PHIL 210: FIRST ASSIGNMENT, SEPTEMBER 2012

Distributed by *MyCourses*: 26 September, 2012
Due in class: 3 October, 2012
Value: 12.5%

Instructions There are 100 points, worth 12.5% of your final mark (and also 7.5 bonus marks for 3(e)); the points available per question are given following the question number. Please WRITE CLEARLY, and make sure you check your answers as best you can. (It would be wise to write in pencil, so that mistakes can be erased, thus avoiding confusion both for you and for us.) When explanations are called for, keep them concise. DO NOT USE COVER PAGES OF YOUR OWN, NOR ANY CARDBOARD OR PLASTIC COVERS: Following the assignment itself is OUR OWN cover sheet; please fill this out and **STAPLE** the pages together with this cover page on top. Please also consult the LATE POLICY outlined on the Course Description.

NOTE. It should be remembered that the Assignments and Mid-Term Exam are not *just* assessment tests, but have a *pedagogical* function, in that they're designed to push you to look into things which you might not already have looked at sufficiently closely. • **NB.**

1. (25) Questions about FOL languages.
   - (10) (a) Suppose we have an FOL with names/constants $a, b, c, d, e,$ function symbols $f(x), g(x), h(x,y)$, predicate symbols: $P(x), Q(x), R(x,y), x = y$, and the three connectives $\neg, \wedge, \vee$. For each of the following, indicate whether they are **simple terms** or **complex terms** or **atomic sentences** or **non-atomic sentences**, and explain why, pointing out *all* the reasons why they fail to be so, if indeed they do. (State also whether there are the right number of brackets. If a bracket is missing, the expression is not properly formed.)
     1. $\wedge R(b)$
     2. e
     3. $Q(Q(a))$
     4. $\neg R(h(a, h(g(c), b)))$
     5. $R(f(b), h(a,e)) \wedge b \neq g(h(b,a))$
     6. $c(d) \vee P(b)$
     7. $c = (c = c)$
     8. $f(g(c)) \neq d$
     9. $f(g(h(g(a), g(b))))$
     10. $Q(b) \wedge \neg((Q(c) \vee R(a,b)) \wedge (P(b) \vee R(b,a)))$
   - (5) (b) Do question 1.17 from p. 36 of the textbook.
   - (5) (c) Do question 1.20 from p. 39 of the textbook.
   - (5) (d) Do question 1.22 from p. 39 of the textbook.

2. (60) Questions from Chapters 2–4 of the textbook. You can use any of the programs **Tarski's World**, **Fitch** and **Boole** in doing these, but please copy out your **Fitch** proofs and your truth-tables by hand (or computer) to submit to us. If you design a counterexample world in **Tarski's World**, you can print a picture (e.g., screen shot) of this world, but then below this picture, explain briefly but carefully why the sentences you're considering are true respectively false in this world. *Remember, you should really study the sections which come before each of these exercises, and perhaps try other, surrounding exercises as practice.*
   - (10) (a) Do question 2.18 from p. 62 of the textbook.
   - (10) (b) Do question 2.24 from p. 66 of the textbook.
   - (10) (c) Do question 3.14 from p. 81 of the textbook.
   - (10) (d) Do question 4.3 from pp. 104 of the textbook.
   - (10) (e) Do question 4.5 from pp. 104 of the textbook.
   - (10) (f) Do question 4.19 from p. 109 of the textbook.

3\. (15) *This sort of question is very useful practice for manipulating truth-values, in particular for looking at the consequences of committments to truth-values* without *using truth-tables.* Read the **NOTE** below before you tackle them; the first two questions are just for practice. Note also that the answers to each part are independent. So, if in (a) you find that Malfoy is a Knight, that doesn't mean he's a Knight in (c).

On a strange island (Voldemort's Island), Voldemort has cast a spell which makes the inhabitants either **Knaves**, who always lie, or **Knights**, who always tell the truth; there he has imprisoned Harry Potter and some of his Hogwarts cronies. Two prisoners on the island are said to be *of the same type* if they are both Knights or both Knaves. Answer the following questions, explaining your answers briefly but as precisely as you can.

(0) (a) Consider three inhabitants (all of them prisoners, so Knights or Knaves), Harry Potter, Draco Malfoy and Pansy Parkinson. Harry says 'All three of us are Knaves', and Draco says 'Exactly one of us three is a Knight'. What are they all?

(0) (b) Suppose a visitor to the island comes across three inhabitants, Ron Weasley, Luna Lovegood and Ginny Weasley, and knows they are either Knights or Knaves. The visitor asks Ron 'How many Knights are among you?', but the answer is indistinct. She then asks Luna 'What did Ron say?', and Luna replies 'Ron said that there is exactly one Knight among us'. Ginny then says 'Don't believe Luna, she's lying!'. What are Luna and Ginny?

(7.5) (c) Suppose you come across Harry Potter, Hermione Granger and Draco Malfoy, who are all, of course, either Knights or Knaves. Harry and Hermione speak, making the following statements:

Harry: Hermione and Malfoy are both Knights.
Hermione: Harry is a Knave and Malfoy is a Knight.

What are Harry, Hermione and Malfoy?

(7.5) (d) Suppose you come across Harry Potter and Hermione Granger. Only Harry speaks, and says:

Harry: Hermione and I are of the same type.

What is Hermione Granger, a Knight or a Knave? Do you know what Harry Potter is?

(7.5) (e) BONUS Suppose a prisoner on the island says 'I am a Knight if and only if the statement P is true'. What can you conclude? (Points only for an explicit explanation.)

NOTE In tackling these questions, there are some things to remember. • **NB.**

1. If A is a not a Knight, then A must be a Knave, and conversely, i.e., A is a Knight if and only if A is not a Knave, and A is a Knave if and only if A is not a Knight. There is a clear parallel between this and our assumption here that a sentence is *true* if and only if it is *not false*, and *false* if and only if it is *not true*.

2. A is a Knight if and only if *everything* A says is true, and correspondingly A is a Knave if and only if *everything* A says is false. Hence, it's important to realise that discovering just *one* truth uttered by A is enough to conclude that A is a Knight, for if A were a Knave, she couldn't even say one true thing. Analogously for Knaves. Thus, if A is a Knight, and A says that B is lying when she says S, then truly B lies, and so S is false (so not-S is true), and B is a Knave. On the other hand, if A is a Knave, and says that B is lying when she says S (in other words, says that S is false), then S must be *true*, and so B is a Knight.

   We might sum this up in the following way: Suppose that A states S. Then we have seen that, whatever S is, A is a Knight if and only if S is true. Notice that S is true if and only if S; so if we now abbreviate the proposition '*A* is a Knight' by *K*, we have *K* if and only if *S*, in other words, that *K* and *S* are logically equivalent. This means that we can quite happily skip back and forth between *K* and *S* (or between $\neg K$ and $\neg S$) in reasoning about truth-values.

3. Moreover, if *A* and *B* contradict each other (as, in effect, Luna and Ginny do in (b)), then it should be clear that *one* of them must be a Knight and the other a Knave. (Think of it in this simple way: if one says *P* and the other $\neg P$, then one of them must speak the truth, and is therefore a Knight, and the other one must speak falsely, and is therefore a Knave. Be clear how this extends to contradictory statements *generally*.)

4. Given 1. and 2., the way to proceed is to pick a character, say *A* (the problem may give you some reason to focus on a certain character from the group, and you may not have any information about some people), *assume* that *A* is a Knight (or a Knave), and then follow the consequences through. If you reach a contradiction, then you know that the original assumption must be wrong, so *A* must be a Knave (or a Knight, if you started from the assumption of Knavehood). Now you can proceed to try to get information about *B* and *C* and so on, possibly now directly, or you might have to adopt another assumption, say that *B* is a Knave. However, if you don't reach a contradiction from your assumption that *A* is a Knight (or whatever you have assumed), then you have not ruled out that *A* is a Knight (if that is what you assumed), so you have to try to see whether the other possibility (*A* is a Knave in this case) leads to a contradiction, which would mean that *A* is bound to be a Knight.

5. Note that what you're doing in proceeding this way is making an *assumption* about the truth-value of a sentence, then analysing and tracking through the consequences of that, just as you do with the games introduced in Chapter 3 of the textbook. If you've made the wrong assumption, then you will eventually come across some incompatibility, just as when you play the game in **Tarski's World**, the computer will try if it can to trip you up, i.e., show that you've committed yourself to something unsustainable. At this point, you know that the opposite assumption must be right, so you turn your attention to another sentence, etc.

As a simple example, consider the following: Suppose *A* says 'Either I am a Knave or *B* is a Knight'; what are *A* and *B*? If *A* is a Knave, then what he says is false, which means that not-(Either *A* is a Knave or *B* is a Knight) is true. This means that *both* '*A* is a Knave' and '*B* is a Knight' are *false*, which means that '*A* is a Knight' and '*B* is a Knave' are both true, which means that *A* is a Knight and *B* is a Knave. But we have assumed that *A* is a Knave, so we have both *A* is a Knave and *A* is a Knight, which is a contradiction.

Hence, the assumption *A* is a Knave leads to a contradiction, so *A* must be a Knight. Now if *A* is a Knight 'Either I am a Knave or *B* is a Knight' said by *A* must be true; but '*A* is a Knave' is false, since we know that *A* is a Knight, so the other disjunct ('*B* is a Knight') *must* be true to make the whole sentence true; hence *B* is a Knight, and the question is fully answered.

PHIL 210A
Introduction to Deductive Logic
First Assignment
Due Date: Wednesday, 3rd October 2012, in class.

NAME (Last, First): ____________________

MCGILL ID: ____________________

TA:

◯ David Chabot

◯ Zoli Filotas

◯ Bruno Guindon

◯ David Weinkauf

Question	*Mark*
1(a)	
1(b)	
1(c)	
1(d)	
2(a)	
2(b)	
2(c)	
2(d)	
2(e)	
2(f)	
3(c)	
3(d)	
3(e)	
Total	
%TOTAL	