

- 2.40** a. By the *mn* rule, the dealer must stock $5(4)(2) = 40$ autos.
 b. To have each of these in every one of the eight colors, he must stock $8 \cdot 40 = 320$ autos.

2.50 Two numbers, 4 and 6, are possible for each of the three digits. So, there are $2(2)(2) = 8$ potential winning three-digit numbers.

2.62 The number of ways to divide the 9 motors into 3 groups of size 3 is $\binom{9!}{3! 3! 3!} = 1680$. If both motors from a particular supplier are assigned to the first line, there are only 7 motors to be assigned: one to line 1 and three to lines 2 and 3. This can be done $\binom{7!}{1! 3! 3!} = 140$ ways. Thus, $140/1680 = 0.0833$.

2.76 Define the events: U : job is unsatisfactory A : plumber A does the job
 a. $P(U|A) = P(A \cap U)/P(A) = P(A|U)P(U)/P(A) = .5 \cdot .1/.4 = 0.125$
 b. From part a. above, $1 - P(U|A) = 0.875$.

2.92 a. The three tests are independent. So, the probability in question is $(.05)^3 = 0.000125$.
 b. $P(\text{at least one mistake}) = 1 - P(\text{no mistakes}) = 1 - (.95)^3 = 0.143$.

2.94 Define the events A : device A detects smoke B : device B detects smoke
 a. $P(A \cup B) = .95 + .90 - .88 = 0.97$.
 b. $P(\text{smoke is undetected}) = 1 - P(A \cup B) = 1 - 0.97 = 0.03$.

2.120 Denote the events G : good refrigerator D : defective refrigerator
 a. If the last defective refrigerator is found on the 4th test, this means the first defective refrigerator was found on the 1st, 2nd, or 3rd test. So, the possibilities are DGGD, GDGD, and GGDD. So, the probability is $\binom{2}{6} \binom{4}{5} \binom{3}{4} \frac{1}{3}$. The probabilities associated with the other two events are identical to the first. So, the desired probability is $3 \binom{2}{6} \binom{4}{5} \binom{3}{4} \frac{1}{3} = \frac{1}{5}$.

b. Here, the second defective refrigerator must be found on the 2nd, 3rd, or 4th test.
 Define: A_1 : second defective found on 2nd test
 A_2 : second defective found on 3rd test
 A_3 : second defective found on 4th test

Clearly, $P(A_1) = \binom{2}{6} \binom{1}{5} = \frac{1}{15}$. Also, $P(A_3) = \frac{1}{5}$ from part a. Note that $A_2 = \{DGD, GDD\}$. Thus, $P(A_2) = 2 \binom{2}{6} \binom{4}{5} \binom{1}{4} = \frac{2}{15}$. So, $P(A_1) + P(A_2) + P(A_3) = 2/5$.

c. Define: B_1 : second defective found on 3rd test
 B_2 : second defective found on 4th test

Clearly, $P(B_1) = 1/4$ and $P(B_2) = (3/4)(1/3) = 1/4$. So, $P(B_1) + P(B_2) = 1/2$.

- 2.125** Define the events for the person: D : has the disease H : test indicates the disease
Thus, $P(H|D) = .9$, $P(\bar{H} | \bar{D}) = .9$, $P(D) = .01$, and $P(\bar{D}) = .99$. Thus,

$$P(D|H) = \frac{P(H|D)P(D)}{P(H|D)P(D) + P(H|\bar{D})P(\bar{D})} = 1/12.$$

- 2.134** Define F as "failure to learn. Then, $P(F|A) = .2$, $P(F|B) = .1$, $P(A) = .7$, $P(B) = .3$. By Bayes' rule, $P(A|F) = 14/17$.

- 2.136** Let A = woman's name is selected from list 1, B = woman's name is selected from list 2.
Thus, $P(A) = 5/7$, $P(\bar{B} | A) = 2/3$, $P(\bar{B} | \bar{A}) = 7/9$.

$$P(A|\bar{B}) = \frac{P(\bar{B} | A)P(A)}{P(\bar{B} | A)P(A) + P(\bar{B} | \bar{A})P(\bar{A})} = \frac{\frac{2}{3}(\frac{5}{7})}{\frac{2}{3}(\frac{5}{7}) + \frac{7}{9}(\frac{2}{7})} = \frac{30}{44}.$$

- 3.36** a. The random variable Y does not have a binomial distribution. The days are not independent.
b. This is not a binomial experiment. The number of trials is not fixed.
- 3.72** Let Y = # of tosses until the first head. $P(Y \geq 12 | Y > 10) = P(Y > 11 | Y > 10) = 1/2$.

- 3.96** a. Let Y = # of attempts until you complete your call. Thus, Y is geometric with $p = .4$.
Thus, $P(Y = 1) = .4$, $P(Y = 2) = (.6).4 = .24$, $P(Y = 3) = (.6)^2.4 = .144$.
b. Let Y = # of attempts until both calls are completed. Thus, Y is negative binomial with $r = 2$ and $p = .4$. Thus, $P(Y = 4) = 3(.4)^2(.6)^2 = .1728$.
- 3.112** Let Y = # of malfunctioning copiers selected. Then, Y is hypergeometric with probability function

$$p(y) = \frac{\binom{3}{y} \binom{5}{4-y}}{\binom{8}{4}}, y = 0, 1, 2, 3.$$

- a. $P(Y = 0) = p(0) = 1/14$.
b. $P(Y \geq 1) = 1 - P(Y = 0) = 13/14$.
- 3.127** Let Y = # of typing errors per page. Then, Y is Poisson with $\lambda = 4$ and $P(Y \leq 4) = .6288$.
- 3.130** Define: Y_1 = # of cars through entrance I, Y_2 = # of cars through entrance II. Thus, Y_1 is Poisson with $\lambda = 3$ and Y_2 is Poisson with $\lambda = 4$.

$$\text{Then, } P(\text{three cars arrive}) = P(Y_1 = 0, Y_2 = 3) + P(Y_1 = 1, Y_2 = 2) + P(Y_1 = 2, Y_2 = 1) + P(Y_1 = 3, Y_2 = 0).$$

$$\text{By independence, } P(\text{three cars arrive}) = P(Y_1 = 0)P(Y_2 = 3) + P(Y_1 = 1)P(Y_2 = 2) + P(Y_1 = 2)P(Y_2 = 1) + P(Y_1 = 3)P(Y_2 = 0).$$

Using Poisson probabilities, this is equal to 0.0521

- 3.136** Let Y = # of *E. coli* cases observed this year. Then, Y has an approximate Poisson distribution with $\lambda \approx 2.4$.
a. $P(Y \geq 5) = 1 - P(Y \leq 4) = 1 - .904 = .096$.
b. $P(Y > 5) = 1 - P(Y \leq 5) = 1 - .964 = .036$. Since there is a small probability associated with this event, the rate probably has changed.

3.145 Using the binomial theorem, $m(t) = E(e^{tY}) = \sum_{y=0}^n \binom{n}{y} (pe^t)^y q^{n-y} = (pe^t + q)^n$.

3.146 $\frac{d}{dt} m(t) = n(pe^t + q)^{n-1} pe^t$. At $t = 0$, this is $np = E(Y)$.

$\frac{d^2}{dt^2} m(t) = n(n-1)(pe^t + q)^{n-2} (pe^t)^2 + n(pe^t + q)^{n-1} pe^t$. At $t = 0$, this is $np^2(n-1) + np$.
Thus, $V(Y) = np^2(n-1) + np - (np)^2 = np(1-p)$.

3.156 a. $m(0) = E(e^{0Y}) = E(1) = 1$.

b. $m_W(t) = E(e^{tW}) = E(e^{t3Y}) = E(e^{(3t)Y}) = m(3t)$.

c. $m_X(t) = E(e^{tX}) = E(e^{t(Y-2)}) = E(e^{-2t} e^{tY}) = e^{-2t} m(t)$.

3.178 Using the binomial, $E(Y) = 1000(.1) = 100$ and $V(Y) = 1000(.1)(.9) = 90$. Using the result that at least 75% of the values will fall within two standard deviation of the mean, the interval can be constructed as $100 \pm 2\sqrt{90}$, or (81, 119).

3.202 Let $W = \#$ of drivers who wish to park and $W' = \#$ of cars, which is Poisson with mean λ .

a. Observe that

$$\begin{aligned} P(W = k) &= \sum_{n=k}^{\infty} P(W = k | W' = n) P(W' = n) = \sum_{n=k}^{\infty} \frac{n!}{k!(n-k)!} p^k q^{n-k} e^{-\lambda} \frac{\lambda^n}{n!} \\ &= \lambda^k e^{-\lambda} \left(\frac{p^k}{k!} \right) \sum_{n=k}^{\infty} \frac{q^{n-k}}{(n-k)!} \lambda^{n-k} = \frac{(\lambda p)^k}{k!} \sum_{j=0}^{\infty} \frac{q^j}{j!} \lambda^j = \frac{(\lambda p)^k}{k!} e^{-\lambda} e^{q\lambda} \\ &= \frac{(\lambda p)^k}{k!} e^{-\lambda p}, k = 0, 1, 2, \dots \end{aligned}$$

Thus, $P(W = 0) = e^{-\lambda p}$.

b. This is a Poisson distribution with mean λp .

3.214 Define: $A =$ accident next year $B =$ accident this year $C =$ safe driver

Thus, $P(C) = .7$, $P(A|C) = .1 = P(B|C)$, and $P(A|\bar{C}) = P(B|\bar{C}) = .5$. From Bayes' rule,

$$P(C|B) = \frac{P(B|C)P(C)}{P(B|C)P(C) + P(B|\bar{C})P(\bar{C})} = \frac{.1(.7)}{.1(.7) + .5(.3)} = 7/22.$$

Now, we need $P(A|B)$. Note that since $C \cup \bar{C} = S$, this conditional probability is equal to

$$\begin{aligned} P(A \cap (C \cup \bar{C}) | B) &= P(A \cap C | B) + P(A \cap \bar{C} | B) = P(A \cap C | B) + P(A \cap \bar{C} | B), \text{ or} \\ P(A|B) &= P(C|B)P(A|C \cap B) + P(\bar{C}|B)P(A|\bar{C} \cap B) = 7/22(.1) + 15/22(.5) = .3727. \end{aligned}$$

So, the premium should be $400(.3727) = \$149.09$.