/14.3

ex III / $y = (2x+1)^5 (x^3 - x + 1)^4$

$= 5(2x+1)^4 (2)(x^3 - x + 1)^4 + 4(x^3 - x + 1)^3 (3x - 1)(2x+1)^5$

<etc> $= 2(2x+1)^4 (x^3 - x + 1)^3 (17x^3 - 9x + 6x^2 + 3)$

Consider

$f(x) = a^x$

$= (e^{\ln a})^x$

$= e^{x \ln a}$

$f'(x) = \frac{d}{dx}\left(e^{\ln a\, x}\right)$

<etc>

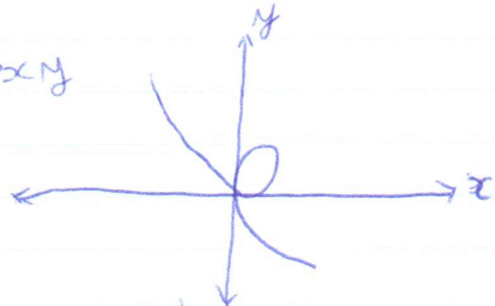
$f(x) = a^x$

$f'(x) = a^x \ln a$

3.5 Implicit Differentiation

10/17

ex/ If $x^3 + y^3 = 6xy$
then $\frac{dy}{dx} = ?$



$$\frac{d}{dx}\left[x^3 + (y(x))^3\right] = \left[6x\,y(x)\right]\frac{d}{dx}$$

y as a fn of x

$$3x^2 + 3y^2\frac{dy}{dx} = 6\left(y + x\frac{dy}{dx}\right)$$

$$= 6y + 6x\frac{dy}{dx}$$

$$3y^2\frac{dy}{dx} - 6x\frac{dy}{dx} = 6y - 3x^2$$

$$\frac{dy}{dx}(3y^2 - 6x) =$$

$$\frac{dy}{dx} = \frac{6y - 3x^2}{3y^2 - 6x}$$

Where is the slope of the tangent to the curve horizontal in the first quadrant?

$$0 = 6y - 3x^2$$

$$x^2 = 2y$$

$$y = \frac{x^2}{2}$$

$$x^3 + \left(\frac{x^2}{2}\right)^3 = 6x\frac{x^2}{2}$$

$$x^3 + \frac{x^6}{8} = 3x^3$$

$$x^6 = 2x^3 \cdot 8$$

$$x^3 = 16$$

$$= 2^4$$

$$x = 2^{4/3}$$

$y =$ <etc>.

10/17/12

$$\sin(x+y) = y^2 \cos x$$

$$\frac{dy}{dx} = ?$$

$$\frac{d}{dx}\sin(x+y(x)) = \frac{d}{dy}\left((y(x))^2 \cos x\right)$$

$$\cos x+y \cdot \left(1+\frac{dy}{dx}\right) = \left(2y\frac{dy}{dx}\cos x \underbrace{- y^2 \sin x}_{\left(+y^2(-\sin x)\right)}\right)$$

<etc>

Higher order derivatives too

$$x^4 + y^4 = 16$$

$$\frac{d^2y}{dx^2} = ?$$

$$4x^3 + 4y^3\frac{dy}{dx} = 0$$

$$y^3\frac{dy}{dx} = -x^3$$ quotent rule.

$$\frac{d^2y}{dx^2} = \frac{3x^2y^3 - x^3 3y^2\frac{dy}{dx}}{y^6}$$

= <etc>

$$\frac{ddy}{dx} = \frac{-48x^2}{y^7}$$

More.

$$y = \sin^{-1} x$$

$$\frac{dy}{dx} = ?$$

$$\sin y = \sin(\sin^{-1} x)$$

$$= x$$

$$\frac{d}{dx}\sin y = 1$$

$$\cos y\frac{dy}{dx} = \text{<etc>}$$

$$\frac{d}{dy}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

exam TC 341 Sec. 009. 10/19

7 pm Friday including 3.6

Student card

HB pencils

eraser

pen.

3.6 Implicit Differentiation.

Look at 3.6

$y(x) = \log_a x$

$y' = ?$

$a^y = a^{\log_a x}$

$= x$

$\frac{d}{dx} a^y = \frac{d}{dx} x$

<fire alarm> 10/20

$\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

Using log can be easiest.

long way

ex. $y = \ln \frac{x+1}{\sqrt{x-2}}$

$y' = 1 \div \frac{x+1}{\sqrt{x-2}} \cdot \frac{d}{dx}\left(\frac{x+1}{\sqrt{x-2}}\right)$

$= \frac{\sqrt{x-2}}{x+1} \cdot \frac{f'g - g'f}{g^2}$

$= \frac{\sqrt{x-2} - \left(\frac{1}{2}\right)(x-2)^{-\frac{1}{2}}(x+1)}{(\sqrt{x-2})}$

~~[illegible]~~ Done Enough

$(x-2)^{\frac{1}{2}}$

$y = \ln \frac{x+1}{\sqrt{x-2}}$

$= \ln(x+1) - \ln(\sqrt{x-2})$

$= \ln(x+1) - \ln((x-2)^{\frac{1}{2}})$

$= \ln(x+1) - \left(\frac{1}{2}\right)\ln(x-2)$

$y' = \frac{1}{x+1} - \left(\frac{1}{2}\right)\left(\frac{1}{x-2}\right)$

10/20.2

$$y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}$$

$$\ln y = \ln\left(\frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\right)$$

$$\ln y = \ln(x^{3/4}) + \ln(x^2+1)^{1/2} + \ln(3x+2)^5$$

$$= (3/4)\ln x + (\tfrac{1}{2})\ln(x^2+1) + 5\ln(3x+2)$$

$$\frac{d}{dx}\ln y = \frac{d}{dx} \text{ " "}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{3}{4x} + \frac{x}{x^2+1} - \frac{5}{3x+2}\cdot 3$$

$$\frac{dy}{dx} = y\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$$

$$= \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}\left[\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right]$$

ex. $f(x) = \ln|x|$

$$f(x) = \begin{cases} \ln x & x > 0 \\ \ln -x & x < 0 \end{cases}$$

$f'(x) = ?$

$f'(x) = \frac{1}{x}$ if $x > 0$

$f'(x) = \frac{1}{x}$ if $x < 0$

ex. $y = x^{\sqrt{x}}$

$$\frac{d}{dx}\ln y = \frac{d}{dx}(\sqrt{x}\ln x)$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{2\sqrt{x}}\ln x + \sqrt{x}\cdot\frac{1}{x}$$

$$= \frac{1}{\sqrt{x}}\,\frac{1}{2}\ln x \text{ <etc>}$$