Final exam I, November 20, 2012

Math 360 Term 1, Winter 2012

Notes:

- This is an "open book" exam. You may consult course notes and lab projects to answer the questions. You may also consult the web (however, it is unlikely that this will be of immediate help).

- You may hand in hard copies of your answers or email digital answers to Mayang Mata at mayang@math.ubc.ca, or you may use a combination of both. Both hard copy and digital answers are due at 10:50 am on Tuesday, Nov 20.

1. Consider a game that is played between four different strategies A, B, C, and D, and assume that the payoff matrix for this game is the following:

	A	B	C	D
A	2	4	5	-3
B	1	-2	2	19
C	-3	4	6	7
D	0	0	1	1

   Recall that the payoff matrix gives the payoff to the strategy indicated by the row when playing against the strategy indicated by the column. Which ones are good strategies in this game (i.e., which strategies are ESS's)? Explain.

2. Consider the 2x2-game between C and D given by the following payoff matrix:

	C	D
C	-4	5
D	1	1

Let $p(t)$ be the frequency of C-players at time t. What is the fitness of the two strategies C and D as a function of $p(t)$?

Formulate the replicator equation, i.e., the differential equation for dp/dt that corresponds to the given payoff matrix.

3. For the differential equation found in 2., find all equilibrium points and determine their stability. Explain.

4. Consider the following system of coupled differential equations:

$$\frac{dx}{dt} = x^2y - x + b \tag{1}$$

$$\frac{dy}{dt} = a - x^2y, \tag{2}$$

where $x(t)$ and $y(t)$ are concentrations of chemical substances, and a and b are parameters. Find the equilibrium points of the dynamical system described by these two differential equations. Explain. (If you use Matlab to answer this question, please provide the Matlab commands used.)

5. In lab project 7, you used Matlab files to solve systems of coupled differential equations numerically. Modify these files to solve the system of differential equations from problem 4 for the following two sets of parameters:

i) $a = 1$ and $b = 0.1$

ii) $a = 0.5$ and $b = 0.1$

Explain what you see in both cases. What can you say about the stability of the equilibrium found in 4. in the two cases? Explain your reasoning.

6. Consider the following chemical reactions between four molecules of type A, B, X and Y:

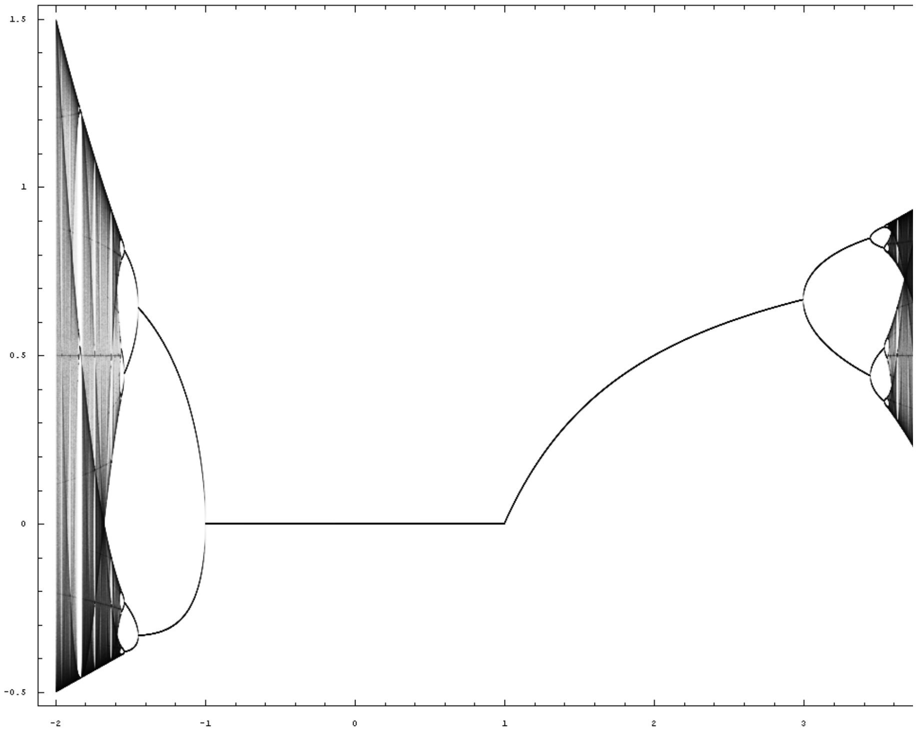
$$\begin{aligned} A + X &\xrightarrow{k_1} 2X \\ X + Y + Y &\xrightarrow{k_2} 3Y \\ Y &\xrightarrow{k_3} B \end{aligned}$$

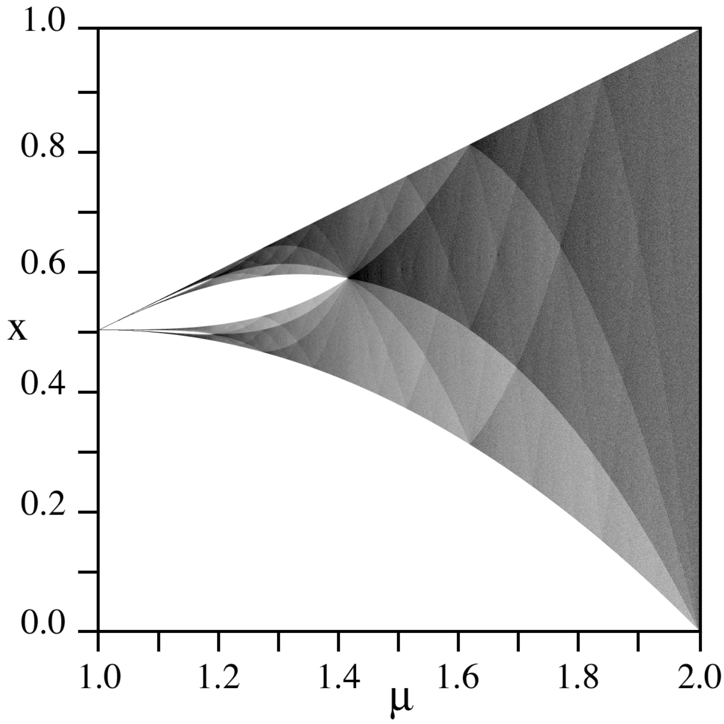
Let $x(t) = [X](t)$ be the concentration of X-molecules at time t, and $y(t) = [Y](t)$ be the concentration of Y-molecules at time t, and assume that the concentrations of A and B are constant.

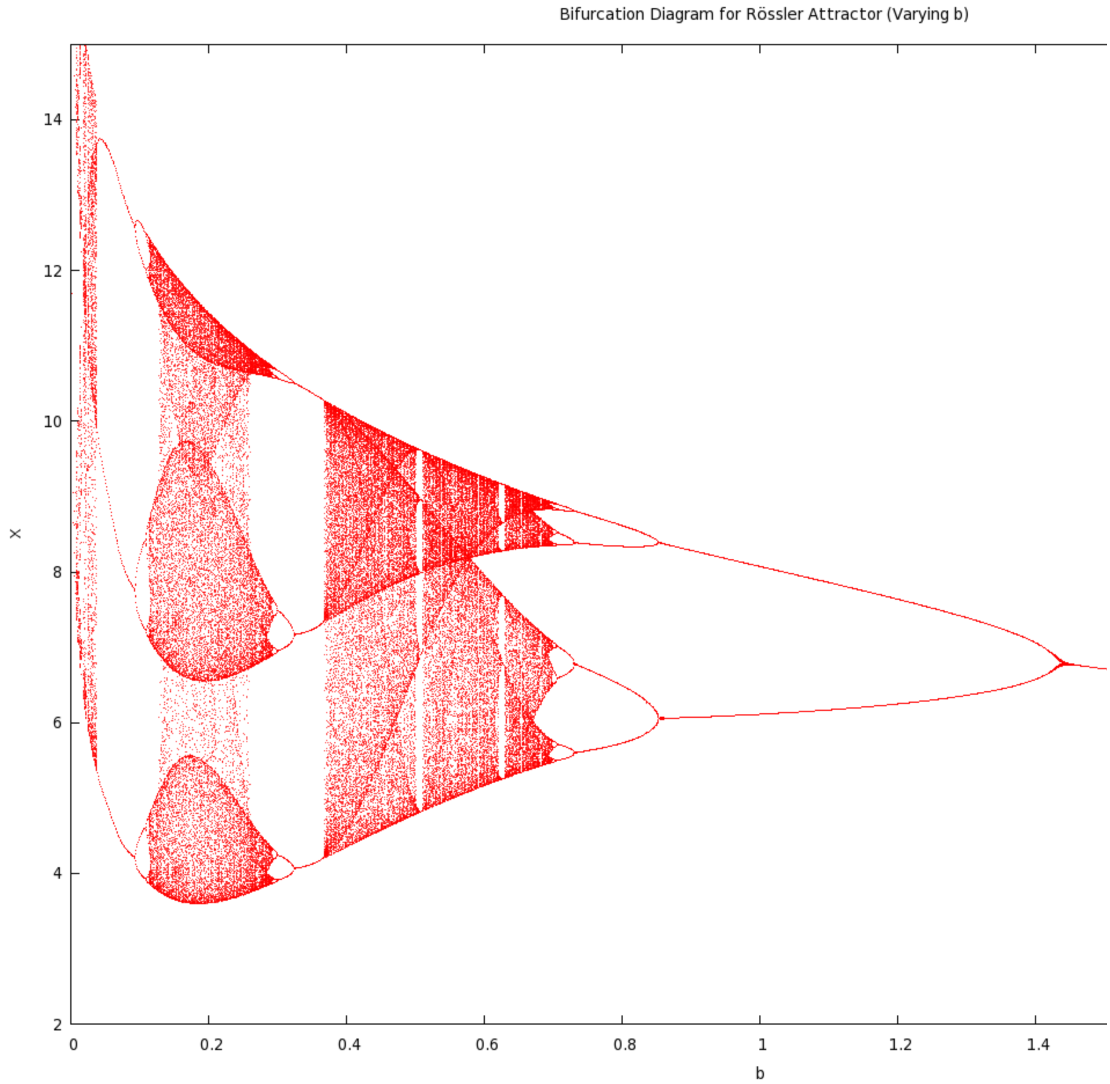
a) Write a set of differential equations for dx/dt and dy/dt using the reactions above.

b) Find all equilibrium points for this dynamical system.

7. Shown in the figures below are three bifurcation diagrams for dynamical systems in discrete time. Where do you see a period-doubling route to chaos? Explain.






Bifurcation Diagram for Rössler Attractor (Varying b)
x
b
2
4
6
8
10
12
14
0
0.2
0.4
0.6
0.8
1
1.2
1.4